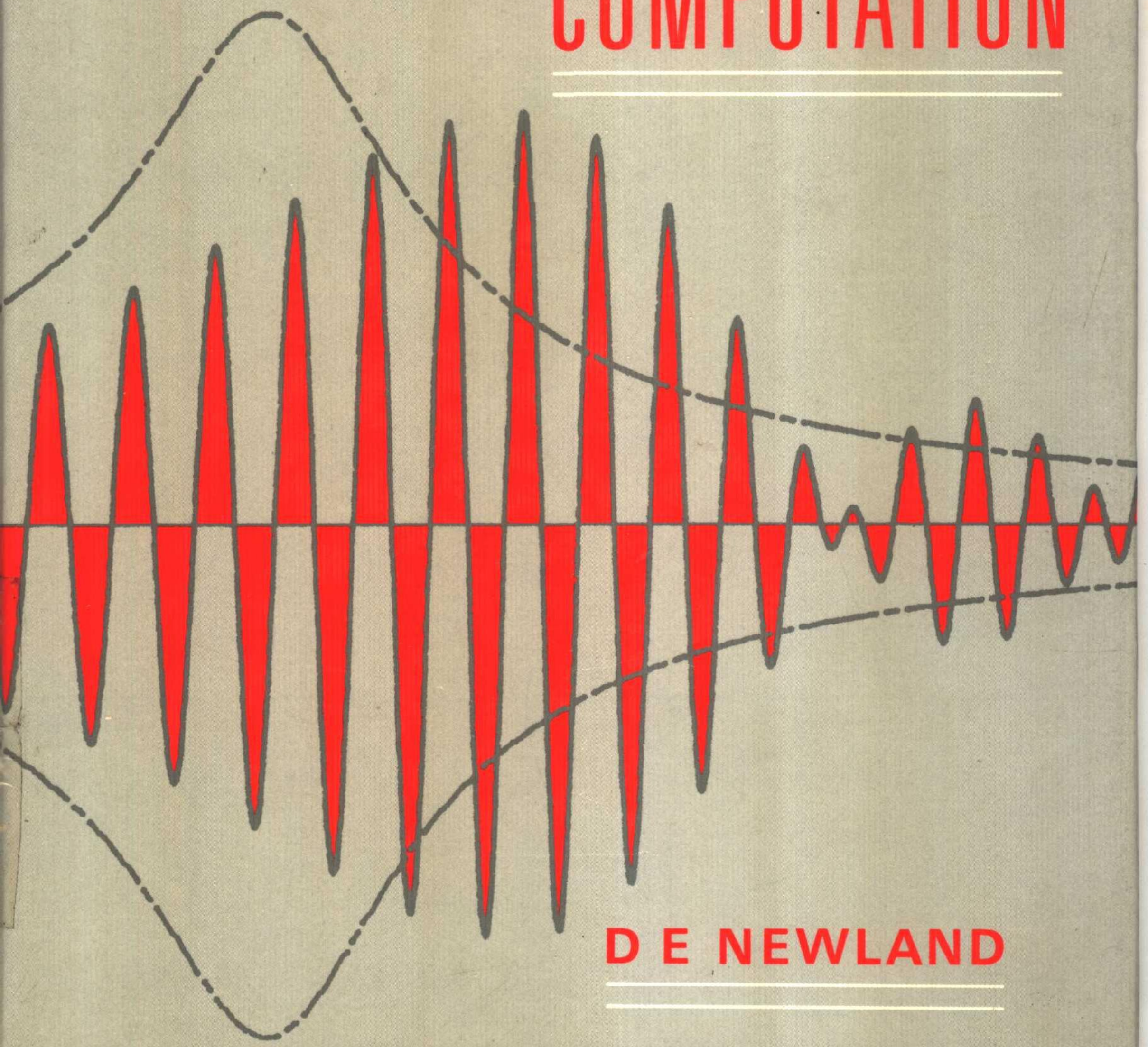
MECHANICAL VIBRATION ANALYSIS AND COMPUTATION



D E NEWLAND

Mechanical vibration analysis and computation

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Preface

This book has arisen from my experience as a teacher and consultant in mechanical vibrations. It is intended for senior undergraduates in mechanical engineering and the applied sciences and for postgraduates who are concerned with vibration problems. It is also designed to be a practical handbook for those who need to carry out vibration calculations; I hope that it will be useful for many engineers in industry.

