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Fuzzy Mathematical Models in Engineering and Management Science

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FUZZY MATHEMATICAL MODELS IN ENGINEERING AND MANAGEMENT SCIENCE

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FUZZY MATHEMATICAL MODELS IN ENGINEERING AND MANAGEMENT SCIENCE

Dedication

To

My Wife, Yvette Kaufmann

- *Arnold Kaufmann*

My Grandparents and Parents
for Instilling within Me
the Thirst for Knowledge
and the Quest for Excellence

and

My Teachers, Research Colleagues and Students
Who have All Taught and Inspired Me

- *Madan M. Gupta*

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इन्द्रियाणि मनो बुद्धिरस्याधिष्ठानमुच्यते ।
एतैर्विमोहयत्येष ज्ञानमावृत्य देहिनम् ॥४०॥

40. *indriyāṇi mano buddhir*
asyā 'dhiṣṭhānam ucyate
etair vimohayaty eṣa
jñānam āvṛtya dehinam

"The senses, the mind and the intelligence are the breeding grounds of desire and lust; they veil the real knowledge of the living entity and bewilder the embodied soul".

इन्द्रियाणि पराण्याहुरिन्द्रियेभ्यः परं मनः ।
मनसस्तु परा बुद्धिर्यो बुद्धेः परतस्तु सः ॥४२॥

42. *indriyāṇi parāṇy āhur*
indriyebhyaḥ paraṁ manaḥ
manasas tu parā buddhir
yo buddheḥ paratas tu saḥ

"The active senses are superior to the passive matter; mind is higher than the senses; intelligence is still superior than the mind, but the soul is the most superior".

"Bhagavad-Gita"

FOREWORD

Life is full of uncertainties and the information and data which are associated with our professional and private activities are often more vague and possibilistic than random and probabilistic. Many of our activities and the systems with which we deal cannot be modelled easily, therefore, with any degree of accuracy using conventional techniques. New tools are required for this purpose and Professor Lofti A. Zadeh, while not necessarily the first to recognize this need, was the first to introduce such mathematical tools based upon fuzzy set theory and fuzzy logic. He did this in 1965 and since that time there has been an exponential growth in this field with applications in engineering and in the management and social sciences.

Professor A. Kaufmann and Professor M.M. Gupta are two of the leading researchers in this field and they are certainly amongst its greatest proponents. This text is the latest in their very substantial contributions to this field. It deals with the notions of fuzzy numbers with levels of perception and levels of presumption. It also provides many interesting and useful examples of applications in engineering and management science. Of particular interest, are their discussions of applications in areas employing zero-based budgeting, the Delphi method, critical path optimization, reliability modelling, filtering and transportation.

Readers will find this book not only interesting, but easily understood. As one of the recent converts to this field, I highly recommend it to all of those who work with data which are somewhat "fuzzy".

April, 1988

Peter N. Nikiforuk
Dean of Engineering
University of Saskatchewan
Saskatoon, Saskatchewan

Professor Arnold Kaufmann

Arnold Kaufmann is a professor and scientific advisor in the field of Operations Research (O.R.) and Management Science (M.S.). He has served as a professor at the Ecole Superieure des Mines de Paris, at the Institute Polytechnique de Grenoble, and at the University of Louvain (Belgium). He is the author or co-author of over sixty-five books in the field of applied mathematics, operations research, sociology, and methods of creativity, etc. His recent famous books are in the area of fuzzy set theory, expert systems, artificial intelligence and fuzzy mathematical modelling.

Professor Kaufmann has received many coveted honours and awards, including many honourary doctorate from Universities around the world.

Professor Madan M. Gupta

Madan Gupta received the B.Eng. (Honors) in 1961, and the M.Sc. in 1962, both in electronics-communications engineering, from BITS Pilani, India. He received the Ph.D. degree for his studies in adaptive control systems in 1967 from the University of Warwick, U.K.

