Network Analysis

Third Edition

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PRENTICE-HALL, INC., *Englewood Cliffs, New Jersey*

Library of Congress Cataloging in Publication Data

VAN VALKENBURG, MAC ELWYN,
Network analysis.

Bibliography: p.
1. Electric networks. I. Title.
TK454.2.V36 1974 621.319'2 73-12705
ISBN 0-13-611095-9

10 9 8 7 6 5 4 3 2

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY. LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*

Preface to the Third Edition

The objective in preparing this third edition of *Network Analysis* has been to retain the basic organization of previous editions but to make additions to reflect changes that have occurred in the teaching of the subject since the second edition appeared in 1964. The topics spanned by the subject of network analysis have increased in number and complexity. To explain my rationale in determining which topics to include and the depth to which each is treated, I must state that my perception of a course in network analysis is as a service course to the subjects that follow in the electrical engineering curriculum. The subject has long enjoyed that rolè, although not all practitioners are anxious to acknowledge that fact. Hence the topics to be considered should introduce material that will be useful in later courses taken by the student, and each should be studied in sufficient depth to make it easy to bridge the gap to the coverage of the next course. Thus the writing of equations to describe a network which is a model of a physical system permeates all of lumped system analysis and it should be treated in depth. The solution of the state equation, including an interpretation of the meaning of e^{At} where A is a matrix can be postponed to a later course.

One of the innovations of this third edition is Appendix E which contains a detailed listing of appropriate topics for computer homework assignments or topics for an associated software laboratory, including a detailed listing of textbook references. At the end of chapters, where appropriate, specific suggestions are made as to computer exercises which will support the topics covered in that chapter. A number of factors are involved in the decision to provide the information in this form, rather than to introduce numerical method material or to present computer printout to illustrate possible computer operations. The most important of these is that the actual use of the computer by the student is greatly influenced by the library of subroutines available to the student at a computer center. Computer center libraries continue to be in a state of rapid change and there is little uniformity from one to the next.

State variables have assumed new importance and are introduced. Those who will become proficient in the analysis of networks will learn *all* of the methods. However, it does not appear likely that the state-variable formulation of equations will replace nodal analysis, for example. It is generally good pedagogy to learn one or two methods well before adding more methods to a "bag of tricks." For the beginner, the traditional node and loop formulations are recommended.

Other additions include a treatment of Tellegen's theorem. The simple elegance of Tellegen's contribution and the wide variety of applications to which it may be applied for new insight and clarity continue to amaze engineers. A new treatment of the Nyquist criterion appears in this edition, after having been omitted from the second edition. The reason for this change is the new emphasis on active networks with their attendant stability problems, and the insight that is provided in understanding these networks through the Nyquist criterion.

This edition contains a significant number of new problems and the revision or updating of others. In addition, the number of solutions to problems given in Appendix G has been increased to make the book better suited for independent study.

The indebtedness of the author to his students, colleagues, and former teachers is great. To these and all others who have given help and encouragement in the preparation of the book, I offer an inadequate acknowledgment of appreciation. In particular, I am indebted to those users of the book who have been kind enough to send me their impressions of the first two editions and suggestions for an improved third edition. Finally, it is a pleasure to record that it has been enjoyable working with Virginia Huebner of the College Book Editorial Department of Prentice-Hall, and to express my gratitude to Evelyn, my wife, who gave invaluable assistance in proofreading the manuscript.

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Preface to the Second Edition

This book is designed for use in an introductory course or a second course in electric network analysis. The student is assumed to have the mathematical sophistication usually associated with the completion of a course in the calculus.

