

# A TEXTBOOK OF NUCLEAR PHYSICS

BY

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## PREFACE

THE main purpose of this book is to provide a general coverage of nuclear physics suitable for honours physics students. An attempt has been made to keep the subject matter as self-contained as possible and, for this reason, the first seven chapters deal with those parts of atomic physics, relativity, and quantum mechanics, which are a necessary prelude to the study of nuclear physics. It is, of course, impossible to give, for example, a comprehensive account of quantum mechanics in a single chapter, and it is hoped that the reader will already have some background to this most important aspect of modern physics, or be prepared to follow up the references given at the end of Chapter 6.

The second part of the book, Chapters 8 to 20 inclusive, is concerned with what may be described as "low energy nuclear processes and nuclear structure" the main field of study of nuclear physics in the post 1945 years. The penultimate chapter, on the other hand, deals with the rapidly developing subject of high energy nuclear physics and the light it throws on our knowledge of those peculiar objects we call "elementary particles". There is a growing inclination, an unfortunate one in the author's opinion, for the physics of finite nuclear matter to be divorced from the physics of elementary particles.† In an attempt to offset this tendency, a fairly long and detailed account of elementary particle physics is given in Chapter 21.

In the final chapter, an account is given of our present day knowledge of thermonuclear reactions in stars and also of one of the more successful theories of the origin of the elements. This chapter also illustrates the importance of not keeping the various branches of physics in rigidly separated compartments.

Throughout the book it has not always been possible to avoid the irritating phrase "it can be shown that"; but when this is inevitable, references to original sources have been given, and there is an adequate and up-to-date bibliography.

A few specific comments on some of the chapters may be helpful. Chapters 11 and 12 are devoted to the art of experimental techniques in nuclear physics: the range of detectors of nuclear radiation is continuously extending (the bubble chamber, the semiconduction counter, the spark chamber . . .); the chapter on "Electronic Techniques" does not attempt to give any detailed account of modern multi-channel pulse height analysers, nor does

† The importance of bearing in mind the properties of finite nuclei in studying elementary particles was the theme of Wilkinson's Rutherford Memorial Lecture (1962).

it include any discussion of the transistor equivalents of the thermionic vacuum tube circuits.

There is no attempt to cover any reactor physics; this is best regarded as a new brand of engineering physics, nor are current researches into laboratory controlled thermonuclear reactions mentioned.

Special emphasis should be made of parts of the subject matter of Chapter 16, "Nuclear Magnetism". Although the study of nuclear magnetic resonance in bulk matter is now mainly a sensitive tool for elucidating certain aspects of the solid and liquid state, the writer believes that it is worthwhile discussing such topics as spin-lattice and spin-spin relaxation effects. This chapter also includes a description of the elegant optical methods of determining nuclear magnetic moments. Finally, an account is given of one of the most refined experiments of recent years, the determination of the anomalous part of the gyromagnetic ratio of the muon at CERN.

The chapters need not be read in strict numerical order and it is possible to omit certain sections, such as that on "Angular Correlations in Nuclear Reactions" in Chapter 20, without any loss of understanding of the rest of the book.

Chapter 20, "Nuclear Reactions and Nuclear Models" attempts the well-nigh impossible task of covering an enormous field of "nuclear activity". For this reason, an extensive bibliography is given at the end of this chapter and the reader is urged to look at some of the review articles listed. The important section on the Nuclear Shell Model can be read separately, and it is probably wise to do this, for this model is referred to on several occasions in the earlier chapters.

There is no account included of cosmic ray physics and the remarkable high energy nuclear chain reactions known as extensive air showers. Here in the energy region from  $10^{12}$  to  $10^{20}$  electron volts we enter a domain which seems to be beyond the range of any foreseeable future advances in particle accelerators. These ultra-high energy reactions are omitted from the text,<sup>†</sup> as they belong to the province of the cosmic ray physicist. Another great problem of cosmic ray physics is also omitted from the book;—"where and how do the primary cosmic ray nuclei attain their enormous energies"?<sup>††</sup>

Two important contributions to nuclear physics in the past decade have been reluctantly omitted from the book because of the mathematical difficulties involved. The first is the theory of Brueckner,<sup>‡</sup> in which a determined attack has been made on the fundamental problem of the properties of nuclear matter, using a self-consistent field approach on the assumption that two-body interactions are dominant in the nucleus. The second is the

<sup>†</sup> The interested reader is referred to the book W. GALBRAITH, *Extensive Air Showers* Butterworth's, London, 1958.