Mechanical Vibration Analysis and Computation is a companion for my other book *Random Vibrations and Spectral Analysis* (second edition, Longman, 1984). Although each can be read separately, the two books go together and, where appropriate, topics in this book are cross-referenced to those in *Random Vibrations*. Taken together, the two books cover the whole field of mechanical vibration analysis, including random and non-linear vibrations and digital data analysis. Although the subject is a mathematical one, and in this new book there is a considerable amount of matrix analysis, I have tried to keep the mathematics as simple as possible; the emphasis is on practical applications of the theory in computation, rather than on rigorous proofs of theoretical results.

Although most readers will probably have had some preliminary introduction to vibration analysis, the basic ideas of single degree-of-freedom vibration theory are reviewed in Chapter 1. This is followed, in Chapter 2, by a detailed study of the frequency response of linear systems. The emphasis is on the interpretation of a system's frequency response in terms of its eigenvalues. In Chapter 3 more general response properties are considered, including alternative measures of damping, and the time for resonant oscillations to build up. These three chapters serve as a background for Chapter 4 which introduces the main ideas of matrix analysis, including the expression of a general linear vibration problem in terms of a set of first-order differential equations. This transformation is necessary in order to use modern computational methods which are organized for sets of first-order equations.

The physical interpretation of natural frequencies and mode shapes is discussed at length in Chapter 5 which includes applications to three major practical problems: the bending vibrations of self-supporting chimneys and masts, the torsional vibrations of a diesel-electric generator system, and the hunting oscillations of a railway vehicle. The general theory of linear vibration is usually developed on the assumption that the eigenvalues of the vibrating system are all different. Sometimes this assumption is not valid, and then there may no longer be a full set of independent modes. Such problems are considered in Chapter 6, and the analysis is illustrated by a torsional vibration problem which has three zero eigenvalues.

Most practical problems of vibration analysis require the application of computer routines to extract eigenvalues and eigenvectors, and Chapter 7 is about how these programs work. The numerical procedures for computing the eigenvalues of a real, unsymmetric matrix by the QR method are explained in detail and corresponding logical flow diagrams are given in the appendices. Although many workers will use library programs for vibration analysis, it is helpful to know how these programs operate and what their shortcomings are. These matters are discussed in detail in Chapter 7. The accompanying flow diagrams will allow those readers who wish to prepare their own programs to do so in whatever language is most convenient.

Chapters 8, 9 and 10 are about methods of numerical calculation for the vibration response of large linear systems. In Chapter 8, the frequency-response function matrix and impulse-response function matrix are expressed in general form in terms of the mass, stiffness and damping matrices. In Chapter 9 these functions are used to generate general input-output relations for a multi-degree-of-freedom linear system. In Chapter 10 methods of discrete calculation are described, including discrete calculations in both the frequency domain and the time domain, discrete finite-difference calculations, and response calculations by numerical integration. The numerical integration section deals with the fourth-order Runge–Kutta method, and a logical flow diagram for one integration step is given in Appendix 7.

When the mass and stiffness matrices of an undamped system are symmetric matrices, the general frequency-response and impulse-response equations are simplified. Chapter 11 is a detailed analysis of such a system. The analysis applies for the general case when there is no restriction on the eigenvalues, and so applies when there are repeated eigenvalues as well as when all the eigenvalues are distinct. Many structural vibration problems have symmetric matrices and the method of modal truncation that can be applied to problems in this class has important practical applications.

Chapters 12 and 13 are concerned with the analysis and vibration properties of continuous systems. General series solutions for frequency-response functions and impulse-response functions are derived and their application is illustrated by the analysis of the longitudinal vibration of an elastic column subjected to displacement inputs at one end. Beam and plate vibrations are considered in detail, including the vibration of beams when the effects of rotary inertia and shear deformation are included.

