SECOND EDITION

MATHEMATICS METHODS
for the Elementary and Middle School

Hatfield · Edwards · Bitter

Mathematics Methods for the Elementary and Middle School

Second Edition

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Allyn and Bacon

Boston, London, Sydney, Toronto, Tokyo, Singapore

This book is dedicated to the preservice and in-service teachers who use it, with our hope that their teaching of mathematics will be successful.

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Preface

Mathematics Methods for the Elementary and Middle School, Second Edition, provides preservice and in-service kindergarten through grade-eight teachers with ideas, techniques, and approaches to teach mathematics. The book emphasizes manipulatives, problem solving, mathematical connections, estimation, mental math, assessment, cultural diversity, calculators, and computers as an integral part of teaching mathematics. These topics are not addressed as separate issues. As is appropriate, they are treated as part of the developmental learning process.

Although the book can be used without the computer-related infusion ideas, the authors do not encourage this practice. In using this text as we developed it, we noted numerous benefits to students from hands-on computer experience. If the book is used with related field experiences, the actual computer use is extremely important. A data disk is provided for the BASIC and Logo programs and database and spreadsheet templates for readers to use without manually entering each of the programs. In addition, commercial software programs are suggested.

The book models the approach of concrete-to-abstract developmental learning. Manipulatives are emphasized throughout the book. Base 10 blocks, geoboards, attribute blocks, Cuisenaire rods, and the balance are a few of the concrete materials emphasized. In addition, paper models of many of the manipulatives are provided in Appendix C for readers to remove for their personal use.

Assessment techniques are included in each chapter where appropriate. Actual children's work is provided for preservice and in-service teachers to review and evaluate. Included in each discussion are different types of learners and methods of correcting common student errors. Teachers who work with special needs students will find these materials helpful, with direct application for classroom use. The Instructor's Manual provides additional worksheets for further evaluation of common errors.

Exercises for preservice and in-service teachers are provided at three levels in each chapter. The levels are related to the cognitive domain of Bloom's taxonomy:

A. **Memorization and Comprehension Exercises**
low-level thought activities

B. **Application and Analysis Exercises**
middle-level thought activities

C. **Synthesis and Evaluation Exercises**
high-level thought activities

Many exercises are provided at each level. The instructor can select those most appropriate on the basis of availability of field experiences, computers, software, calculators, manipulatives, and similar factors. This option makes it possible to match student work to unique situations.

The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) are provided in each chapter to help the user of this book relate topics in the elementary mathematics curriculum to these standards. They are integrated into the development of the mathematics concepts as appropriate. Activities are provided throughout the book for the following categories:

Problem Solving—emphasizes strategies and higher-order thinking

Manipulatives—uses concrete experiences for developing concepts

Calculators—emphasizes problem solving and deemphasizes tedious computations

Mental Math—builds methods of doing computation mentally

Computers, *Logo and BASIC programming*—uses exploration to learn mathematical concepts

Cultural Relevance—emphasizes activities to illustrate historical cultural contributions to mathematics

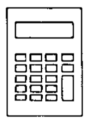
Computers, *spreadsheets, databases, and graphing*—uses exploration in problem solving

Estimation—emphasizes techniques to determine reasonable answers

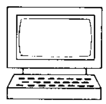
Mathematical Connections—encourages interdisciplinary approaches to learning mathematics

The following pictures will be used to alert the reader to the emphasis of each category when it appears throughout the text.

CALCULATORS

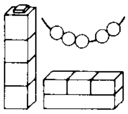


COMPUTERS



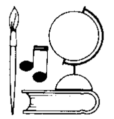
ESTIMATION

MANIPULATIVES



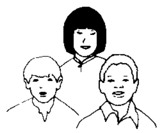
PROBLEM SOLVING

MATHEMATICAL CONNECTIONS



MENTAL MATH

CULTURAL RELEVANCE



This book is a resource book for the teaching of mathematics in the elementary and middle school. Problem solving, cooperative learning, and manipulatives are emphasized at all levels. Different learning styles are emphasized, and concrete-to-abstract materials are a prerequisite for each development.

