COMPUTER ASSISTED DECISION MAKING

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Expert Systems, Decision Analysis, Mathematical Programming

edited by

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1986

NORTH-HOLLAND
AMSTERDAM • NEW YORK • OXFORD

ISBN: 0 444 87887 4

Published by:
ELSEVIER SCIENCE PUBLISHERS B.V.
P.O. Box 1991
1000 BZ Amsterdam
The Netherlands

Sole distributors for the U.S.A. and Canada:
ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
52 Vanderbilt Avenue
New York, N.Y. 10017
U.S.A.

Library of Congress Cataloging-in-Publication Data
Main entry under title:

Computer assisted decision making.

Proceedings of a seminar organised by UNICOM in the spring of 1984.
Includes bibliographies.
1. Expert systems (Computer science)--Congresses.
2. Decision-making--Data processing--Congresses.
3. Programming (Mathematics)--Congresses. I. Mitra, Gautam. II. Unicom Consultants.
QA76.76.E95C66 1986 006.3'3 85-25215
ISBN 0-444-87887-4 (Elsevier)

PRINTED IN THE NETHERLANDS

PREFACE

In the spring of 1984 UNICOM organized a three day seminar on Computer Assisted Decision Making. The seminar was supported by the British Computer Society, Operational Research Society and Queen Mary College IT Centre and took place in London from 2 to 4 April 1984. An analysis of the background of the eighty or more delegates who attended the seminar showed an industry bias. It also demonstrated the strong interest which exists in industry and also in the academic circles for the newly emerging methodologies for decision making. In planning the seminar our primary objective was to disseminate information covering recent developments and applications of three major problem solving methodologies, namely, expert systems (set against the broad context of artificial intelligence), decision analysis, and mathematical programming. The use of computer support to solve real life decision problems is the common theme that unites these topics. There are methodological aspects such as the representation of uncertainty, logical inference mechanisms which are also common to these seemingly different decision tools. Demonstration of a number of software products employing these techniques took place and these stimulated informal discussions and complemented the formal presentations. In general the seminar provided a forum for discussion and debate among the contributors and participants.

In contrast to seminars which are addressed mainly to the academic community, this seminar was set out to present market developments, technological updates and progress reports on research results. It is well known to practitioners closely associated with the research, development, and exploitation of information technology, that in most sectors industrial research and development continue to stay ahead of the developments in the universities. Technology development cycles are getting progressively shorter and academic departments find it difficult to rapidly reorganize their strategy to deal with this climate of continual change. In industry, on the other hand, a realization of market opportunities, complemented by entrepreneurial drive lead to suitable deployment of research and development funding. Set against this background a seminar of this type offers considerable scope. For instance it allows participants to evaluate market trends and technological gaps. In order that the proceedings of such a seminar provide similar value to a wider audience it is essential that the written contributions are printed and published over a short time cycle. This poses a genuine difficulty to the publishers and the editors in preparing a volume which is well balanced in its contents and which is clearly and accurately laid out. The quality and novelty of a well edited academic volume relies heavily on the formal refereeing procedure which is well known in academic circles. We place great emphasis on timeliness and this is coupled with our belief that there is need for a volume in which research results and market trends are set side by side and not seen entirely divorced from each other. We have therefore digressed from the rigorous refereeing procedure and replaced this with broad comments and criticisms by the contributors and other specialists. As an editor I have simply strived to achieve clarity, accuracy and an early publication date. We have also made a special effort to include edited transcripts from the tape recordings of two presentations and backed these by copies of slides which were used to support the original presentations.

The participants of the seminars were requested to complete a seminar appraisal questionnaire. We were encouraged by the overall assessment of the respondents who considered attendance at the seminar was worthwhile.

It is expected that this report of the proceedings will provide useful reference material and summary information on present developments and likely developments in the near future. In the intervening years I hope the readers will find similarly worthwhile reference material and summary information contained in these proceedings.

