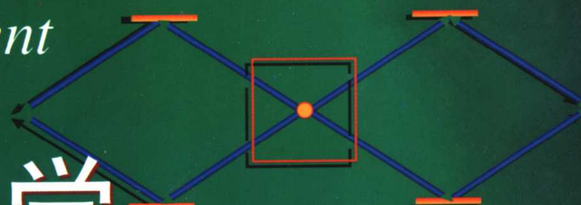
Quantum Mechanics

A Modern Development



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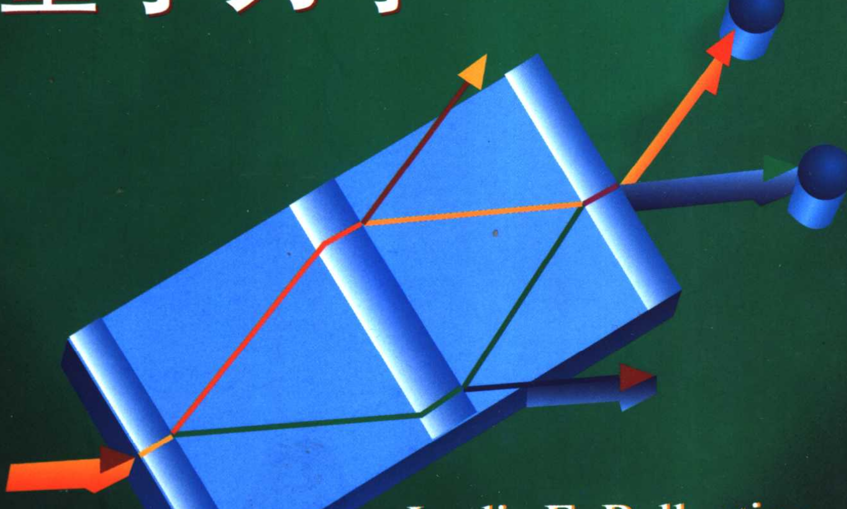

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QUANTUM MECHANICS: A MODERN DEVELOPMENT

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Preface

Although there are many textbooks that deal with the formal apparatus of quantum mechanics and its application to standard problems, before the first edition of this book (Prentice–Hall, 1990) none took into account the developments in the foundations of the subject which have taken place in the last few decades. There are specialized treatises on various aspects of the foundations of quantum mechanics, but they do not integrate those topics into the standard pedagogical material. I hope to remove that unfortunate dichotomy, which has divorced the practical aspects of the subject from the interpretation and broader implications of the theory. This book is intended primarily as a graduate level textbook, but it will also be of interest to physicists and philosophers who study the foundations of quantum mechanics. Parts of the book could be used by senior undergraduates.

The first edition introduced several major topics that had previously been found in few, if any, textbooks. They included:

- A review of *probability theory* and its relation to the quantum theory.
- Discussions about *state preparation* and *state determination*.
- The Aharonov–Bohm effect.
- Some firmly established results in the theory of *measurement*, which are useful in clarifying the interpretation of quantum mechanics.
- A more complete account of the *classical limit*.
- Introduction of *rigged Hilbert space* as a generalization of the more familiar Hilbert space. It allows vectors of infinite norm to be accommodated within the formalism, and eliminates the vagueness that often surrounds the question whether the operators that represent observables possess a complete set of eigenvectors.
- The *space–time symmetries* of displacement, rotation, and Galilei transformations are exploited to derive the fundamental operators for momentum, angular momentum, and the Hamiltonian.
- A charged particle in a magnetic field (Landau levels).

- Basic concepts of *quantum optics.*
- Discussion of modern experiments that test or illustrate the fundamental aspects of quantum mechanics, such as: the direct measurement of the momentum distribution in the hydrogen atom; experiments using the single crystal neutron interferometer; quantum beats; photon bunching and antibunching.
- Bell's theorem and its implications.

This edition contains a considerable amount of new material. Some of the newly added topics are:

- An introduction describing the range of phenomena that quantum theory seeks to explain.
- Feynman's *path integrals.*
- The adiabatic approximation and Berry's phase.
- Expanded treatment of state preparation and determination, including the *no-cloning theorem* and *entangled states.*
- A new treatment of the *energy-time* uncertainty relations.
- A discussion about the influence of a measurement apparatus on the environment, and vice versa.
- A section on the quantum mechanics of rigid bodies.
- A revised and expanded chapter on the *classical limit.*
- The *phase space* formulation of quantum mechanics.
- Expanded treatment of the many new interference experiments that are being performed.
- Optical homodyne tomography as a method of measuring the quantum state of a field mode.
- Bell's theorem without inequalities and probability.