<sup>††</sup> P. MORRISON, *Rev. Mod. Phys.* **29**, 235, 1957.

<sup>‡</sup> A useful introduction to Brueckner theory is given in the book D. J. THOULESS, *The Quantum Mechanics of Many Body Systems*, Academic Press, 1961.

method of dispersion relations, which has become the cornerstone of theoretical elementary particle physics. A very brief reference to this method is given in Chapter 21.

The author is greatly indebted to Dr. J. E. Evans of A.E.R.E., Harwell, for his encouragement in writing this book. He has read most of the chapters and has provided much helpful constructive criticism. Dr. Evans is also responsible for writing the Section on "Angular Correlations in Nuclear Reactions" in Chapter 20.

It is also a pleasure to acknowledge the help of many of my colleagues at the College of Technology, Oxford, who have read several of the chapters.

The photographic plates numbers I, II, IV, were kindly provided by the Rutherford High Energy Laboratory, Harwell. I should like to thank Mr. T. F. Gubbins for his help in obtaining permission to include these photographs.

Oxford

C. M. H. S.

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Fig. 5, p. 262; Fig. 11, p. 277; Fig. 13, p. 287; Fig. 22, p. 304; Fig. 23, p. 307, of *Ann. Rev. Nucl. Sci.* 9.

Fig. 9, p. 20, of *Ann. Rev. Nucl. Sci.* 10.

Fig. 16, p. 726, *Nuclear Spectroscopy*, Part B, Editor F. Ajzenberg-Selove, Academic Press, New York.

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Fig. 15, p. 95, from L. I. Schiff, *Quantum Mechanics*, 2nd edition, McGraw-Hill, New York, 1955.

Fig. 1, p. 293; Fig. 2, p. 294, *Proc. Phys. Soc. A* 62, 1949.

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Diagram on p. 125 of *J. de Phys. et Rad.* 16, 1955.

Diagram on p. 544 of *Phil. Mag.* 7, 1904.

From the *Handbuch der Physik*, Springer-Verlag, Berlin: Vol. XLIV, Fig. 17, p. 147;

Table 1, p. 162; Table 2, p. 166; Table 1, p. 231.

Vol. XXXIX, Table 5, p. 258.

Fig. 1, p. 639, from *Rev. Sci. Instr.* 27, 1956.

Questions from the Graduateship of the Institute of Physics examination papers.

Many figures are redrawn from *The Physical Review*, *Physical Review Letters*, and *Reviews of Modern Physics*.

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## CHAPTER 1

# INTRODUCTION

### 1.1. KINETIC THEORY OF MATTER

The atomic theory of matter discussed more than 2000 years ago by the Greek philosophers Democritus and Lucretius lay dormant until the beginning of the nineteenth century. John Dalton in 1808 pointed out that the quantitative laws of chemical composition, derived from the gravimetric measurements of the substances involved in chemical reactions, could be explained in terms of an atomic theory. The fact, for example, that gases combine chemically in simple proportions, was interpreted in terms of atoms of certain elements combining to form molecules. Atoms of the same element were postulated to be identical in all respects: size, shape, weight, chemical affinity. In spite of the great advance in chemistry brought about by the Dalton atomic theory there existed as late as 1900 a school of scientists who were reluctant to accept the reality of atoms. Their main objection was the lack of direct evidence of individual atoms and at the time it seemed unlikely that such evidence would ever be found.

The molecular theory of matter was developed in physics in relation to the properties of gases. The fact that the common gases obey the simple laws of Boyle, Charles and Dalton is a reflection of the gaseous state rather than of the individual properties of the constituent molecules. Maxwell and independently Boltzmann, in the second half of the nineteenth century, developed with great skill the statistical theory known as the "Kinetic Theory of Gases". In dealing with the enormous number of molecules in one cubic centimetre of gas, about  $10^{19}$  at S.T.P., a mathematical theory dealing with the motion of each individual molecule is impossible. The only hope of deriving any useful theoretical results is to deal with a large collection of molecules by statistical methods.

#### Pressure of a Gas

The pressure exerted by a gas on the walls of its containing vessel is a macroscopic manifestation of the rate at which momentum is communicated to unit area of the walls by the rebounding molecules. The pressure  $p$  of an ideal gas is given by the expression

$$p = \frac{1}{3} m n \bar{c^2} \quad (1)$$

where  $m$  is the mass of a single molecule,  
 $n$  is the number of molecules per unit volume,  
 $mn = \rho$ , where  $\rho$  is the density of the gas.  
 $\overline{c^2}$  is the mean square speed of the molecules.