I would like to thank Professor M.A. Laughton of Queen Mary College for first suggesting that we hold a seminar on this topic. We were very ably supported by Mrs. J. Valentine in the organization and administration of the seminar. She also managed to keep the contributors and the participants generally satisfied and well humoured both before and during the event. Mr. R. Parslow in his turn provided public relation and administrative support. I also thank my wife Dhira for her patience in proof reading and the rest of the family for their general understanding and for accepting continued rescheduling of priorities. At the final stages of completing the editorial work Mrs. P. Denham worked wonders with the word-processor. She carried out many revisions efficiently and patiently and still managed to keep calm and good humoured. I am glad to avail this opportunity to thank the whole team and many well wishers.

Gautam Mitra
Wentworth, Surrey
October, 1984

Since writing the preface and the editorial introduction I had the opportunity of discussing the contents of the volume with a number of people from Europe and the USA. As a result Dr. Hammond, Professor Prade, Dr. Kohout, Professor Gaines, and Dr. Efstathiou were also able to provide further solicited contributions. Finally, I would like to thank all the contributors who cooperated in revising their material based on editorial comments and criticisms.

Gautam Mitra
Wentworth, Surrey
10 May 1985

INTRODUCTION AND OVERVIEW

PERSPECTIVES

It is the task of the editor to set the scene and provide some explanation as to the purpose and contents of the volume. The main focus of operational research and management science has been to aid the decision making process in a managerial setting. Throughout their development as number crunching and information processing machines, computers have been progressively harnessed by the operational research and management science analysts to support the representation and solution of their quantitative models. Systems analysts and data processing specialists starting from another end of the spectrum have again strived to support the decision making process. Business analysts and behavioural scientists have also focussed their attention on managerial and organizational problems and today the multi-disciplinary contributions towards decision support aids is well accepted. Indeed an enterpreneur in today's IT market place not only employs funds and (hardware) technology but also seeks research and development support from psychologists, mathematicians, business analysts and computer specialists towards the creation and exploitation of new thought products.

The purpose of this seminar on Computer Assisted Decision Making (CADM) was to bring practitioners and academics from the fields of mathematical programming, artificial intelligence and decision analysis to share the same platform. The main theme that unites them is their preoccupation to solve decision problems with computer support. Preoccupation with a chosen methodology often leads to a natural progression from working knowledge to specialized expertize. However, the pitfall of such single mindedness is also well known and is strikingly expressed by Professor Weizenbaum [1] who quotes Maschler ... "to a man who has a hammer the whole world looks like a nail."

A TAXONOMY OF DECISION PROBLEMS

From the arguments set out above it follows that there is a case for studying and understanding the range and nature of decision problems in their own right prior to devising methodologies for solving these. Decision problems are closely related to the nature of the work with which various staff members at different levels of an organizational echelon are involved [2], [3], [4]. From entirely independent standpoints Anthony and Jacques arrive at a classification that is based on the time frame and Simon puts forward a classification based on the structured and unstructured nature of decision problems.

Classification Based on Time Frame

Anthony's classsification [2] is based on strategic, tactical, and operational decision problems and may be naturally extended to include problems of industrial and process control, Table 1.

TABLE 1

Problem Characteristics / Type	Objectives	Time Horizon	Level of Management Involvement	Scope	Source of Information	Nature of Information	Degree of Uncertainty
Strategic Planning	Resource Acquisition (Development)	Long 5 years	Top	Broad	External and Internal	Highly Aggregate	High
Tactical Planning	Resource Utilization	Medium 6 months - 1 year	Medium	Medium	External and Internal	Moderately Aggregate	Moderate
Operations Control	Optimal Execution	monthly weekly daily?	Low	Narrow	Internal	Detailed in specific Aspects	Low
Monitoring Control of - Industrial Processes - Financial Fund Transfer and Management	Optimal and Robust Execution	daily hourly more frequent	Low + machine support	Narrow	External and Internal	Detailed in in specific Aspects	Low

Strategic, Tactical, Operational Decision Problems and Industrial Control Problems

Classification based on Anthony's Framework.