The material in this book is suitable for a two-semester course. Chapter 1 consists of mathematical topics (vector spaces, operators, and probability), which may be skimmed by mathematically sophisticated readers. These topics have been placed at the beginning, rather than in an appendix, because one needs not only the results but also a coherent overview of their theory, since they form the mathematical language in which quantum theory is expressed. The amount of time that a student or a class spends on this chapter may vary widely, depending upon the degree of mathematical preparation. A mathematically sophisticated reader could proceed directly from the Introduction to Chapter 2, although such a strategy is not recommended.

The *space–time symmetries* of displacement, rotation, and Galilei transformations are exploited in Chapter 3 in order to derive the fundamental operators for momentum, angular momentum, and the Hamiltonian. This approach replaces the heuristic but inconclusive arguments based upon analogy and wave–particle duality, which so frustrate the serious student. It also introduces *symmetry* concepts and techniques at an early stage, so that they are immediately available for practical applications. This is done without requiring any prior knowledge of group theory. Indeed, a hypothetical reader who does not know the technical meaning of the word "group", and who interprets the references to "groups" of transformations and operators as meaning sets of related transformations and operators, will lose none of the essential meaning.

A purely pedagogical change in this edition is the dissolution of the old chapter on approximation methods. Instead, stationary state perturbation theory and the variational method are included in Chapter 10 ("Formation of Bound States"), while time-dependent perturbation theory and its applications are part of Chapter 12 ("Time-Dependent Phenomena"). I have found this to be a more natural order in my teaching. Finally, this new edition contains some additional problems, and an updated bibliography.

Solutions to some problems are given in Appendix D. The solved problems are those that are particularly novel, and those for which the answer or the method of solution is important for its own sake (rather than merely being an exercise).

At various places throughout the book I have segregated in double brackets, [[$\cdots$]], comments of a historical comparative, or critical nature. Those remarks would not be needed by a hypothetical reader with no previous exposure to quantum mechanics. They are used to relate my approach, by way of comparison or contrast, to that of earlier writers, and sometimes to show, by means of criticism, the reason for my departure from the older approaches.

Acknowledgements

The writing of this book has drawn on a great many published sources, which are acknowledged at various places throughout the text. However, I would like to give special mention to the work of Thomas F. Jordan, which forms the basis of Chapter 3. Many of the chapters and problems have been "field-tested" on classes of graduate students at Simon Fraser University. A special mention also goes to my former student Bob Goldstein, who discovered a simple proof for the theorem in Sec. 8.3, and whose creative imagination was responsible for the paradox that forms the basis of Problem 9.6. The data for Fig. 0.4 was taken by Jeff Rudd of the SFU teaching laboratory staff. In preparing Sec. 1.5 on probability theory, I benefitted from discussions with Prof. C. Villegas. I would also like to thank Hans von Baeyer for the key idea in the derivation of the orbital angular momentum eigenvalues in Sec. 8.3, and W. G. Unruh for point out interesting features of the third example in Sec. 9.6.

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Contents

Introduction

The Phenomena of Quantum Mechanics

Quantum mechanics is a general theory. It is presumed to apply to everything, from subatomic particles to galaxies. But interest is naturally focussed on those phenomena that are most distinctive of quantum mechanics, some of which led to its discovery. Rather than retelling the historical development of quantum theory, which can be found in many books,* I shall illustrate quantum phenomena under three headings: *discreteness*, *diffraction*, and *coherence*. It is interesting to contrast the original experiments, which led to the new discoveries, with the accomplishments of modern technology.

It was the phenomenon of *discreteness* that gave rise to the name "quantum mechanics". Certain dynamical variables were found to take on only a

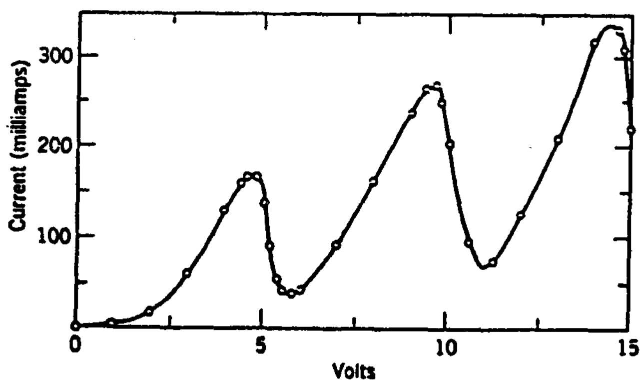

Fig. 0.1 Current through a tube of Hg vapor versus applied voltage, from the data of Franck and Hertz (1914). [Figure reprinted from *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles*, R. Eisberg and R. Resnick (Wiley, 1985).]

