A SECOND COURSE IN STOCHASTIC PROCESSES

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SAMUEL KARLIN
STANFORD UNIVERSITY

HOWARD M. TAYLOR
CORNELL UNIVERSITY

ACADEMIC PRESS, INC.
Harcourt Brace Jovanovich, Publishers

Boston San Diego New York
London Sydney Tokyo Toronto

This book is printed on acid-free paper. ∞

ACADEMIC PRESS, INC.
1250 Sixth Avenue, San Diego, California 92101

United Kingdom Edition published by
ACADEMIC PRESS LIMITED
24-28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Karlin, Samuel, Date.
A second course in stochastic processes.

Continues A first course in stochastic processes.
Includes bibliographical references and index.
1. Stochastic processes. I. Taylor, Howard M., joint author. II. Title.
QA274.K372 519.2 80-533
ISBN 0-12-398650-8

PRINTED IN THE UNITED STATES OF AMERICA

91 92 10 9 8 7 6 5

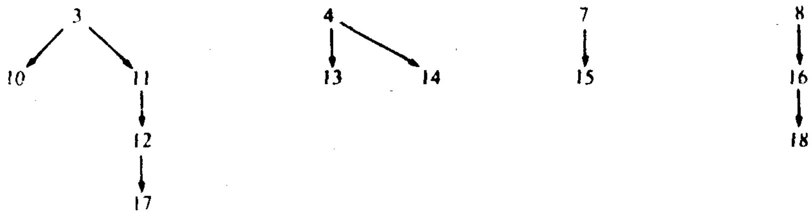
PREFACE

This *Second Course* continues the development of the theory and applications of stochastic processes as promised in the preface of *A First Course.* We emphasize a careful treatment of basic structures in stochastic processes in symbiosis with the analysis of natural classes of stochastic processes arising from the biological, physical, and social sciences.

Apart from expanding on the topics treated in the first edition of this work but not incorporated in *A First Course,* this volume presents an extensive introductory account of the fundamental concepts and methodology of diffusion processes and the closely allied theory of stochastic differential equations and stochastic integrals. A multitude of physical, engineering, biological, social, and managerial phenomena are either well approximated or reasonably modeled by diffusion processes; and modern approaches to diffusion processes and stochastic differential equations provide new perspectives and techniques impinging on many subfields of pure and applied mathematics, among them partial differential equations, dynamical systems, optimal control problems, statistical decision procedures, operations research, studies of economic systems, population genetics, and ecology models.

A new chapter discusses the elegant and far-reaching distributional formulas now available for a wide variety of functionals (e.g., first-passage time, maximum, order statistics, occupation time), of the process of sums of independent random variables. The identities, formulas, and results in this chapter have important applications in queueing and renewal theory, for statistical decision procedures, and elsewhere.

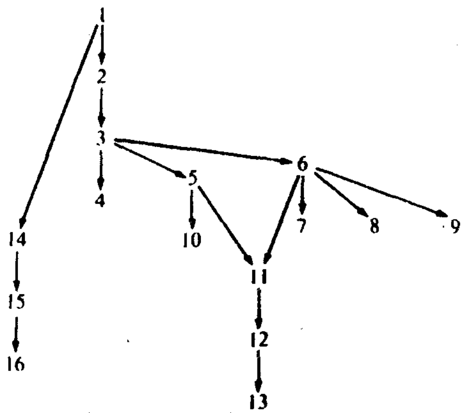
The logical dependence of the chapters in *A Second Course* is shown by the diagram below (consult also the preface to *A First Course* on the relationships of Chapters 1–9).



The book can be coupled to *A First Course* in several ways, depending on the background and interests of the students. The discussion of Markov chains in *A First Course* can be supplemented with parts of the more advanced Chapters 10–12, and 14. The material on fluctuation theory of sums of independent random variables (Chapters 12 and 17), perhaps supplemented by some parts of the chapter on queueing processes (Chapter 18), may be attractive and useful to students of operations research and statistics. Chapter 16, on compounding stochastic processes, is designed as an enticing introduction to a hierarchy of relevant models, including models of multiple species population growth, of migration and demographic structures, of point processes, and compositions of Poisson processes (Lévy processes).