From equation (1), the root mean square speed,  $c_{\text{R.M.S.}} = \sqrt{\overline{c^2}}$  is given by

$$c_{\text{R.M.S.}} = \sqrt{\frac{3p}{\rho}} \quad (2)$$

Typical values at S.T.P. are 1,840 and 460 m per sec for hydrogen and oxygen respectively.

### Equipartition of Energy

Combining equation (1) with the equation of state for 1 mole of ideal gas  $PV = RT$ , the average kinetic energy of a molecule can be expressed in terms of Boltzmann's constant  $k$

$$\frac{1}{2} m \overline{c^2} = \frac{3}{2} kT \quad (3)$$

where  $k = R/N$ ,  $N$  being Avogadro's number. Equation (3) is a special case of Boltzmann's Equipartition of Energy theorem which states that the average energy per degree of freedom of a molecule is equal to  $\frac{1}{2}kT$ . For a monatomic gas, each atom has three degrees of freedom corresponding to the three independent components of translational momentum of the centre of mass of the atom. Diatomic molecules in a gas have five degrees of freedom, the two extra degrees being associated with two independent internal rotations about axes perpendicular to the chemical bond joining the two atoms. Classical Physics cannot account for the absence of rotation of the molecule about the chemical bond or why, below 60°K, hydrogen gas molecules cease to rotate. This behaviour can only be explained in terms of the quantum theory. The equipartition theory is based on the assumption that as a result of many collisions between the particles of a system, a steady state is established with the total available energy equally divided between the individual degrees of freedom. A bold application of the theorem has been made by Fermi to explain the mechanism of acceleration of cosmic ray particles (mainly protons and other light atomic nuclei). There is strong astronomical evidence for the existence of moving clouds of gas in interstellar space. Statistically the protons and other cosmic ray nuclei gain energy in colliding with the magnetic fields of these clouds, until the total available energy is shared between the cosmic ray particles and the moving gas clouds† according to the equipartition principle.

† The "temperature" of the assembly of gas clouds is very high.



### The Distribution of Molecular Speeds

The individual kinetic energies of the molecules of a gas are spread out over a wide range. This is not surprising in view of the fact that even if at some instant all the molecules had the same energy this state of affairs would not persist for long. At normal pressure and temperature a single molecule experiences about  $10^9$  intermolecular collisions per second. Some of the molecules will gain energy in collisions, some will lose energy, the average energy depending on the temperature. At each collision the energy of a molecule will in general change; nevertheless the number of molecules in

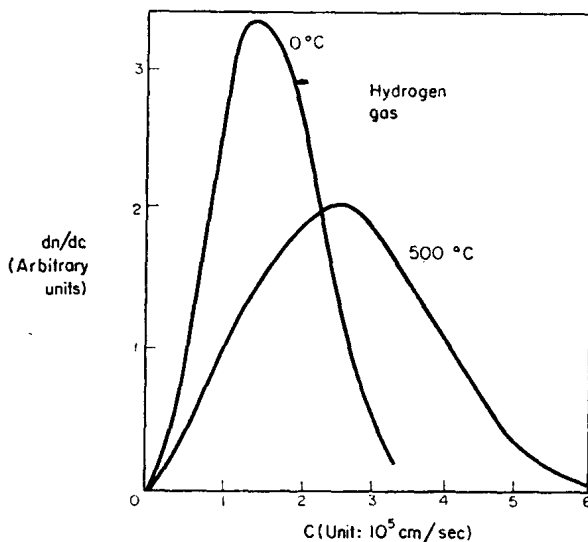


FIG. 1. Maxwell distribution of molecular velocities.

a definite small energy range will not alter, because, for every molecule scattered out of this narrow energy range one on average will be scattered into it. The steady state distribution of speeds amongst a large number of molecules in a gas at temperature  $T$  is given by the famous Maxwell law (1869).

The number of molecules  $dn$  out of a total number  $n$ , whose speed lies between  $c$  and  $c + dc$  is given by

$$\frac{dn}{dc} = \frac{4n}{\sqrt{\pi}} \left( \frac{m}{2kT} \right)^{\frac{3}{2}} c^2 e^{-mc^2/2kT} \quad (4)$$

For small speeds,  $mc^2/2kT \ll 1$ ,  $dn/dc$  is small as the term  $c^2$  dominates the distribution; for high speeds the value of  $dn/dc$  is again small because the negative exponent of the exponential factor, which is the kinetic energy of a molecule of speed  $c$  divided by the thermal energy  $kT$ , is numerically