*See, for example, Eisberg and Resnick (1985) for an elementary treatment, or Jammer (1966) for an advanced study.

discrete, or *quantized*, set of values, contrary to the predictions of classical mechanics. The first direct evidence for discrete atomic energy levels was provided by Franck and Hertz (1914). In their experiment, electrons emitted from a hot cathode were accelerated through a gas of Hg vapor by means of an adjustable potential applied between the anode and the cathode. The current as a function of voltage, shown in Fig. 0.1, does not increase monotonically, but rather displays a series of peaks at multiples of 4.9 volts. Now 4.9 eV is the energy required to excite a Hg atom to its first excited state. When the voltage is sufficient for an electron to achieve a kinetic energy of 4.9 eV, it is able to excite an atom, losing kinetic energy in the process. If the voltage is more than twice 4.9 V, the electron is able to regain 4.9 eV of kinetic energy and cause a second excitation event before reaching the anode. This explains the sequence of peaks.

The peaks in Fig. 0.1 are very broad, and provide no evidence for the sharpness of the discrete atomic energy levels. Indeed, if there were no better evidence, a skeptic would be justified in doubting the discreteness of atomic energy levels. But today it is possible, by a combination of laser excitation and electric field filtering, to produce beams of atoms that are all in the same quantum state. Figure 0.2 shows results of Koch *et al.* (1988), in which

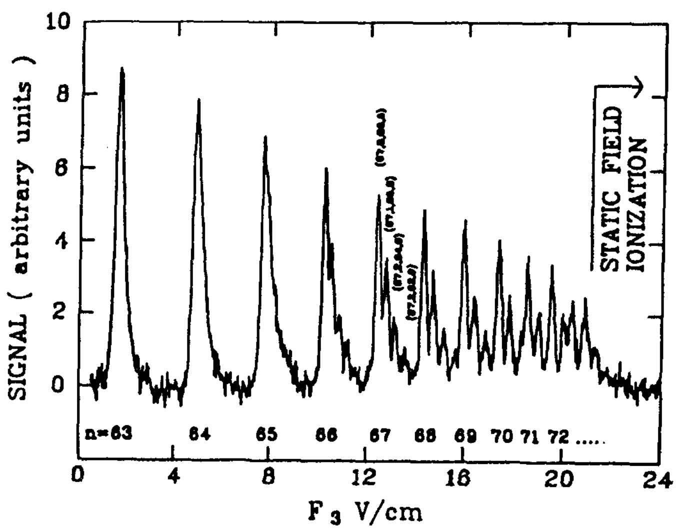

Fig. 0.2 Individual excited states of atomic hydrogen are resolved in this data [reprinted from Koch *et al.*, *Physica Scripta* **T26**, 51 (1988)].

the atomic states of hydrogen with principal quantum numbers from $n = 63$ to $n = 72$ are clearly resolved. Each n value contains many substates that would be degenerate in the absence of an electric field, and for $n = 67$ even the substates are resolved. By adjusting the laser frequency and the various filtering fields, it is possible to resolve different atomic states, and so to produce a beam of hydrogen atoms that are all in the same chosen quantum state. The discreteness of atomic energy levels is now very well established.

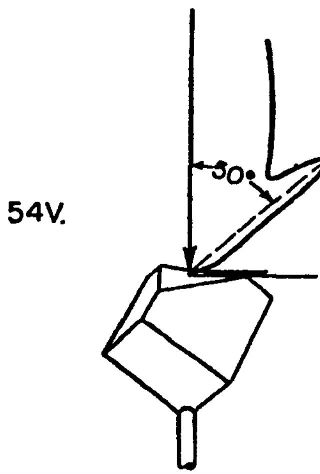

Fig. 0.3 Polar plot of scattering intensity versus angle, showing evidence of electron diffraction, from the data of Davisson and Germer (1927).

The phenomenon of *diffraction* is characteristic of any wave motion, and is especially familiar for light. It occurs because the total wave amplitude is the sum of partial amplitudes that arrive by different paths. If the partial amplitudes arrive in phase, they add constructively to produce a maximum in the total intensity; if they arrive out of phase, they add destructively to produce a minimum in the total intensity. Davisson and Germer (1927), following a theoretical conjecture by L. de Broglie, demonstrated the occurrence of diffraction in the reflection of electrons from the surface of a crystal of nickel. Some of their data is shown in Fig. 0.3, the peak at a scattering angle of 50° being the evidence for electron diffraction. This experiment led to the award of a Noble prize to Davisson in 1937. Today, with improved technology, even an undergraduate can easily produce electron diffraction patterns that are vastly superior to the Nobel prize-winning data of 1927. Figure 0.4 shows an electron