Table 2

Basic structure of work in organisations
and associated decision support systems

Time Span	Stratum	Organisational Level	Main Activity	Capabilities of Decision Support Systems
	VIII	Super Corporation	Shaping Society	
50 years				
	VII	Corporation	Providing overall strategic direction	
20 years				
	VI	Corporate Group of Subsidiaries	Creating strategy and translating it into business direction	
10 years				
	V	Corporate Subsidiary & Top Specialists	Redefining goals and determining field of operations	Articulation of principles guiding goal setting
5 years				
	IV	General Management and Chief Specialists	Creating methods of operation	Selecting from types eg. generating new decision support systems
2 years				
	III	Departmental Managers & Chief Specialists	Organising programmes and systems of work	Restructuring within fixed structure, eg. establishing new criteria
1 year				
	II	Front Line Managerial Professional & Technical	Generating programmes of work	Altering judgement on a variable within a fixed structure, eg. 'What if?' models
3 months				
	I	Office & Shop Floor	Doing concrete tasks	Judgement within fixed structure, eg. with information retrieval service
1 day				

Strategic planning is carried out by top management and the main goal is to acquire and develop productive resources. The time frame of decision making is long five years or more and there is a broad scope of development and growth of the organization. Most of the information is considered in a highly aggregated form and the problem is characterized by high degree of uncertainty concerning the future taking into account both external and internal circumstances.

Tactical Planning is undertaken by the middle managers, it is concerned mainly with efficient resource utilization and the time horizon of the decisions are shorter around 6 months to 1 year. Decision making is based on moderately aggregated information collated from internal and external sources and there is a reduced level of uncertainty coming mainly from external circumstances. The outcomes of these decisions are more limited in their scope of influencing the organization's present performance and future development.

Operations control is focussed on the optimal execution utilizing resources which are deployed by the tactical planning exercise. These correspond to well specified clerical or industrial tasks carried out by employees lower down the organization. The scope of influence at the operations level is narrow, the degree of uncertainty is low; however, the information as required for decision making may be extremely detailed in some specific aspect of the problem.

Industrial control problems are those where decisions are made and control action are taken regularly for continued operation of a plant. In the financial sector funds may be transferred or managed against fluctuating exchange rates, interest rates, etc. The time frame of decision making in this case is daily or hourly and in addition to optimality consideration one looks for robust decision making.

L. Phillips [7] has taken E. Jaques' [3] analysis of nature of work as set out in Table 2 and used it to illustrate the same hierarchical structure of decision problems and the scope of decision support tools.

Simon's Taxonomy

Simon [4] classifies decision problems into two types: programmed and nonprogrammed, which are also referred to as structured and unstructured decisions respectively. This classification is set out in Table 3.

Programmed decisions can be structured into specific procedural instructions; they can be delegated to the lower echelons of the organisation and do not require extensive supervision. These decision problems occur routinely, repetitively and are normally in the domain of the clerks.

Nonprogrammed decisions cannot be handled by well established and well defined treatment. Their solution requires considerable creativity, judgemental input and abstract thinking. They are complex and unstructured and are usually dealt with by top managers.

Table 3

Simon's Taxonomy of Decision Problems

Types of Decisions		Techniques: Conventional		Techniques: Modern
Programmed: Structured	1.	Clerical Routine	1.	Mathematical Models
	2.	Operating Procedures	2.	Electronic Data Processing
Routine Repetitive	3.	Habit Forming		
Non Programmed:	1.	Judgment, intuition creativity	1.	Decision Theory
Unstructured Complex	2.	Rules of Thumb	2.	Heuristic Problem Solving
Abstract	3.	Executive Training	3.	Other AI Methods

Programmed = Structured Decisions

They are carried out routinely, repetitively. They can be structured into specific procdedural instructions ... delegated lower down the strata.