There is also a careful consideration of the application of Rayleigh's principle. This principle is used to calculate the whirling speed of a rotating cantilever shaft of variable area when subjected to external pressure. It is a problem with no known exact solution. The corollaries of Rayleigh's principle give some extremely useful practical results and they are explained carefully. These corollaries are not as well known as they deserve to be.

Finally, Chapter 14 is about parametric and nonlinear vibrations. It begins with the theory of the Mathieu equation and the exact calculation of stability boundaries, first without damping and then in the presence of damping. This leads to a consideration of autoparametric systems and the phenomenon of internal resonance. The Mathieu analysis is also used to investigate the stability of the forced periodic vibration of systems with nonlinear stiffness. This introduces nonlinear jump behaviour, when a periodic response suddenly jumps from one steady amplitude to another steady amplitude.

When a system with large nonlinearity is subjected to large-amplitude excitation, there may be conditions under which periodic motion is impossible. The system then undergoes chaotic motion which never repeats itself. This behaviour is illustrated by a typical time history computed by numerical integration of the equations of motion. However, for weakly nonlinear systems, the main engineering interest is in their periodic response, and several methods of finding the forced periodic response of weakly nonlinear systems are discussed. To illustrate how these methods may be applied, Galerkin's method is used to calculate the forced response of a system with a centrifugal pendulum vibration absorber.

In order to help readers who are studying the subject in detail for the first time and to guide lecturers who may be using the book, a suggested choice of topics for a first course on vibration analysis and computation is given after this preface.

The book includes a detailed list of references, and interested readers will be able to use these references to pursue many of the topics that are discussed here. Also there is a set of specially chosen problems intended to illustrate the theoretical ideas and methods in each chapter. The author hopes that teachers and students will find these problems helpful. Where possible, answers are given at the end of the problems, but in some cases the answers are too complicated or too lengthy (for example a graph or a numerical output from a computer calculation) so that it has not been practicable to print all the results here. A solutions manual is being prepared and it is hoped that this will be available shortly after the book is published. Details can be obtained from the author.

Lastly, it is necessary to say that no warranty is given that the calculation methods and procedures are free from error. The reader's attention is drawn particularly to the legal disclaimer printed below. In solving any new problem, it is always desirable to begin with special cases for which solutions are known and work up progressively to the full problem. Also, test calculations should be carried out on the full problem using extreme sets of parameter values (for example very large masses or very small stiffnesses) to test that the calculation procedure works properly on asymptotic cases of the full problem. It is possible that major errors will be found in the book and certain that there will be minor ones. The author will be most grateful for the notification of all of these and will be very pleased to hear from readers who care to write with their comments and with suggestions for improving later editions.

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D. E. NEWLAND
November, 1988

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Selected topics for a first course on vibration analysis and computation

All of Chapters 1, 2, 3 and 4.

All of Chapter 5. The intention is to illustrate the properties of eigenvalues and eigenvectors without requiring detailed knowledge of the examples.

Chapter 6 deals with the special properties of singular and defective matrices, and the details may be omitted on first reading. However, the reader should understand that a matrix cannot necessarily be diagonalized if its eigenvalues are repeated and should know the form of the Jordan matrix.

Chapter 7 describes numerical methods for the extraction of eigenvalues and related calculations. The details may be omitted other than to see in general terms how eigenvalues are computed.

All of Chapter 8 is needed except for the following sections:

Frequency-response functions when the eigenvector matrix is defective.
  Example 8.2: Frequency-response function for a system with repeated eigenvalues.
Impulse-response functions when the eigenvector matrix is defective.
Use of the matrix exponential function.
Application to the general response equation.

All of Chapter 9.

The discrete response methods in Chapter 10 should be included if time permits because they are the basis of practical calculations of vibration behaviour. However, the whole of this chapter may be omitted without interfering with the rest of the course.

Chapter 11 includes Lagrange's equations and a detailed analysis of the properties of systems with symmetric matrices, which are extremely important in structural vibration and in many mechanical problems. It is possible to begin reading at the section entitled 'Alternative proof of orthogonality when the eigenvalues are distinct' and to omit all the earlier part of this chapter if time does not allow the detailed analysis in the first part of the chapter to be absorbed.

All of Chapter 12 except the last section on general response equations.