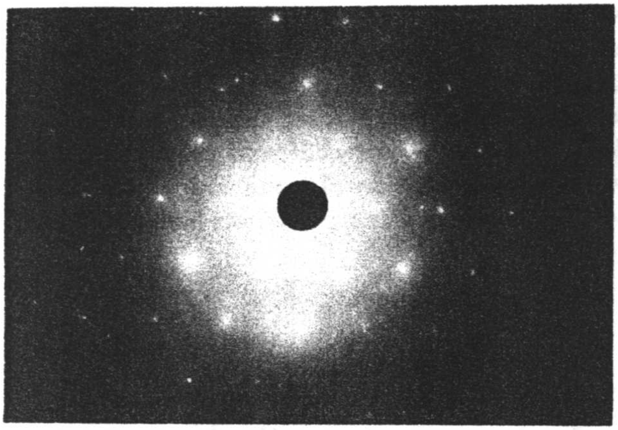

Fig. 0.4 Diffraction of 10 kV electrons through a graphite foil; data from an undergraduate laboratory experiment. Some of the spots are blurred because the foil contains many crystallites, but the hexagonal symmetry is clear.

diffraction pattern from a crystal of graphite, produced in a routine undergraduate laboratory experiment at Simon Fraser University. The hexagonal array of spots corresponds to diffraction scattering from the various crystal planes.

The phenomenon of diffraction scattering is not peculiar to electrons, or even to elementary particles. It occurs also for atoms and molecules, and is a universal phenomenon (see Ch. 5 for further discussion). When first discovered, particle diffraction was a source of great puzzlement. Are "particles" really "waves"? In the early experiments, the diffraction patterns were detected holistically by means of a photographic plate, which could not detect individual particles. As a result, the notion grew that particle and wave properties were mutually incompatible, or *complementary*, in the sense that different measurement apparatuses would be required to observe them. That idea, however, was only an unfortunate generalization from a technological limitation. Today it is possible to detect the arrival of individual electrons, and to see the diffraction pattern emerge as a statistical pattern made up of many small spots (Tonomura *et al.*, 1989). Evidently, quantum particles are indeed particles, but particles whose behavior is very different from what classical physics would have led us to expect.

In classical optics, *coherence* refers to the condition of phase stability that is necessary for interference to be observable. In quantum theory the concept of coherence also refers to phase stability, but it is generalized beyond any analogy with wave motion. In general, a *coherent* superposition of quantum states may have properties than are qualitatively different from a mixture of the properties of the component states. For example, the state of a neutron with its spin polarized in the $+x$ direction is expressible (in a notation that will be developed in detail in later chapters) as a coherent sum of states that are polarized in the $+z$ and $-z$ directions, $|+x\rangle = (|+z\rangle + |-z\rangle)/\sqrt{2}$. Likewise, the state with the spin polarized in the $+z$ direction is expressible in terms of the $+x$ and $-x$ polarizations as $|+z\rangle = (|+x\rangle + |-x\rangle)/\sqrt{2}$.

An experimental realization of these formal relations is illustrated in Fig. 0.5. In part (a) of the figure, a beam of neutrons with spin polarized in the $+x$ direction is incident on a device that transmits $+z$ polarization and reflects $-z$ polarization. This can be achieved by applying a strong magnetic field in the z direction. The potential energy of the magnetic moment in the field, $-\mathbf{B}\cdot\boldsymbol{\mu}$, acts as a potential well for one direction of the neutron spin, but as an impenetrable potential barrier for the other direction. The effectiveness of the device in separating $+z$ and $-z$ polarizations can be confirmed by detectors that measure the z component of the neutron spin.

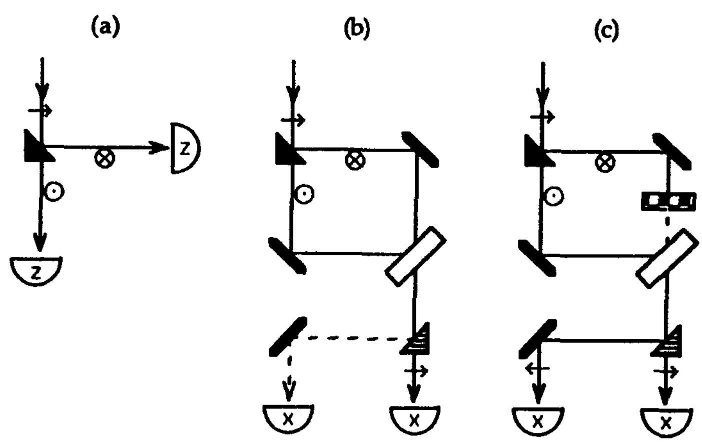

Fig. 0.5 (a) Splitting of a $+x$ spin-polarized beam of neutrons into $+z$ and $-z$ components; (b) coherent recombination of the two components; (c) splitting of the $+z$ polarized beam into $+x$ and $-x$ components.

In part (b) the spin-up and spin-down beams are recombined into a single beam that passes through a device to separate $+x$ and $-x$ spin polarizations.