We strongly recommend devoting a semester to diffusion processes (Chapter 15). The dependence relationships of the sections of Chapter 15 are diagrammed below. Section 1 provides a generalized description of various characterizations of diffusion. The examples of Section 2, which need not be absorbed in their totality, are intended to hint at the rich diversity of natural models of diffusion processes; the emphasis on biological examples reflects the authors' personal interests, but diffusion models abound in other sciences as well. Sections 3–5 point up the utility and tractability of diffusion process analysis. Section 6 takes up the boundary classification of one-dimensional diffusion processes; Section 7, on the same topic, is more technical. Section 8 provides constructions of diffusions with different types of boundary behavior. Sections 9 and 10 treat a number of topics motivated by problems of population genetics and statistics. The formal (general) theory of Markov processes with emphasis on applications to diffusions is elaborated in Sections 11 and 12. Section 13 exhibits the spectral representations for several classical diffusion models, which are of some interest because of their connections with classical special functions. The key concepts, a host of examples, and some methods of stochastic differential equations and stochastic integrals are introduced in Sections 14–16.

Chapter 15



As noted in the earlier prefaces, we have drawn freely on the thriving literature of applied and theoretical stochastic processes without citing specific articles. A few representative books are listed at the end of each chapter and may be consulted profitably as a guide to more advanced material.

We express our gratitude to Stanford University, the Weizmann Institute of Science in Israel, and Cornell University for providing a rich intellectual environment and facilities indispensable for the writing of this text. The first author is grateful for the continuing grant support provided by the National Science Foundation and the National Institutes of Health that permitted an unencumbered concentration on a number of the concepts of this book and on its various drafts. We are also happy to acknowledge our indebtedness to many colleagues who have offered constructive criticisms, among them Professor M. Taqqu of Cornell, Dr. S. Tavaré of the University of Utah, Professors D. Iglehart and M. Harrison of Stanford, and Professor J. Kingman of Oxford. Finally, we thank our students P. Glynn, E. Cameron, J. Raper, R. Smith, L. Tierney, and P. Williams for their assistance in checking the problems, and for their helpful reactions to early versions of Chapter 15.

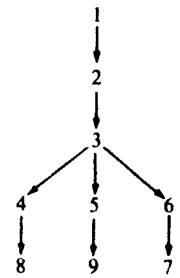
PREFACE TO *A FIRST COURSE*

The purposes, level, and style of this new edition conform to the tenets set forth in the original preface. We continue with our tack of developing simultaneously theory and applications, intertwined so that they refurbish and elucidate each other.

We have made three main kinds of changes. First, we have enlarged on the topics treated in the first edition. Second, we have added many exercises and problems at the end of each chapter. Third, and most important, we have supplied, in new chapters, broad introductory discussions of several classes of stochastic processes not dealt with in the first edition, notably martingales, renewal and fluctuation phenomena associated with random sums, stationary stochastic processes, and diffusion theory.

Martingale concepts and methodology have provided a far-reaching apparatus vital to the analysis of all kinds of functionals of stochastic processes. In particular, martingale constructions serve decisively in the investigation of stochastic models of diffusion type. Renewal phenomena are almost equally important in the engineering and managerial sciences especially with reference to examples in reliability, queueing, and inventory systems. We discuss renewal theory systematically in an extended chapter. Another new chapter explores the theory of stationary processes and its applications to certain classes of engineering and econometric problems. Still other new chapters develop the structure and use of diffusion processes for describing certain biological and physical systems and fluctuation properties of sums of independent random variables useful in the analyses of queueing systems and other facets of operations research.

The logical dependence of chapters is shown by the diagram below. Section 1 of Chapter 1 can be reviewed without worrying about details. Only Sections 5 and 7 of Chapter 7 depend on Chapter 6. Only Section 9 of Chapter 9 depends on Chapter 5.



An easy one-semester course adapted to the junior-senior level could consist of Chapter 1, Sections 2 and 3 preceded by a cursory review of Section 1, Chapter 2 in its entirety, Chapter 3 excluding Sections 5 and/or 6, and Chapter 4 excluding Sections 3, 7, and 8. The content of the last part of the course is left to the discretion of the lecturer. An option of material from the early sections of any or all of Chapters 5–9 would be suitable.

The problems at the end of each chapter are divided into two groups: the first, more or less elementary; the second, more difficult and subtle.