Teaching experience in the decade that has passed since the preparation of the first edition of the book has reinforced my conviction that the student's introduction to network analysis should begin with the so-called "transient" case and proceed to the sinusoidal steady state and related topics. I do not argue which order is the more basic or important, but rather that the beginning student will attain real understanding more readily by this route. Thus the basic organization of the second edition remains essentially that of the first:

The first three chapters of the book are concerned with an introduction of the elements that serve to model electrical devices, with definitions pertaining to networks, and to the formulation of the Kirchhoff equilibrium equations. There follows a study of the behavior of networks in terms of natural modes or the natural frequencies of response due to arbitrary excitation. The Laplace transformation provides the means by which this natural behavior of the network is unified with the characteristics of the excitation (or signal) with each represented by a transform and then studied in terms of the poles and zeros of the transform. Once this is accomplished, the case of sinusoidal excitation under steady-state operation is introduced. This, in turn, leads to the study of a number of topics important to the electrical engineer such as Bode plots, average power, insertion loss, and the various signal spectra. The early introduction of the Laplace transformation without proof has not presented any difficulties for students, in my experience. I believe that Whitehead* was correct in observing that, "*. . . it is not essential that proof of the truth should* constitute the first introduction to the idea."

At the time the first edition was prepared, it was felt that it was necessary to demonstrate the utility of the pole-and-zero approach by including material on a number of important applications such as *LC* one-port networks, image-parameter filters, amplifier-networks of the stagger-tuned variety, and some topics relating to automatic control. The pole-and-zero approach has since become firmly established in the curriculum and these chapters have accordingly been replaced. More examples are provided throughout. Some topics are treated in more depth than in the first edition including convolution, sinusoidal steady state analysis, the Routh-Hurwitz criterion, Fourier series and the Fourier integral. New topics added to this edition include the two-port parameters, complex loci and Bode plots, average power, power transfer, and insertion loss. Three appendices provide coverage of complex numbers, matrices, and magnitude and frequency scaling.

A few words are necessary concerning notation and conventions. The units for element values are assumed to be given in ohms, henrys, farads, volts, and amperes unless otherwise stated—both in the text and in the figures. I have used limits on integrals when I felt that it would add to the clarity or student understanding of the presentation; otherwise I have preferred to regard the integral sign as a symbol representing a rather detailed word statement and implying that the limits over which integration is performed must be known. Similarly, I have used $i(t)$ rather than i, and $I(s)$ rather than I when it seemed important to identify the variable. In every case, capital letters are used for variables in the frequency domain and lower-case letters for the time domain.

A number of decisions have been made painfully. I have used *order* in discussing algebraic equations relating to the time domain and then switched to ***degree*** when discussing algebraic equations in the frequency domain. This usage agrees with present technical practice by engineers; the use of either one or the other exclusively gives rise to strange-sounding expressions. For the symbol for transfer functions for two-port networks, I have used the subscript order 12, as in Z_{12}, knowing that some teachers have strong preferences for Z_T, Z^T, or Z_{21}. It will be clear from the text and the figures which

* Alfred North Whitehead, *The Aims of Education*, paperback edition by Mentor Books, New York, 1949, p. 15.

particular transfer function is meant so that there is little real danger of confusion. I hope that no great difficulty will arise in interchanging 1 and 2 or in replacing 12 by T. I have felt helpless in trying to give full credit to all contributors in a field in the *Further Reading* sections at the end of each chapter, and so have chosen to select only references that supplement the material given or provide an alternative approach that might lead to better understanding of the subject.

One of the rewards in writing a textbook such as this is that it provides an excuse for discussing and testing ideas with both colleagues and students. I must express great indebtedness to students who have, wittingly or unwittingly, helped me in fixing the order of presentation and the pattern of emphasis. Most of the revision for the second edition was accomplished while I was a member of the visiting faculties at the University of California, Berkeley and the University of Colorado. I am indebted to friends at these two schools as well as at the University of Illinois for most helpful discussions.