Non Programmed = Unstructured Decisions

They are complex, unique. Do not lend themselves to well defined treatment requires deep analysis, judgmental input, evaluation of risk.

In some sense the two frameworks are not entirely independent of each other. For instance a large number of operations control and industrial control problems are in the nature of structured decision problems. The strategic and tactical planning on the other hand lead to usually unstructured problems. However, the boundaries of the structured and unstructured problems and those separated by time frame are not entirely sharp. For instance a number of scheduling problems such as crew scheduling [5], vehicle scheduling [6] are operations control problem. However, the manual method of solving these problems require considerable amount of mental abstraction and reasoning skill. It is well known that good human schedulers develop a high degree of specialist skill focussed to their problem and also call upon their experience and intuitions. Some of these are definitely nonprogrammed tasks.

Another important conclusion that emerges from the analysis of this framework can be summarized as follows. If we consider the number of instances of problem solving it becomes obvious, how often we need to solve the problems taken from the different groups. This frequency of problem solving is clearly based on the time frame of these problems themselves. For instance industrial control and operations control problems are solved

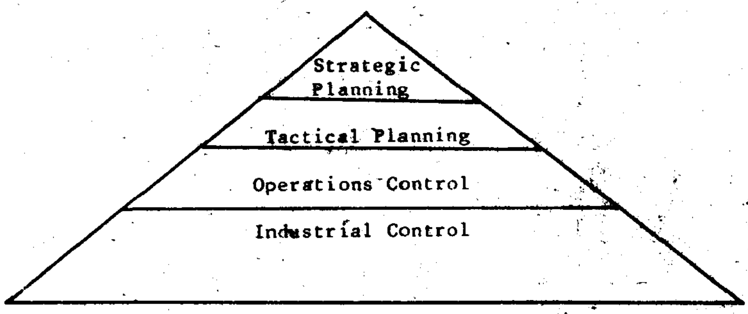

Diagram 1

on hourly, daily, weekly basis and hence there are more instances of these than those of tactical planning and strategic planning which are solved on a quarterly or half yearly basis, Diagram 1. Thus the scope for applying computer supported problem solving methodologies is greater with the groups lower down the scale of time span.

Turning to the methodologies we find that decision analysis is well applied to the strategic and tactical planning problems [7] L. Phillips. Mathematical programming is most successful in its application to tactical planning problems [8]. It can also be applied to a lesser degree to operations control and industrial control problems. The uncertainty aspect of the strategic planning problem does not make mathematical programming the most appropriate tool: its only use in this area is restricted to generating alternative scenarios. Expert systems are claimed to have applications across the whole range of problems and the most successful applications are reported in operations control and industrial control [9]. The contents of the papers presented at the seminar broadly support these observations. The contributions by Phillips, Conway, Turtle and Williams are in the field of decision analysis and are solely concerned with strategic planning. The papers by Shapiro, Christofides, Rowland and Sparrow constitute the mathematical programming contribution and relate mainly to tactical problems. However, Rowland outlines the scope of mathematical programming within EXXON and confirms its application to strategic planning and operations control problems as well. Sparrow, provides a case study of an industrial control application of mathematical programming. The AI based tools span all these domains of applications and Benson, Keen, Jones, Ritchie, Williams, discuss a range of applications which confirm this claim. Worden provides examples of unstructured decision problems with a continuous inference making requirement ranging from satellite surveillance to financial ticker tape market. The blackboard system described by him handles these problems and falls clearly in the realm of (complex) industrial control.

METHODOLOGICAL ISSUES

Developments in CADM are multidisciplinary in nature, and the topic has received major contributions primarily from two groups: the information processing and the mathematical modelling specialists. The central motivation here is to create computer (hardware) based tools and products to aid the decision process, see Diagram 2. An analysis of recent developments provide further insight. Computer scientists, DP specialists, management scientists, all have been involved in the solution of decision problems.