The scope of the book is quite extensive, and on this account, it has been divided into two volumes. We view the first volume as embracing the main categories of stochastic processes underlying the theory and most relevant for applications. In *A Second Course* we introduce additional topics and applications and delve more deeply into some of the issues of *A First Course*. We have organized the edition to attract a wide spectrum of readers, including theorists and practitioners of stochastic analysis pertaining to the mathematical, engineering, physical, biological, social, and managerial sciences.

The second volume of this work, *A Second Course in Stochastic Processes*, will include the following chapters: (10) Algebraic Methods in Markov Chains; (11) Ratio Theorems of Transition Probabilities and Applications; (12) Sums of Independent Random Variables as a Markov Chain; (13) Order Statistics, Poisson Processes, and Applications; (14) Continuous Time Markov Chains; (15) Diffusion Processes; (16) Compounding Stochastic Processes; (17) Fluctuation Theory of Partial Sums of Independent Identically Distributed Random Variables; (18) Queueing Processes.

As noted in the first preface, we have drawn freely on the thriving literature of applied and theoretical stochastic processes. A few representative references are included at the end of each chapter; these may be profitably consulted for more advanced material.

We express our gratitude to the Weizmann Institute of Science, Stanford University, and Cornell University for providing a rich intellectual environment and facilities indispensable for the writing of this text. The first author is grateful for the continuing grant support provided by the Office of Naval Research that permitted an unencumbered concentration on a number of the concepts and drafts of this book. We are also happy to acknowledge our indebtedness to many colleagues who have offered a variety of constructive criticisms. Among others, these include Professors P. Brockwell of La Trobe, J. Kingman of Oxford, D. Iglehart and S. Ghurye of Stanford, and K. Itô and S. Stidham, Jr. of Cornell. We also thank our students M. Nedzela and C. Macken for their assistance in checking the problems and help in reading proofs.

SAMUEL KARLIN
HOWARD M. TAYLOR

PREFACE TO FIRST EDITION

Stochastic processes concern sequences of events governed by probabilistic laws. Many applications of stochastic processes occur in physics, engineering, biology, medicine, psychology, and other disciplines, as well as in other branches of mathematical analysis. The purpose of this book is to provide an introduction to the many specialized treatises on stochastic processes. Specifically, I have endeavored to achieve three objectives: (1) to present a systematic introductory account of several principal areas in stochastic processes, (2) to attract and interest students of pure mathematics in the rich diversity of applications of stochastic processes, and (3) to make the student who is more concerned with application aware of the relevance and importance of the mathematical subleties underlying stochastic processes.

The examples in this book are drawn mainly from biology and engineering but there is an emphasis on stochastic structures that are of mathematical interest or of importance in more than one discipline. A number of concepts and problems that are currently prominent in probability research are discussed and illustrated.

Since it is not possible to discuss all aspects of this field in an elementary text, some important topics have been omitted, notably stationary stochastic processes and martingales. Nor is the book intended in any sense as an authoritative work in the areas it does cover. On the contrary, its primary aim is simply to bridge the gap between an elementary probability course and the many excellent advanced works on stochastic processes.

Readers of this book are assumed to be familiar with the elementary theory of probability as presented in the first half of Feller's classic *Introduction to*

Probability Theory and Its Applications. In Section 1, Chapter 1 of my book the necessary background material is presented and the terminology and notation of the book established. Discussions in small print can be skipped on first reading. Excercises are provided at the close of each chapter to help illuminate and expand on the theory.

This book can serve for either a one-semester or a two-semester course, depending on the extent of coverage desired.

In writing this book, I have drawn on the vast literature on stochastic processes. Each chapter ends with citations of books that may profitably be consulted for further information, including in many cases bibliographical listings.

I am grateful to Stanford University and to the U.S. Office of Naval Research for providing facilities, intellectual stimulation, and financial support for the writing of this text. Among my academic colleagues I am grateful to Professor K. L. Chung and Professor J. McGregor of Stanford for their constant encouragement and helpful comments; to Professor J. Lamperti of Dartmouth, Professor J. Kiefer of Cornell, and Professor P. Ney of Wisconsin for offering a variety of constructive criticisms; to Dr. A. Feinstein for his detailed checking of substantial sections of the manuscript, and to my students P. Milch, B. Singer, M. Feldman, and B. Krishnamoorthi for their helpful suggestions and their assistance in organizing the exercises. Finally, I am indebted to Gail Lemmond and Rosemarie Stampfel for their superb technical typing and all-around administrative care.

SAMUEL KARLIN

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