It is a pleasure to thank the following who have given valuable assistance in one or both of the editions: Don A. Baker of Los Alamos, Doran Baker of Utah State University, Joseph Chen of IBM, Jose B. Cruz, Jr. of Illinois, L. Dale Harris of the University of Utah, Shlomo Karni of the University of New Mexico, Wan Hee Kim of Columbia University, Jack Kobayashi of Hughes Aircraft Company, Franklin F. Kuo of Bell Telephone Laboratories, Philip C. Magnusson of Oregon State University, Wataru Mayeda of Illinois, William R. Perkins of Illinois, Ronald A. Rohrer of Illinois, Thomas M. Stout of Thompson-Ramo-Wooldridge, Glen Wade of Cornell University, and Philip Weinberg of Bradley University. Herbert M. Barnard and Edwin C. Jones, Jr. made valuable contributions in improving the text and checking the galley proofs. Editorial assistance from W. L. Everitt and Robert W. Newcomb is also acknowledged. Finally, I express my appreciation to my wife Evelyn and children for their patience and understanding during the preparation of the book.

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Development of the Circuit Concept 1

1-1. INTRODUCTION

One of the characteristics of the scientific method is the continual bringing together of a wide variety of facts to fit into a simple, understandable theory that will account for as many observations as possible. The name *conceptual scheme* has been used by the American chemist and educator James Conant for the theory or picture that results.[1] Perhaps the most familiar conceptual scheme to students of science and engineering is that of the atomic theory from which we take our pictures of the electron and of electric charge. Other important conceptual schemes are conservation of energy and conservation of charge.

Although electricity and magnetism were recognized early in the history of man—the charging of amber by friction, the use of the lodestone in navigation—it was not until the nineteenth century that significant progress was made in developing a conceptual scheme. The discovery by Galvani and Volta about 1800 that electricity could be produced by chemical means greatly simplified experimentation. Important discoveries were made in a relatively short interval of time after Volta. In 1820, Oersted identified the magnetic field with current, and Ampère measured the force caused by the current. In 1831 Faraday and, independently, Henry discovered electric induction.

[1] James B. Conant, *Science and Common Sense* (Yale University Press, New Haven, Conn. 1951).

These and other experiments were brought together to form a successful conceptual scheme by the English physicist James Clerk Maxwell in 1873. In Maxwell's equations, as the scheme has come to be known, all electric and magnetic phenomena are explained in terms of fields resulting from charge and current. The success of Maxwell's conceptual scheme is evidenced by the persistent agreement of results deduced from Maxwell's equations with observations for a period of over 100 years.

In view of Maxwell's success, why do we now embark upon a study of *another* conceptual scheme for the same phenomena, the electric circuit? Equally important as a question, how are the two concepts related? The answer to the first of our questions is the practical utility of the circuit concept. As a practical matter, we are not often interested in fields so much as we are in voltages and currents. The circuit concept favors analysis in terms of voltage and current from which other quantities—such as charge, fields, energy, power, etc.—can be computed if desired. The answer to our second question will require a longer answer and justification. Briefly, circuit concepts arise from the same basic experimental facts as do Maxwell's equations. However, the circuit involves approximations that are not included in the more general concept of field theory. It is important that we understand the nature of these approximations—the limitations of circuit theory—before we develop our subject.

It will be helpful to define the function of the circuit in terms of two basic building blocks: charge and energy. We regard charge and energy as the least common denominators in describing electrical phenomena, the primitive quantities in terms of which we can build our conceptual scheme of the electric circuit. A physical circuit is a system of interconnected apparatus. Here we use the word *apparatus* to include sources of energy, connecting wires, components, loads, etc. A circuit functions to transfer and transform energy. Energy transfer is accomplished by charge transfer. In the circuit, energy is transferred from a point of supply (the source) to a point of transformation or conversion called the load (or sink). In the process, the energy may be stored.

1-2. CHARGE AND ENERGY

Thales of Greece is credited with the discovery, about 600 B.C., that when briskly rubbed with a piece of silk or fur, amber becomes "electrified" and is capable of attracting small pieces of thread. This same technique for producing electricity was used centuries later by Coulomb in France (and independently by Cavendish in England) in establishing the inverse square law of attraction of charged bodies.