• The book has thirteen chapters. Chapter 1 includes the past, present, and future of elementary mathematics. The philosophy is explored for emphasizing problem solving, manipulatives, the computer, the calculator, estimation, and mental math. Introductory computer experiences are provided in Logo and BASIC to ensure an anxiety-free introduction to the computer.

• Chapter 2 discusses how children learn mathematics and solve problems. Learning theories are briefly reviewed in relation to effective teaching models and lesson plan development. Problem-solving strategies are discussed and illustrated. Characteristics of good problem solvers are outlined and related to lesson planning. The chapter emphasizes the importance of good lesson planning in effective teaching.

• Chapter 3 discusses culturally relevant mathematics with the acknowledgment that all cultures of our world have contributed to the discoveries of mathematics. The book attempts to bring students' cultural heritage to various teaching situations. Culture-relevant assessment materials are included throughout the book.

• Chapter 4 discusses geometry. Its placement in the beginning of the book is to encourage teachers to use geometric ideas in their teaching. Concrete experiences with objects are the key to developing informal, plane, and solid geometry. Hands-on exploration is the theme of the chapter and is emphasized throughout the remainder of the book.

• Chapter 5, on measurement, follows geometry and emphasizes the metric and customary systems. The topics of linear, area, volume, capacity, temperature, mass, and time are carefully developed, including their related units of measurement. Activities are provided for each topic. Many of the activities encourage exploration to develop frames of reference for various units of measurement.

• Chapter 6 discusses number readiness and the beginning of the numerical aspects of mathematics. Number readiness is the initial introduction of numbers, including one-to-one correspondence, counting, and the general concept of number.

• Chapter 7 discusses the whole number system in relation to the basic facts of addition, subtraction, multiplication, and division. Concrete materials and mental arithmetic are emphasized in the development process.

• Chapter 8 covers numeration and the development of counting systems. The base 10 numeration system is discussed in the context of the more general idea of numeration as it applies to any base system. Base 10 blocks and chip trading activities are developed as examples of proportional and nonproportional materials to use with children.

• Chapter 9 discusses the algorithms for addition, subtraction, multiplication, and division using base 10 blocks followed by chip trading activities. The calculator and computer are an integral part of the development.

• Chapter 10 covers decimals and rational numbers, specifically common fractions, using concrete materials. The operations are explored with several models. The approach emphasizes understanding the algorithm rather than memorized procedures.

• Chapter 11 discusses ratio, proportion, percent, and rate. Pattern blocks are used to develop an understanding of percent. The calculator, estimation, and mental math are stressed in relation to ratio, proportion, percent, and rate.

• Chapter 12 develops patterns, functions, algebra, and number theory with various divisibility rules and numerical explorations. The middle school curriculum will find the topics extremely useful in developing number "sense."

• Chapter 13 explores graphing, statistics, and probability. Types of graphs and their interpretation are discussed and illustrated with examples. Computer activities are suggested for the graphing presentations. Statistics and probability theory are highlighted. Real-world examples are provided, and activities to motivate students are presented. Mathematical connections are emphasized throughout the chapter.

Throughout the book, preservice and in-service teachers will be referred to as *the teacher*. The book has been class tested in college courses, and teachers have successfully used the activities with children. Using the computer as a tool offers many learning experiences for teachers as well as elementary and middle school children. The authors hope this careful, realistic presentation makes mathematics meaningful to elementary and middle school teachers and their students.