From 1962 to 1964, he served as a lecturer in electrical-communications engineering at the University of Roorkee, India. He joined the faculty of the College of Engineering at the University of Saskatchewan in 1967 as a sessional lecturer, becoming a full professor in July 1978. His present research interests are in the areas on non-invasive methods in medical diagnosis, medical imaging, intelligent robotic systems, cognitive information, neural networks, and computer vision.

He is a co-author with A. Kaufmann, of the books *Introduction to Fuzzy Arithmetic: Theory and Applications* (Van Nostrand Reinhold, 1985) and *Fuzzy Mathematical Models in Engineering and Management Science* (North-Holland, in press) and is the editor of the books *Fuzzy Automata and Decision Processes* (1977), *Advances in Fuzzy Set Theory and Applications* (1979), *Approximate Reasoning in Decision Analysis* (1982), *Fuzzy Information, Knowledge Representation, and Decision Analysis* (1983), *Approximate Reasoning in Expert Systems* (1985), *Fuzzy Computing* (1988), *Fuzzy Logic in Knowledge-Based Systems Decision and Control* (1988), all with North-Holland, and *Adaptive Methods for Control Systems Design* (IEEE Press, 1986). He is the subject editor for the *Encyclopedia of Systems and Control* (Pergamon Press, Oxford, 1987) and has authored or co-authored over 200 research papers. He is an advisory editor for the *International Journal of Fuzzy Sets and Systems* (IFSA) and other journals in the field.

PREFACE

The universe, as described by the mathematical sciences, is not one that is closed. Each day brings innovative directions in theory and applications. Mathematics, the queen of all sciences, is also the most generous in novel theories and applications, the recent theory of fuzzy sets being a good example of this generosity. In one of our recent books, "Introduction to Fuzzy Arithmetic, Theory and Applications" (Van Nostrand, Reinhold, 1985), we discussed the state-of-knowledge of fuzzy numbers, and fuzzy sets on the real line. We also presented much new work in this book, but since its completion we have realized the need for an additional book in this field due to the many interesting developments, both in theory and mathematical modelling, that have appeared. Many of these developments are described in the present book.

We designed the first book as a text book for students, for researchers by giving many new results and research ideas, and for practitioners by giving examples of applications. The present volume on "Fuzzy Mathematical Models in Engineering and Management Science" reflects the same objectives. It presents many new results, examples and novel applications. This approach, in our opinion, makes the book interesting, easy to understand and has immense pedagogical value.

Fuzzy arithmetic is not difficult to learn; firstly because it is only an extension of ordinary arithmetic and secondly, because the "fuzzification" is a phenomenon that is natural and inherent in the human thinking and cognitive processes. In human sciences, data and processes may or may not be vague, may or may not be measureable, may be subjective or objective. However, when a mathematical model is used in decision making processes its validity must be questioned, especially if the actual model must be reduced to one that is deterministic even when the environment is fuzzy. If our knowledge of the environment is imprecise, as happens in medical diagnosis, engineering, management decision making, etc, the model must include the notion of the level of presumption. Fuzzy numbers have been created to reflect the vagueness of human perception and thus the notion of the level of presumption. These fuzzy numbers thus reflect the human cognitive process.

This book deals with the theory of fuzzy mathematical models with applications in engineering and management science. Part I, which contains ten chapters, is devoted to the theoretical basis for these mathematical models. Part II, which contains eleven chapters, is devoted to a variety of applications in engineering and management science. There are also seven appendices which contain some special mathematical operations

(Minkowaski's operations) on fuzzy quantities and detailed biographical material.

The first part of the book is devoted to theoretical considerations which lay the foundations for fuzzy mathematical modelling. We briefly introduce the theory of fuzzy sets and fuzzy numbers in Chapters 2 and 3, but a more extensive discussion may be found in our other book on fuzzy arithmetic. Fuzzy numbers are a special kind of fuzzy sets which are normal and convex. Although these numbers can be described using many types of shapes, for practical applications it is best to use triangular and trapezoidal shapes to describe valuation data with certain levels of presumption.

