# INTERMEDIATE QUANTUM MECHANICS

## Third Edition

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# EDITORS' FOREWORD

Everyone concerned with the teaching of physics at the advanced undergraduate or graduate level is aware of the continuing need for a modernization and reorganization of the basic course material. Despite the existence today of many good textbooks in these areas, there is always an appreciable time-lag in the incorporation of new viewpoints and techniques which result from the most recent developments in physics research. Typically these changes in concepts and material take place first in the personal lecture notes of some of those who teach graduate courses. Eventually, printed notes may appear, and some fraction of such notes evolve into textbooks or monographs. But much of this fresh material remains available only to a very limited audience, to the detriment of all. Our series aims at filling this gap in the literature of physics by presenting occasional volumes with a contemporary approach to the classical topics of physics at the advanced undergraduate and graduate level. Clarity and soundness of treatment will, we hope, mark these volumes, as well as the freshness of the approach.

Another area in which the series hopes to make a contribution is by presenting useful supplementing material of well-defined scope. This may take the form of a survey of relevant mathematical principles, or a collection of reprints of basic papers in a field. Here the aim is to provide the instructor with added flexibility through the use of supplements at relatively low cost.

The scope of both the lecture notes and supplements is somewhat different from the "Frontiers in Physics" series. In spite of wide variations from institution to institution as to what comprises the basic graduate course program, there is a widely accepted group of "bread and butter" courses that deal with the classic topics in physics. These include: Mathematical methods of physics, electromagnetic theory, advanced dynamics, quantum mechanics, statistical mechanics, and frequently nuclear physics and/or solid-state physics. It is chiefly these areas that will be covered by the present series. The listing is perhaps best described as including all advanced undergraduate and graduate courses which are at a level below seminar courses dealing entirely with current research topics.

Finally, because the series represents something of an experiment on the part of the editors and the publisher, suggestions from interested readers as to format, contributors, and contributions will be most welcome.

DAVID PINES

# PREFACE TO THE FIRST EDITION

This book is intended to serve as a text for a second course in quantum mechanics, for graduate students in both theoretical and experimental physics. It is assumed that the student has a knowledge of the principles of quantum mechanics, equivalent to the first eight chapters of Schiff's *Quantum Mechanics,* or the entire book of Merzbacher. I believe that the general exposition of the theory as given in these books should be followed by a discussion of the applications to problems in which the basic physics is essentially known and well understood, notably the structure of atoms and the theory of atomic collisions, so that the solidity of the theory becomes apparent. After this, the student will be better prepared to study nuclear physics, where the forces are unknown, or solid-state physics, where the physical approximations are often tentative. I have stressed the connection with experimental information and with the physical picture rather than the formal development of the theory. Some recent books have stressed all too much the formal side.

Good books are available for study at this level, e.g., Condon and Shortley, *Theory of Atomic Spectra;* Slater, *Quantum Theory of Atomic Structure;* Mott and Massey, *Theory of Atomic Collisions;* Heitler, *Quantum Theory of Radiation.* However these books are mainly intended for the expert, or at least the student who wishes to specialize in one particular field of quantum mechanics. The present book is intended to give to the graduate student in physics enough knowledge in at least one of the fields, namely, atomic structure, so that he can then intelligently follow discussions on various coupling schemes in atoms, nuclei, and fundamental particles. It will give him a working knowledge of Clebsch-Gordan coefficients. It also gives a detailed treatment of optical transition probabilities, including quantitative calculation.

H. A. BETHE

*Ithaca, New York*
*January 1964*

# PREFACE TO THE SECOND EDITION

In preparing this Second Edition, the chapters on theory of atomic structure have essentially been retained from the First Edition, although we hope to have made some improvements in presentation. We have added to the book a fairly extensive section on collision theory. Some of this material cannot be found in other textbooks. For a more extensive treatment, the reader is referred to the classic book by Mott and Massey, *Theory of Atomic Collisions.*

We wish to point out that the present edition, like the First Edition, is not intended to be used as the sole textbook discussing the fundamental theory in a second course of quantum mechanics. Rather, it is intended as a supplement to a text stressing fundamental theory, such as Messiah's *Quantum Mechanics*, Volume II. A few chapters on field theory which were in the First Edition have been omitted because this subject is covered adequately in other books.

H. A. BETHE
R. W. JACKIW

*April 1968*

# PREFACE TO THE THIRD EDITION

In our Third Edition we have made revisions, which were suggested during the almost quarter century that *Intermediate Quantum Mechanics* has been in print and in use. They are mainly of pedagogical nature, reflecting our own teaching experience and the many useful comments we have received from colleagues, for which we are very grateful. Also a brief description of recent progress in the treatment of electron correlations is included. These new developments, based on the Buckner, Bethe, Goldstone method in nuclear physics, help substantially in calculating accurate atomic energy levels.

H.A. BETHE
R. JACKIW

*August, 1985; Aspen, Colorado*

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# Part I

# THEORY OF ATOMIC STRUCTURE

There are at least three good reasons for undertaking a careful study of atomic structure. First, on the basis of the quantum theory every known feature of the electronic structure of atoms can be explained. Knowledge of this structure is important for chemistry, solid state physics, spectroscopic determination of nuclear properties (hyperfine structure, etc.), and many other applications. The quantitative validity of these explanations is limited only by computational difficulties. Second, the excellent agreement between theoretical and experimental results in the tremendously wide range of atomic phenomena provides a crucial test for the validity of quantum mechanics. Finally, the theory of atomic structure is a convenient "theoretical laboratory" in which one can become acquainted with many physical ideas and mathematical tools which are relevant to other branches of physics. Some aspects of the theory of nuclear structure, for example, parallel quite closely atomic theory.