We wish to thank the following for their invaluable comments on the text: Jerry P. Becker, Southern Illinois University; Susan Brown, University of Houston; Linda Ghion, Berkeley County Public Schools; Hiram Johnston, Georgia State University; Lloyd Joyal, University of Wisconsin–Eau Claire; Theodor Korithoski, University of Wisconsin–Green Bay; and Gerry Rossi, Salisbury State University. We wish to acknowledge our past students whose actual work appears as illustrations in the book. We thank Dorothy S. Strong, Gwendolyn Long, and Telkia Rutherford of the Chicago Public Schools Mathematics Bureau and the Board of Education of the Chicago Public Schools for their willingness to share their instructional form and the student results. We also thank Diana Moore Winston for her contribution to Chapter 2. Finally, we thank Nancy Forsyth, Christine Nelson, and especially Violet J. Tanner for her careful reading of the initial manuscript.

Our special gratitude goes to the professors and students of Arizona State University and Missouri Western State College who participated in the two-year pilot study and gave valuable suggestions for improvement.

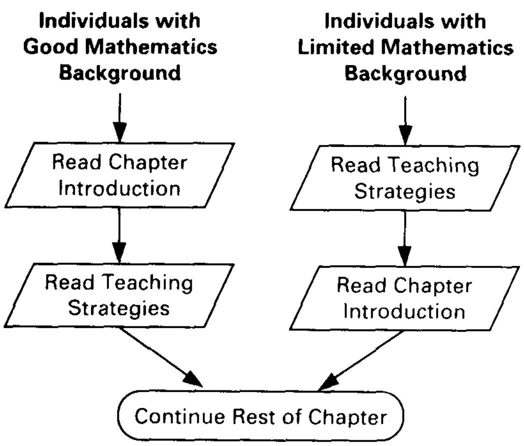
How to Use This Book

This book is to be used with classes that emphasize the teaching of elementary and middle school mathematics. Field experiences in relation to learning from the book are encouraged. Generally, the book is organized into seven sections per chapter—introduction, teaching strategies, assessment, summary, integrating technology, exercises, and bibliography.

The *introductions* present a brief review of the concepts that have been covered in prerequisite college content courses. This is especially true in Chapters 6 through 13, where teachers need to see the mathematical understandings to which children will be exposed from an "adult" viewpoint. The mathematical content is presented in a more symbolic, formal structure. Definitions of mathematical terms have been checked with James and James (1976), *Mathematics Dictionary*, 4th edition, and with West, Griesbch, Taylor, and Taylor (1982), *The Prentice-Hall Encyclopedia of Mathematics*, as appropriate.

The *teaching strategies* present developmental activities that can be used with elementary and middle school students. These emphasize a variety of techniques in concrete/pictorial experiences, problem solving, cooperative learning, and computer/calculator technologies.

Because some readers may have a major in mathematics while others may have a more limited background, the authors suggest that the book be used in the following manner:



The *assessment* sections present techniques for evaluating what the student actually knows and for correcting common error patterns and other frequently occurring difficulties of elementary and middle school students. Sample student work is included and analyzed for error patterns, a practice which we encourage in all teachers.

The *exercises* are written on three levels of difficulty. There are many exercises so that course instructors can choose which assignments are most appropriate for their course objectives. It is not the intent of the authors that everyone complete every exercise. The exercises are to encourage understanding of the content covered in the text as well as to prepare a preservice teacher for classroom instruction and to strengthen the in-service teacher's mathematics presentations. Many of the exercises can be used in future teaching, so teachers are encouraged to save them.

The *bibliographies* present quick references for teachers interested in further information on a topic. Readers who are asked to search professional journals for assignments will find the bibliography a good starting point.

The computer programs are to be integrated into the curriculum as tools to understand and teach mathematics. The BASIC and Logo programs are available on disk to be copied by each user, thus avoiding the tedious task of typing in the program. Readers need to be encouraged to explore with different responses and record results, to look for a pattern or discover a formula. Please note that a Logo disk is needed to run Logo and a spreadsheet and database program is needed to do the spreadsheets and databases. The assumption of the computer activities is that they will have direct transfer to the children in the classroom. Readers are not expected to do their own programming; however, individuals who have had more extensive work with computers may try extending computer activities or writing a new program to go along with a topic. The computer and calculator activities can be done as explorations in an individual setting or in cooperative learning (small, noncompetitive) groups where work is shared among students to draw conclusions.