Our present-day understanding of the nature of charge is based on the conceptual scheme of the atomic theory. We picture the atom as composed of a positively charged nucleus surrounded by negatively charged electrons. In the neutral atom, the total charge of the nucleus is equal to the total charge of the electrons. When electrons are removed from a substance, that substance becomes positively charged. A substance with an excess of electrons is negatively charged.

The basic unit of charge is the charge of the electron. The MKS unit of charge is the *coulomb.* The electron has a charge of 1.6021×10^{-19} coulomb.

The phenomenon of transferring charge from one point in a circuit to another is described by the term *electric current.* An electric current may be defined as the time rate of net motion of electric charge across a cross-sectional boundary. A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time.

In equation form, the current[2] is

$$i = \frac{dq}{dt} \tag{1-1}$$

If the charge q is given in coulombs and the time t is measured in seconds, the current is measured in *amperes* (after the French physicist André Ampère). Since the electron has a charge of 1.6021×10^{-19} coulomb, it follows that a current of 1 ampere corresponds to the motion of $1/(1.6021 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons past any cross section of a path in 1 second.

In terms of the atomic-theory conceptual scheme, all substances are pictured as made up of atoms. In a solid, some electrons are relatively free of the nucleus; the attractive forces on these electrons are exceedingly small. Such electrons are distinguished by the name *free electrons.* An electric current is the time rate of flow of these free electrons, passing from one atom to the next as pictured in Fig. 1-1.

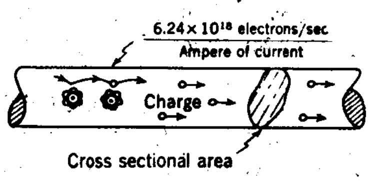

Fig. 1-1. Representation of the motion of charge in a conductor.

In some materials there are many free electrons, so that large currents are easily attained. Such materials are known as *conductors.* Most metals and some liquids are good conductors. Materials with relatively few free electrons are known as *insulators.* Common insulating materials include glass, mica, plastics, etc. Other materials

[2]The symbol i for current is taken from the French word *intensité.*

called *semiconductors* play a significant role in electronics. Two common semiconductors are germanium and silicon.

There is a common misconception that since some electric waves propagate at approximately the speed of light the electrons in a conductor travel with this same velocity. The actual mean velocity of free electron drift is but a few millimeters per second! (See Prob. 1-2 for a numerical example.)

Another conceptual scheme upon which our thinking is based is the *conservation of energy*. By our training in the methods of science, we immediately become suspicious of any scheme that claims to create energy. The law of conservation of energy states that energy cannot be created, nor destroyed, but that it can be converted in form. Electric energy is energy converted from some other form. There are a number of ways in which this is accomplished. Some of them are as follows:

(1) *Electromechanical energy conversion*. The rotating generator, patterned after the invention of Faraday in 1831, produces electrical energy from mechanical energy of rotation. Usually the mechanical energy is converted from thermal energy by a turbine and, in turn, the thermal energy is converted from chemical energy by burning fossil fuel or from nuclear fuel. Sometimes the conversion is from hydraulic energy by hydroelectric generation.

(2) *Electrochemical energy conversion*. Electric batteries produce electric energy by the conversion of chemical energy. A potentially important use of such batteries is in the electric car. Fuel cells are in this general classification.

(3) *Magnetohydrodynamics (MHD) energy conversion*. These devices generate electric energy from the mechanical energy of a high velocity ionized gas.

(4) *Photovoltaic energy conversion*. A class of devices are able to convert light energy directly into electric energy. The best known device of this type is the *solar cell*.

The function of each of these different sources of electric energy is the same in terms of energy and charge. In one form of battery, for example, two metallic electrodes—one of zinc and one of copper—are immersed in dilute sulfuric acid. The formation of zinc and copper ions causes negative charge to accumulate at the electrodes. Energy is supplied to the charge by the difference in the energy of ionization of zinc and copper in the chemical reaction. Once the battery circuit is closed by an external connection, as shown in Fig. 1-2, the chemical energy is expended as work for each unit of charge in transporting the charge around the external circuit. The quantity "energy per unit