When using fuzzy numbers, it is necessary to determine their order (ranking). The problem of linear ordering is discussed in Chapter 4. The imprecision or vagueness associated with a fuzzy number is an important piece of information and it is discussed in Chapter 5. In Chapter 6 we give particular attention to triangular fuzzy numbers (T.F.N.'s) and their use in the approximation of several functions of triangular fuzzy numbers. In fuzzy mathematical modelling, we often encounter the deconvolution (solution) of the equations of the type $\mathbf{A}\ (+)\ \mathbf{B} = \mathbf{C}$, and $\mathbf{A}\ (\cdot)\ \mathbf{B} = \mathbf{C}$, with given **A** and **C**, and unknown **B**. This subject is discussed extensively in Chapter 7. T-norms and T-conorms; that is, operators for fuzzy variables which belong to [0, 1], and their respective attributes, are studied in Chapter 8. A fuzzy number in [0, 1] is a special kind of fuzzy set which can be used in modelling knowledge and is discussed in Chapter 9. We give a detailed discussion of fuzzy numbers in [0, 1] with higher order interval of confidence in Chapter 10. We also discuss in this chapter the solution of simultaneous equations with equalities and inequalities. This first part of the volume, thus, lays the basic fuzzy mathematical background essential to the modelling of systems with fuzzy (soft) data.

The second part of this volume is devoted to some very important applications of fuzzy set theory in engineering and management science. The first application, given in Chapter 12, is taken from a model in management science which traditionally assumes a model with either deterministic or probabilistic variables. In this chapter, we show how to treat a zero-base budgeting (Z.B.B.) model using triangular fuzzy number. A considerable amount of criticism has been made of the zero-base budgeting method because it conventionally uses the deterministic approach. The approach presented in this volume, however, utilizes fuzzy data, and this fuzzy modelling of the problem seems to be more realistic. We apply the same treatment to the Delphi technique and develop a fuzzy Delphi approach in Chapter 13. As is well known, the classical Delphi approach has been used with a large success by corporation such as Rand Corporation and the approach presented here makes use of triangular fuzzy numbers. This variation to the classical Delphi approach may prove to be more powerful in that it can make use of realistic data which are vague in nature.

Discounting with fuzzy numbers and fuzzy smoothing problems are illustrated using several examples in Chapters 14 and 15 respectively. Chapter 16 deals with the important topic of reliability which conventionally utilizes a probabilistic approach. This problem deserves deep study and wide explanation since a large scale system is usually composed of many components and/or sub-systems with non-objective survival curves rather than objective ones. We use intervals of confidence in the evaluation of reliability of systems and give a possibility theory for the study of reliability models.

Among many other indicators for economical choices, cost efficiency quotients are very useful. In Chapter 17 we show how to utilize this criterion when data are not very well known or not well defined. When criticial path methods (C.P.M.) are used, litigation between experts is not rare and we show in Chapter 18 how to make a convenient aggregation using experts' subjective opinions. In Chapter 19 we discuss the problem of investment with fuzzy data. Of course, we did not forget the application of the dynamic programming method developed by Richard Bellman, the co-developer of fuzzy mathematics. The dynamic programming method with fuzzy data can be applied to various problem situations. We also give some classical but illustrative examples of this method. Finally, in Chapter 20 we show how to solve a well known problem in transportation using the fuzzy stepping-stone method when the unitary costs are not precise.

In this part of the book we limit our studies to building models using fuzzy data. In fact, the use of fuzzy numbers is not limited to only engineering and management science. Fuzzy mathematics concerns all domains of human sciences and also even the so-called "exact sciences" when the information may be exact but not necessarily precise. Chapter 21 presents a general view of fuzzification of models in engineering and management science.