An index is included for quick reference to key definitions. When in doubt, look up the word in the index. Paper models and patterns for several of the manipulatives are included in Appendix C and may be copied for individual practice or whole classroom use with children. Observation forms are provided for field experiences. These materials are referenced as needed in the text.

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1

The Past, Present, and Future of Mathematics Education

Key Question: *What insights must teachers have to survive the present and prepare for the future in mathematics education?*

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Introduction

The content and instructional methods by which you were taught mathematics may not be appropriate for children living most of their lives in the twenty-first century. As mathematics educators, you must help prepare children for an ever-changing world by being aware of the expanding field of knowledge in mathematics. This text has three main goals: to build on the strengths of past mathematics education, to model the successful techniques of the present, and to prepare for changes in the future. Mathematics education has always been and will continue to be an expanding field of knowledge. As human beings grow in understanding and in awareness of new possibilities, mathematics education changes. It is not the stagnant body of knowledge that many people assume. Some basic mathematical principles, such as the commutative and associative properties, have remained the same for generations, but others, such as fact strategies, are emerging as new insights. The ways to teach both the old and the new principles will change as more effective methods and new technologies develop. For example, the teacher of the early 1900s would be astonished to see today's students learning mathematics with a computer, creating their own mathematical equations, and checking to see if their answers are correct—all within a few seconds.

This chapter presents an overview of the pertinent course topics and the philosophy of the text. The ideas briefly presented here will be studied in greater detail in subsequent chapters.

Past

Reviewing past influences on mathematics by Thorndike, Pestalozzi, Grube, Dewey, Brownell, Spitzer, Polya, and others is beyond the scope of this book. Instead, the book begins with the change in mathematics brought on by the launching of Sputnik by the Soviet Union in

1957. Our country scrambled to keep up with the Russians by accelerating the pace of curriculum change in science and mathematics. What followed was the *New Math* revolution of the early 1960s that was promoted as a means of helping children understand the *why* behind computational processes. Emphasis was on the underlying structure of mathematics with new content being added on such topics as set theory, number systems, and number properties. In addition to the New Math, a strategy known as *discovery learning* was introduced as the necessary balance to understanding computations, as it was designed to provide the how-to facet of learning mathematics.

Attention soon focused on the vocabulary and expressions used in teaching mathematics, as they were often ambiguous. Changes within the educational terminology, delineating more clearly the differences between curriculum, instruction, and teaching, were begun with MacDonald and expanded under Piaget. The developmental level of the child became a factor in the content of the curriculum.

During the past two decades, Piaget's theory dealing with the intellectual development of a child prevailed over most educational research. Many inferences were drawn from Piaget's theory regarding teaching and instruction: the classroom structure, organization, and management; the nature and sequence of curricular content; and the learning of mathematics. Much of the revision work done in mathematics textbooks in the 1970s reflected Piaget's effect upon the scope of children's education (Driscoll, 1982).

Present

In today's educational system, the Piagetian theory is being challenged by an information-processing theory, which suggests that understanding and comprehension are the mainsprings upon which cognitive science is based. Research in the learning of mathematics is related more to the intake of information, the working memory, and the interaction between working and stored information in the long-term memory. An information-processing system is often described as relating to computer or microcomputer systems.

Research (Capper, 1984) suggests that young children invent or construct much of their own mathematical knowledge. Although their thinking is immature, they enter school with some well-developed ways of using mathematics. Some educators feel that using a child's own conceptions of mathematics in programs of teaching and instruction will reduce a child's tendency to develop anxiety toward mathematics later. Very often it is the view that educators hold about the importance of mathematics in the classroom that affects the anxiety which students develop toward it. Learning associated with problem solving is strengthened, according to research, by the student's heightened awareness of metacognitive elements of the problem-solving process. *Metacognition* is one's knowledge of how one's own cognitive processes work. Evidence implies that metacognitive skills may be the reason for the difference between expert and novice problem solvers (Driscoll, 1982).

