HANDBOOK OF BOOLEAN ALGEBRAS

VOLUME 1

SABINE KOPPELBERG

Edited by

J. DONALD MONK

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Freie Universität Berlin

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Introduction to the Handbook

The genesis of the motion of a Boolean algebra (BA) is, of course, found in the works of George Boole; but his works are now only of historical interest – cf. HAILPERIN [1981] in the bibliography (elementary part). The notions of Boolean algebra were developed by many people in the early part of this century – Schröder, Löwenheim, etc. usually working with the concrete operations union, intersection, and complementation. But the abstract notion also appeared early, in the works of Huntington and others. Despite these early developments, the modern theory of BAs can only be considered to have started in the 1930s with works of M.H. Stone and A. Tarski. Since then there has been a steady development of the subject.

The present Handbook treats those parts of the theory of Boolean algebras of most interest to pure mathematicians: the set-theoretical abstract theory and applications and relationships to measure theory, topology, and logic. Aspects of the subject *not* treated here are discussion of axiom systems for BAs, finite Boolean algebras and switching circuits, Boolean functions, Boolean matrices, Boolean algebras with operators – including cylindric algebras and related algebraic forms of logic – and the role of BAs in ring theory and in functional analysis.

The Handbook is divided into two parts (published in three volumes). The first part (Volume 1) is a completely self-contained treatment of the fundamentals of the subject, which mathematicians in various fields may find interesting and useful. Here one will find the main results on disjointness (the Erdös–Tarski theorem), free algebras (the Gaifman–Hales, Shapirovskii–Shelah, and Balcar–Franěk theorems), and the basic decidability and undecidability results for the elementary theory of BAs, as well as the systematic development of the abstract theory (ultrafilters, representation, subalgebras, ideals, topological duality, free algebras, free products, measure algebras, distributivity, interval algebras, superatomic algebras, tree algebras).

The second part of the Handbook (Volumes 2 and 3) is intended to indicate most of the frontiers of research in the subject; it consists of articles which are more or less independent of each other, although most of them assume a knowledge of at least the easier portions of Part I. The second part is arranged in four sections, with two appendices and a bibliography. Section A, Arithmetical properties of BAs, has two chapters: on distributive laws and their relationships to games on BAs, and on disjoint refinements, treating extensively this elementary notion discussed in Part I. Section B, Algebraic properties of BAs, treats subalgebras, particularly the lattice of all subalgebras and the existence of complements in this lattice; cardinal functions on Boolean spaces; the number of BAs of various sorts; endomorphisms of BAs, including the existence of endo-rigid BAs; automorphisms groups; reconstruction of BAs from their automorphism groups; embeddings and automorphisms, especially for complete rigid

BAs; rigid BAs; and homogeneous BAs. Section C is devoted to special classes of BAs: superatomic algebras, mainly thin-tall and related BAs; projective BAs; and two lengthy chapters on countable BAs, with Ketonen's theorem; and on measure algebras, giving an extensive survey of this topic which is perhaps the most important subfield of the theory of BAs for most mathematicians. Section D deals with logical questions: decidable and undecidable theories of BAs in various languages; recursive BAs; Lindenbaum–Tarski algebras; and Boolean-valued models of set theory. Two appendices, on set theory and on topology, explain some more advanced notions used in some places in the Handbook. There is a chart of topological duality. Finally, there is a comprehensive Bibliography on the aspects of the theory of Boolean algebras treated in the Handbook.

Many people contributed to the Handbook by checking manuscripts for mathematical and typographical errors; in addition to several of the authors of the Handbook, the editor is indebted to the following for help of this sort: Hajnal Andréka, Aleksander Błaszczyk, Tim Carlson, Ivo Düntsch, Francisco J. Freniche, Lutz Heindorf, Istvan Németi, Stevo Todorčević, and Petr Vojtaš. Thanks are also due to the North-Holland staff, especially Leland Pierce, for their editorial work.

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