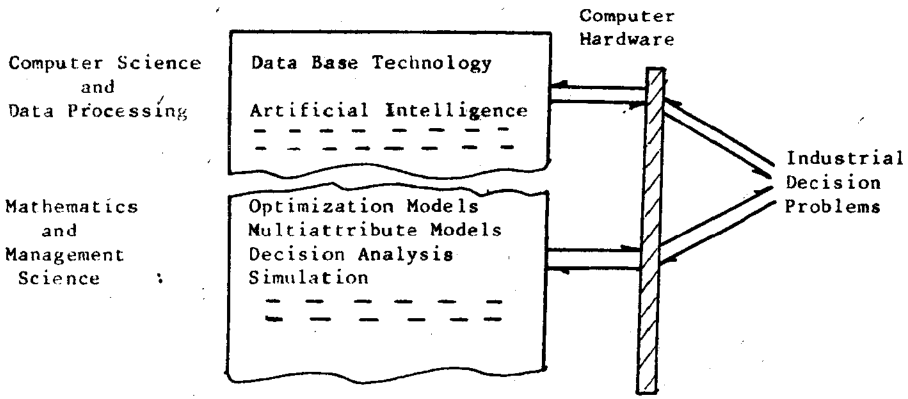

Diagram 2

The boundary between these specializations are becoming less sharp and there is a natural acceptance of a complementary contribution from the other field. For instance Whinston et al [10] outline the introduction of models within database technology and they highlight the importance of these models for processing the problems (applications). Worden in this volume sets out a similar argument. Starting from a personnel data base he argues that a natural extension is to introduce a model (or intelligent processing in AI terms) whereby benefit, promotions etc can be worked out by applying a set of simple rules. At the other end of the spectrum we find that mathematical modellers are becoming increasingly aware of the need for data processing support. Shapiro makes a case in the presentation of his LOGS system that the solution of the optimization models is not the only issue. A tool is needed to define and construct planning problems and then to create management reports from the solution. In short a complete data processing environment is equally important. Along these same lines Rowland reports on the design aspects and the experience of a data and model management system in EXXON.

Classification of Models

Management scientists and decision analysts classify mathematical models in three broad categories described below [11].

Descriptive models as defined by a set of mathematical relations which simply predicts how a physical, industrial or a social system may behave.

Normative models constitute the basis for (quantitative) decision making by a superhuman following an entirely rational and logically consistent set of arguments. Hence quantitative decision problems and idealized decision makers are postulated in order to define these models.

Prescriptive models involve systematic analysis of problems as carried out by normally intelligent persons who apply intuition and judgement. Two distinctive features of this approach are uncertainty analysis and a preference (or value or utility) analysis.

From an analytic standpoint there are three major model characteristics which influence the nature and the applicability of a model to a given problem area. These characteristics may be summarized as: Equation Solving, Preference Analysis, Uncertainty Representation, see Diagram 3.

Major Modelling Issues

From an analytic standpoint there are three major issues which influence the nature and applicability of a model to a given problem area. These may be summarised as Deterministic Relations, Preference Relations and Uncertainty Representation, see Diagram 3.

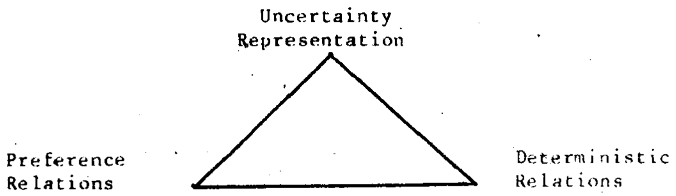

Diagram 3

Deterministic Relations

Deterministic relations (constraints or restrictions) are usually turned into a set of linear or nonlinear equalities. By solving a set of equations linear or nonlinear one would satisfy the quantitative restrictions as required by the real life problem. This is equivalent to finding a point in a point set and all linear and nonlinear programming (optimization) methods incorporate this feature, [12]. Thus the central theme of mathematical programming is to solve a set of equations and to find the extremum of one objective function. The introduction of multiple objective functions compels one to consider preference relations which are discussed in the next subsection. The main issue in mathematical programming is to compute solutions to intractable and difficult problems within a deterministic framework. Over the last decade there has been considerable progress in devising methods and supporting software which process substantial linear and nonlinear programming problems.