Teaching and learning mathematics is a lifelong process. Technological change outdates old skills and increases the importance of others. Slide rules, log and trig tables, and mechanical calculators have given way to programmable and graphing calculators and sophisticated electronic computers.

Technology is also changing the teaching profession, moving the teacher from chalk and blackboard to manipulatives, interactive multimedia, and computer-assisted instruction. Research into education and psychology provides new insights into the ways in which children learn mathematics. Mathematics should be a complete curriculum with all students using the full spectrum of instructional aids—computers, videotapes and videodiscs, multimedia, and calculators.

Future

Despite the increasing importance of mathematics in a world that is both highly technological and highly competitive, the United States lags behind other developed nations in student mathematics achievement (Dossey et al., 1988; National Research Council, 1989). In an international study, McKnight and colleagues (1987) found that average Japanese high school students performed better in mathematics than all but the top 5 percent of American high school students. The *Wall Street Journal* reported that a Japanese semiconductor company opening a plant in the U.S. had to hire American graduate students to perform statistical procedures that were performed by high school students in Japan (Gilder, 1987). Frank Press, president of the National Academy of Sciences and chairman of the National Research Council, has stated that it is "crucial . . . for science, technology, and the economy of the nation that *all* students receive high-quality education in mathematics" (National Research Council, 1989, page 1).

In response to the data which indicate that U.S. students rank low on global tests, President George Bush (1991) recommended a set of national goals to be achieved by the year 2000. These goals are listed in Box 1.1.

The national goals and continuing headlines like "U.S. pupils trailing in math and science" have led educators and government leaders to call for radical reform of the mathematics curriculum and the methods of teaching mathematics. For example, the Mathematical Sciences Education Board and National Research Council (1990) have concluded, "What is required is a complete redesign of the content of school mathematics and how it is taught" (p. 7). In their study of U.S. mathematics education, Dossey and colleagues (1988) found that mathematics instruction continues to be dominated by teacher explanations, chalkboard presentations, and reliance on textbooks and workbooks.

Teacher education has been put forth as one of the causes of public school ills. Among the demands for reform in teacher education are calls for increased field experience and clinical work. Critics and advocates of teacher education seem to agree that there is an increased need for preservice teachers to observe master teachers and to practice recently learned techniques.

Methods of teaching mathematics have been the focus of many research studies in mathematics education. Overall, the research has been inconclusive, suggesting that there is no one best pedagogical method or medium. Prospective teachers should have access to and become familiar with a variety of instructional tools and techniques as different students respond differently to different methods (Copeland, 1970; Flavell, 1963; Harrison & Harrison, 1986; Heddens, 1986; Inhelder & Chipman, 1976; Kamii, 1985).

The use of concrete objects to teach mathematics has been urged in the educational literature for more than 100 years. Mathematics educators find that many children seem to learn better and more easily when they are given actual objects or structural models to manipulate (Beattie, 1986; Bruni & Silverman, 1986; Canny, 1983; Moser, 1986; Parham, 1983; Schultz, 1986; Suydam, 1986; Thornton & Wilmot, 1986; Trueblood, 1986; Williams, 1986). In more modern terminology, the use of concrete materials helps learners construct schemes for new mathematical concepts and skills. Concrete materials for mathematics instruction were widely advocated in the 1960s; today, after a period of relative neglect, their importance is again being recognized.