Some important information is provided in the appendices. Appendix A deals with the properties of triangular and trapezoidal fuzzy numbers and Appendix B gives the details of Minkowski's operations on ordinary subsets. These operations have extensive applications in the morphological description of signals and images. We define fuzzy quantities in Appendix C, and give the theory of Minkowski's operations on fuzzy quantities in Appendix D. These operations may find applications in the manipulation of such data as gray-level and coloured images and cognitive information arising in decision making processes. Finally, we give an extensive list of selected books, and some major current bibliographical sources on fuzzy sets and systems.

This book thus presents many new theoretical developments and innovative applications. It provides useful mathematical tools to our readers which they may find useful in the study of their own problems. Hopefully, the readers will contribute to this new field of applied mathematics by their own research and will generate new applications in the fields dealing with soft data.

April, 1988

Arnold Kaufmann
Madan M. Gupta

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ACKNOWLEDGMENTS

A vision of this book has been long in our mentation and cognition and we are indeed pleased to present it now to our readers.

This book is based on the unpublished works of the authors and the readers will find very little in the published literature dealing with these new theoretical developments and applications in engineering and management science. However, this work would not have been possible without the research efforts of many individuals in the fuzzy community. It was Professor Lotfi A. Zadeh who introduced us to the concept of graded membership in 1965 and this concept has revolutionized the logic of mathematical modelling, computing and analyzing the information which arises from human mentation and cognition. We owe our gratitude to Professor Lotfi A. Zadeh and many other individuals who followed his work over the last two decades.

One of the authors, Madan M. Gupta, acknowledges the stimulating and intellectual environment that the College of Engineering at the University of Saskatchewan has provided to him over the past two decades - ever since he was first introduced to the beautiful mathematics of fuzzy sets and fuzzy logic. This energetic and inspiring environment has been largely responsible for his many publications in this field. This author also wishes to acknowledge the tireless support that he has received from his wife Suman, his three sons Anu, Ashu, and Amit, and his mother during the preparation of this manuscript and while carrying out other equally important tasks.

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Finally, we record our appreciation to Miss Elizabeth Nikiforuk, the Research Assistant in the Intelligent Systems Research Laboratory, for her helpful and smiling attitude. It is she who was responsible for the crystallization of the many cognitive uncertainties in the manuscript into the useful information which appears in this book. It has been a great pleasure to work with her. We also acknowledge the help of Mr. Wayne McMillan, the Manager of the Central Shop at the University of Saskatchewan, for the preparation of the diagrams.

April, 1988

Arnold Kaufmann
Madan M. Gupta

LIST OF PRINCIPAL SYMBOLS

A Fuzzy subset, fuzzy number

$\underset{\sim}{\mathbf{A}}$ Fuzzy quantity

A Ordinary subset

A_α Interval of confidence at level α, α – cut of **A**

$\mathbf{A}_\alpha$ Ordinary subset of a fuzzy subset **A** at level α

$\overline{\mathbf{A}}$ Complement of **A**

$\overline{A}$ Complement of A

$A \times B$ Cartesian product of two sets A and B

$\mathbf{A} \times \mathbf{B}$ Product of two fuzzy subsets **A** and **B**

$[a, b]$ Interval of confidence

(a, b) Interval of **R** "open on the left and on the right," thus, $(x \mid a < x < b)$

$(a, b]$ Interval of **R** "open on the left and closed on the right," thus, $(x \mid a < x \leq b)$

$[a, b)$ Interval of **R** "closed on the left and open on the right," thus, $[x \mid a \leq x < b)$

$[a, b]$ Interval of **R** "closed on the left and on the right," thus, $[x \mid a \leq x \leq b)$

(a_1, a_2, a_3) Representation of a triangular fuzzy number (T.F.N.)

(a_1, a_2, a_3, a_4) Representation of a trapezoidal fuzzy number (Tr.F.N.)