Preference Relations

Whereas making choices or deriving preference relations using one criterion involves straight forward scalar comparison and leads to the relatively simple concept of optimality the situation is naturally complicated with the multiattribute (value) problem in which it is necessary to compare value vectors and determine trade offs. In recent times there has been an enormous upsurge of interest in the problem of multiattribute decision making under certainty and most methods essentially employ some way of logically ordering the solutions. The key concept in this context is that of nondominated set of solutions which is also known as the Pareto optimal set or the efficient frontier. A solution from the Pareto optimal set has the property that no other solution can dominate it. In other words no other solution can be found which is superior in all respects. However, for a pair of distinct solutions from the set, there will be trade off between attributes. Thus one solution will be worse off in one given attribute but superior in at least one other. In developing methods to deal with conflicting objectives the major concern is to devise ways of ranking the solutions. Cost benefit analysis (Frost) [15], indifference mapping (Rivett) [16], utility theory (Fishburn) [17], outranking relations (Roy) [18], hierarchy method (Saaty) [19], are different approaches to this essentially central problem of ranking the solutions. In multicriteria mathematical programming (Zionts) [20] both equation solving and ordering are required. We note that in devising the ranking of the solutions two key concepts of decision theory are always applied: these are concepts of dominance and transitivity. In short the ordering procedure is required to be logically consistent. Statements in natural languages when considered in their essential form express truth values TRUE or FALSE. The preferences stated by decision makers in natural language can thus be turned into logical forms for the purpose of ordering solutions.

The concepts of fuzzy sets and fuzzy reasoning have also been applied to the question of multiattribute decision making. In mathematical programming objectives and constraints are traditionally considered to be two different classes of linear relationships. The concepts of goal programming introduced some unifying feature. Subsequently Bellman and Zadeh [21] proposed an aproach which put together goals and constraints within the framework of fuzzy logic: "Goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined ... thus decision is a confluence of goals and constraints". Multiattribute decision making and goal seeking are also major themes in fuzzy systems and AI research [22].

Uncertainty Representation

Until the mid seventies uncertainty representation in decision problems was dominated by the Bayesian approach. The arguments in favour of this were as follows. The formal probability methods (the so called objective approach) require a large amount of data to be collected in (particularly) structured forms. Whereas in reality incomplete and unstructured historical data is available in many application domains such as medical diagnosis, geological prospecting, etc. The subjective Bayesian approach takes into consideration information presented simply as judgment, intuition or as practitioners' views. The difficulty of collecting exact information as required in the objective approach is thus avoided and the resulting model uses causal independence which leads to an additive form for the representation of uncertainties. Each term of this additive form has the effect of 'weight of evidence' of that particular component.

However, this is still based on a frequency representation of uncertainty. From an entirely behavioural point of view another type of uncertainty occurs which is based on similarity. Thus the degree of belief of a practitioner who is credited with a mental uncertainty model is couched in the language in which he expresses it. From the mid seventies the emergence of fuzzy set theory based on the concept of degree of membership of a set (connected closely with the degree of belief) has been used to represent such uncertainty or inexactness [23]. This is now accepted as a way of dealing with the "linguistic hedge". Medical diagnosis is a particularly relevant domain where both these methodologies have been applied side by side and there is a considerable debate (Phelps, Spiegelhalter) [24], [25] as to the context, rationale and appropriateness of these alternative approaches. From the viewpoint of system building, however, it is essential that a tool often called an 'inference network' (Benson) [26] should be present. The roles of these alternative approaches are in some sense complementary in constructing "inference networks".