Box 1.1 The National Education Goals

By the year 2000:

1. All children in America will start school ready to learn.
2. The high school graduation rate will increase to at least 90 percent.
3. American students will leave grades four, eight, and twelve having demonstrated competency in challenging subject matter including English, mathematics, science, history, and geography; and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning, and productive employment in our modern economy.
4. U.S. students will be first in the world in science and mathematics achievement.
5. Every adult American will be literate and will possess the knowledge and skills necessary to compete in a global economy and exercise the rights and responsibilities of citizenship.
6. Every school in America will be free of drugs and violence and will offer a disciplined environment conducive to learning.

Source: Bush, *America 2000—Source Book*

The technological revolution is causing educators to reconsider their previous goals in determining future needs. Societies themselves are in a state of flux. New demands are being made on the populace. There are new expectations and new employment patterns. Mathematics is affected by both the new technology and the subsequent rapid sociological change it produces. Pedagogically, the future looks promising. It is becoming apparent that education must be a lifelong pursuit. No longer will education be completed when a student graduates from high school or college.

Although teachers will continue to play a central role in education, their role will change. Technology will affect the way teachers present their material in the classroom. How great an impact technology has on learning will depend on whether teachers prepare themselves to use technology to help students learn.

To assure that all students have the best education possible, educators must anticipate and guide the educational change. Educators must learn how to use technology effectively to achieve the highest learning potential for all students. They must set educational and technological goals that will influence designers of educational software and developers of hardware. Teachers should prepare for future developments as they go into preservice and in-service training. As students gain more experience on their computers, their parents will exert more pressure for technology in the classroom. Students themselves will begin to demand more and better instruction as their world calls for greater knowledge and understanding.

As the twenty-first century approaches, educators must consider how to proceed with the teaching and learning of mathematics. Should mathematics instruction proceed with categories of arithmetic, algebra, and geometry, or should it change to concentrate on dichotomies such as finite-infinite, continuous-discrete, and exact-approximate? There is no doubt that the structure of the mathematics curriculum should be renovated. The renovation should include curricula to develop proof and deductive reasoning, which are highly characteristic of logical thinkers. Mathematics curricula should be designed to build an accepted body of mathematical knowledge that can serve as a basis for further learning. However, the future aim of teaching mathematics is to increase the body of mathematics knowledge so students will be able to apply it. The ability to apply mathematics will be vital in the 1990s and into the next century (National Council of Teachers of Mathematics [NCTM], 1984).

Curricular reform in school mathematics K–12 is the intent of three major national projects. The first is NCTM's document *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989), which will have far-reaching effects on shaping the mathematics curriculum. The standards are a set of benchmarks by which schools might measure their mathematics curriculum. The work focuses on major strands of the K–12 curriculum and their sequential development. These standards will be integrated throughout this book. Grade levels have been grouped as K–4, 5–8, and 9–12. The standards address specific needs at these grade levels. The recommendations are directed at the ways schools can change the manner in which mathematics instruction occurs as well as the emphasis different contents receive.

The NCTM *Standards* create a vision of what it means to do mathematics in terms of what students need to do in learning mathematics and what teachers should do in teaching mathematics. The *Standards* call for instruction to be based on problem situations whereby students gain mathematical power by investigating, exploring, making conjectures, verifying answers, and communicating results. This constructivist view of learning means a redefined role for the teacher, from a dispenser of knowledge to a facilitator of learning. An assumption about instruction is that all students should experience the full range of topics addressed in the *Standards.* The focus is on what it means to be mathematically literate in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly changing and extensively being applied in many fields. The goals for all students across grades K–12 are to become a mathematical problem solver, to learn to communicate mathematically, to learn to reason mathematically, to learn to value mathematics, and to become confident in one's own ability.

Another position was taken by the National Council of Supervisors of Mathematics. This group recommended "Twelve Components of Essential Mathematics" (1989). Their position is given in Box 1.2.

The third major effort is the Project 2061 (Blackwell & Henkin, 1989) mathematics work of the American Association for the Advancement of Science. This project looked at what eighteen-year-olds might need to know for