A $= (a, b, c)$ Triangular fuzzy number with parameters: minimum levels of presumption $= a, c$; maximum level of presumption $= b$

$\hat{A}$ Ordinary number associated with a fuzzy number **A**

$\hat{A}_{\mathbf{k}}$ Weighted associated fuzzy number

$\overset{m}{\mathbf{A}}$ Mean of the sheaf $\{A_i\}$

$\mu_{\mathbf{A}}(x)$ Membership function for the element x with respect to the fuzzy subset **A** (level of presumption).

$i(\mathbf{A})$ Index of specificity

iff If and only if

l (**A**) Length of a segment $A = [a_1, a_2]$

$r_k(i_k, m_k, s_k)$ Fuzzy discount rate over the period k

$\mathbf{F}_{ij}(l)$ Return on combined policy on investment i and j

k_n (**A**) Generalized form of energy function

k_f (**A**) Generalized form of imprecision index

max (X, Y), or $X \vee Y$ Maximum of X and Y

min (X, Y), or $X \wedge Y$ Minimum of X and Y

N Set of natural numbers, $N = \{0, 1, 2, 3, \cdots\}$

$\mathbf{N}_0$ The set **N** but excluding 0

η_s Relative index of fuzziness

P Triangular approximation of f (**A**)

R Set of real numbers

R^+ Set of non-negative real numbers

R_0 Set of real numbers excluding 0

R_0^+ Set of positive real numbers

R^- Set of non-positive real numbers

R_0^- Set of negative real numbers

R(**A**, k) Removal of fuzzy number **A** with respect to ordinary subset k

$X \leqslant Y$ Order relation (Y is preferred to X)

$X < Y$ Strict order relation

Z Set of integers, $Z = \{0, +1, -1, +2, -2, +3, -3 \cdots\}$

Z_0 The set of **Z** excluding 0

$\underline{Z}$ Cost function with fuzzy data

$(+)$ Addition of fuzzy numbers by max-min convolution

$\underset{m}{(+)}$ Minkowski addition

$\hat{+}$ Symbol for an algebraic sum, $a \hat{+} b = a + b - a \cdot b$

$(-)$ Subtraction of fuzzy numbers by max-min convolution

$\underset{m}{(-)}$ Minkowski subtraction

$(\cdot)$ Product of fuzzy numbers by max-min convolution

$\underset{m}{(\cdot)}$ Minkowski multiplication

$(:)$ Division of fuzzy numbers by max-min convolution

$\underset{m}{(:)}$ Minkowski division

$(\wedge)$, (V) Minimum (maximum) of fuzzy numbers by max-min convolution

$<$ Strict total order relation

$\leq$ Non-strict total order relation

$\subset\subset$ Strict inclusion

$\subset$ Non-Strict inclusion

$\not\subset$ Non-inclusion

$\cup$ Union

$\cap$ Intersection

ϕ Empty subset

$\delta(\mathbf{A}, \mathbf{B})$ Divergence between **A** and **B**

Γ_n Cumulative discounted investment

$\Pi(t)$ Possibility of failure at time t

θ Time to failure

$\Psi(t)$ Possibility of failure over $[0, t]$
[cumulative failure possibility distribution]

$\mathbf{K}(t)$ $1 - \Psi(t)$, the law of survival possibility

$\lambda(t)$ Failure rate (hazard rate)

$\Lambda(t)$ Cumulative failure rate

$\bar{t}$ Mean failure rate

$\upsilon(x)$ State function of system structure

$\upsilon(\mathbf{A})$ Index of fuzziness of the fuzzy subset **A**

$\Longrightarrow$ Meta-implication (one also says, usually but improperly, implication)

$\Longleftrightarrow$ Logical equivalence

$\forall x$ Universal quantifier [for *all* x)

$\exists\ x$ Existential quantifier (there exists an x)

$\top(x, y)$ Triangular norm (T - norm)

$\perp(x, y)$ Triangular conorm (T - conorm)

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