INTERFACE ISSUES

Reductions in cost and the availability of computers as personal or professional work stations has led to substantial developments in the area of human computer interface. There has been a fundamental change in the respective roles of the problem owner, the analyst and the computer. Diagram 4 sets out the simple relationships connecting the computer, the analyst and the problem owner which have been accepted in practice until recently.

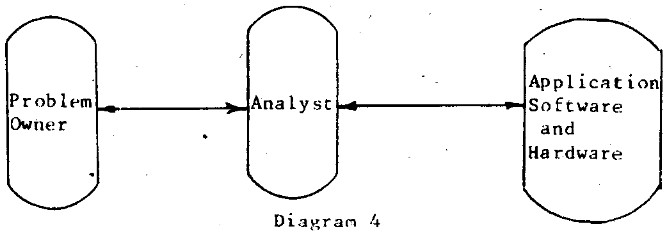

Diagram 4

In this mode of working the problem owner always had to go through the analyst who had the role of a broker and was involved in model definition as well as problem solving.

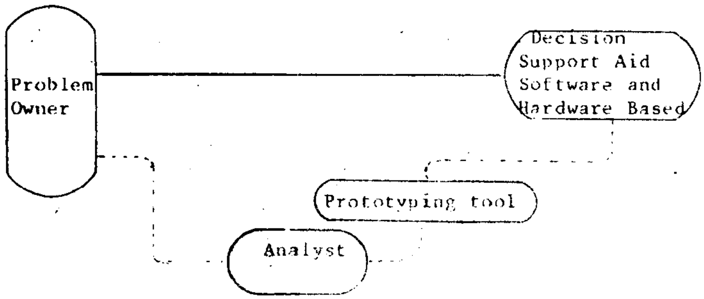

Diagram 5

The role of an IT analyst in recent times has been more of a system builder than a problem solver. Thus he uses a prototyping tool or an application development system to construct a decision support aid which the problem owner uses directly. In the ideal environment there is some communication between the analyst and the problem owner in the form of documentation, training and feedback. Thus in any particular application the role of the analyst is a nonessential one in contrast to that found earlier. In the past the management science community always felt uneasy about the distancing of the problem owner and the problem solution process. This has been referred to as the gap between the real world and the model itself. There has been various approaches towards bridging this gap. For instance Mumford [27] advocates (user) participative system design in the construction of application systems. Mathematical programming systems have been developed with high level model description languages [28], [29] in which the problem owners can directly construct and present their problems for solution. There are many examples [30] of query language facilities being supplied on top of a DBMS system. Many members of the AI community are of course most conscious of the interface issue. Two major themes of research have emerged in this area and these are natural language understanding and Icon driven intelligent front ends. In all these cases the primary purpose remains the same, that is, communication of problem owner's requirements to the model processing, inference making core of the application system. The Alvey program has identified intelligent front end as a key research theme [31]. Their statmement of the theme succinctly captures the main requirements and is set out below.

"A front end to an existing software package, for example a finite element package, or mathematical modelling system, which provides a user-friendly interface (a "human window") to packages which without it are too complex and/or technically incomprehensible to be accessible to many potential users. An intelligent front end builds a model of the user's problem through user-oriented dialogue mechanisms based on menus or quasi-natural language, which is then used to generate suitably coded instruction for the package."

The construction of such front ends would hopefully narrow the gap between models and the real world of the problem owners. The Japanese fifth generation research also starts from the premise that computers as we know today are not well set up to communicate naturally with humans [32]. One major aspect of this research is therefore to improve man machine communication. In concluding discussions relating to the interface issue, one should observe that the interface and modelling parts of a system do not fall in areas which are sharply separated from each other. In many systems interface and model support to some extent overlap with each other and are together focussed on the particular context of the problem.

CONCLUDING REMARKS

Computer assisted decision making falls in a very sensitive area where new developments in information technology, artificial intelligence and mathematical modelling play key roles in problem solving in business and industry. All the trends in research, development and experience of product implementation indicate that these methodologies will come closer together and will continue to exploit hardware, software and man-machine communications tools to produce progressively refined decision support products.

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