Chapter 13 contains standard results on beams and plates in its first four sections, and the sections on Rayleigh's method are extremely important. The following sections may be omitted on a first reading:

Timoshenko beam.
- Effect of rotary inertia only.
- Effect of rotary inertia and shear together.
- Beam with a travelling load.

Example of the whirling of a shaft subjected to external pressure.

Chapter 14 includes much important material but time may prevent all of it being studied in detail. In that case, the sections to be included are:

Autoparametric systems.
Internal resonance.
Nonlinear jump phenomena.
Stability of forced vibration with numerical stiffness.
Numerical integration: chaotic response.

These can be studied alone provided that the equations of motion in the section 'Autoparametric systems' are derived by applying Newton's laws rather than by applying Lagrange's equations. There is reference back to the stability charts for the Mathieu equation, but if these are accepted without proof the material in the sections listed may be followed without further explanation.

A syllabus for a course based on this selection of material is given below together with the numbers of some appropriate problems selected from the list at the back of the book.

Response properties

	Chapter	Problems
Introduction	1	1.1–1.4
Frequency response; expansion in partial fractions; composite systems	2	2.1–2.6
Receptance and mobility graphs; measures of damping; forced vibration with hysteretic damping	3	3.1–3.4
Time for resonant oscillations to build up; acceleration through resonance	3	

Matrix analysis

First-order formulation; normal coordinates; calculation of eigenvalues and eigenvectors	4	
Natural frequencies and mode shapes; examples of the vibration of a chimney, diesel-electric generator, and railway bogie	5	5.1–5.5
Numerical methods for modal analysis; the QR transform (outline only)	7	
Frequency-response function and impulse-response function matrices and their computation	8	8.1, 8.3

Time-frequency transformations

Fourier transforms; time-domain to frequency-domain transformations; general input-output relations; discrete calculations (outline only)	9, 10	9.1, 9.2, 9.4

Systems with symmetric matrices

Special properties of systems with symmetric matrices	11	11.4, 11.6, 11.7

Continuous systems

Normal modes; impulse-response and frequency-response functions; application to the longitudinal vibration of a damped elastic column; properties of beams and plates	12, 13	12.1, 12.2, 12.6, 12.8, 13.1, 13.4

Approximate methods

Approximate calculations of natural frequency; the methods of Rayleigh and of Rayleigh–Ritz; corollaries of Rayleigh's principle	13	13.5, 13.8

Parametric and nonlinear effects

Autoparametric systems; internal resonance; nonlinear jump phenomena; stability of forced vibration with nonlinear stiffness; chaotic vibrations	14	14.1, 14.8, 14.9

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The first draft of this book grew out of my teaching notes for a final-year undergraduate course in vibrations at Cambridge. Several of my colleagues have been involved with this and other vibrations courses, particularly Dr R. W. Gregory, Dr J. D. Smith, Mr J. J. Thwaites and Dr J. Woodhouse, and I am grateful for their indirect contributions to this book. Dr Woodhouse kindly reviewed the final manuscript and made many helpful suggestions. Four of my former graduate students have also contributed significantly. The development of the subject-matter into the computational field stemmed from research done when I was at Imperial College with Mr W. Nelson Caldwell. Later this work was continued at Sheffield with Dr R. W. Aylward; more recently I have had the help of Dr David Cebon and Dr Hugh E. M. Hunt at Cambridge. I am very grateful to all of them. Also, I wish to mention particularly my secretary, Mrs Margaret Margereson, who prepared numerous typed drafts. She shouldered this extra burden with a ready smile however boring the work and however heavy her daily work schedule.

I should not have been able to complete this book without the opportunities provided by two spells of sabbatical leave from my job in Cambridge: a year during 1983–84 and a term in 1986. Without these opportunities to devote myself single-mindedly to the task, it would certainly not be finished now.

It would also not be finished had I not had the interest and support throughout of my wife, Tricia. The mental application that is required for writing a theoretical textbook can only be achieved by eschewing other more routine duties. I have been especially fortunate that Tricia has encouraged me throughout the endeavour and enabled me to bring this project to a successful conclusion.

D.E.N.

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