FUZZY AUTOMATA AND DECISION PROCESSES

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NORTH-HOLLAND · NEW YORK

NEW YORK · AMSTERDAM · OXFORD

Elsevier North-Holland, Inc.
52 Vanderbilt Avenue, New York, New York 10017

North-Holland Publishing Company
P.O. Box 211,
Amsterdam, The Netherlands

Library of Congress Cataloging in Publication Data

Main entry under title:
Fuzzy automata and decision processes.
Bibliography: p.
Includes index.
1. Set theory. 2. Machine theory. 3. Automata.
4. Decision-making. I. Gupta, Madan M. II. Saridis, George N. 1931- III. Gaines, Brian R.
QA248.F85 511'.3 77-534
ISBN 0-444-00231-6

Manufactured in the United States of America

FOREWORD

The natural world in which we live is a world of imprecision and inexactitude. It is a pervasive, nonquantitative world in which there are few sharp boundaries; one in which the transition from membership in one set to another is gradual rather than abrupt. It is a world in which fuzziness is the rule rather than the exception. The human brain has been accustomed to think and to reason in such an environment with varying degrees of success for a long time. However, in dealing with this real world, whether by means of analytical models or actual systems, engineers, scientists, mathematicians, and others have largely ignored this fact. Far too frequently they tried to apply the precision of classical mathematics to this imprecise, nonquantitative pervasive world of ours; when failure resulted, they wondered what went wrong. Many still try.

Obviously, a new approach is needed in this type of work—an approach in which fuzziness is accepted as an essential reality, a reality that cannot be overlooked or ignored. It is especially gratifying, therefore, to acknowledge the work of the researchers in this publication for their valuable contributions to this very important and rapidly developing field. It is also gratifying to acknowledge the devotion of many individuals that has helped to explore the new areas of applications, to stimulate the exchange of scientific information, and to reinforce international cooperation in this important field of *Fuzzy Automata and Decision Processes*.

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PREFACE

The problem of control and decision-making in a fuzzy environment presents one of the most fundamental and challenging issues in the design and analysis of man-machine systems. At present, the behavior of such systems is usually analyzed by the use of methods rooted in classical mathematics. It is becoming increasingly clear, however, that classical mathematics, with a conceptual structure that rests on two-valued logic, is not well-suited for dealing with systems that manifest a high degree of fuzziness—as all man-machine systems do. To deal with fuzziness effectively, we must be prepared to lower our standards of precision and be tolerant of approaches that are approximate in nature. This is the essence of fuzzy logic and is the spirit in which one should apply the theory of fuzzy sets to the analysis of man-machine systems much too complex to be susceptible of description in numerical terms.

Commenting on this issue in some of his papers, Zadeh writes:

> In a large measure, our inability to design such machines stems from a fundamental difference between human intelligence, on the one hand, and machine intelligence on the other. The difference lies in the ability of the human brain—an ability which present day digital computers do not possess—to think and reason in imprecise, nonquantitative, fuzzy terms. It is this ability that makes it possible for humans to decipher sloppy handwriting, understand distorted speech, and focus on that information which is relevant to a decision. It is the lack of this ability that makes even the most sophisticated large scale computers incapable of communicating with humans in natural—rather than artificially-constructed—languages.

> Fuzzy set theory rests on the notion that the key elements in human thinking and human decision-making are based not on numbers but on fuzzy sets—classes of objects in which transition from membership to nonmembership is gradual rather than abrupt. The pervasiveness of fuzziness in human thought processes suggests that much of the logic behind human reasoning is not the traditional two-valued or multivalued logic, but a logic with fuzzy truths, fuzzy connectives, and fuzzy rules of inference.

> The theory of 'fuzzy sets' (or subsets) is, in effect a step toward a rapprochement between the precision of classical mathematics and the pervasive imprecision of the real world—a rapprochement born of the incessant human quest for a better understanding of mental processes and cognition.

Today, the theory initiated by Zadeh is slightly more than a decade old. Although still in its formative stages, it is clear that the theory of fuzzy sets offers a new and highly promising direction for the study of the behavior of complex man-machine systems and, more generally, human decision processes and cognition. In growing numbers, investigators in many countries are exploring possible applications of the theory and are contributing new concepts and techniques both to the theory proper and its uses in various fields. Thus, the coming decade is likely to witness a rapid growth in the literature of fuzzy set theory and its evolution into an important field of scientific methodology.

To become familiar with the theory of fuzzy sets and develop an understanding of its applications is not, at present, an easy task. The papers collected in this volume are intended to provide the reader with a broad view of the field and expose him/her to a representative collection of concrete problems to which the theory of fuzzy sets has been applied.

More specifically, in Part I of the volume, the introductory papers provide a broad perspective view, while the survey papers present an overview of fuzzy mathematics, fuzzy measures and fuzzy integrals, with applications to control systems and fuzzy reasoning.

The papers in Part II are addressed, for the most part, to the basic aspects of the theory and recent theoretical developments, while the papers in Part III are given over to applications in such fields as process control, pattern classification, cluster detection, group consensus formation and decision-making in prosthetic devices. To aid the reader in furthering his/her study of the theory and its applications, an annotated bibliography of the literature since 1965 is included in the volume.

The contributors to this volume will feel that their efforts have been rewarded if their papers provide a stimulus to others to contribute to the theory of fuzzy sets and extend its applications in various directions.

MADAN M. GUPTA
Editor

PROLOGUE

Fuzzy set theory originated in the work of Lotfi A. Zadeh about a dozen years ago. Since then, it has blossomed into a many-faceted field of scientific inquiry, drawing on and contributing to a wide spectrum of areas ranging from pure mathematics and physics to medicine, linguistics and philosophy.

The papers appearing in this volume were contributed in part by participants in a round table discussion on Fuzzy Automata and Decision Processes, held at the Sixth IFAC World Congress at MIT, Cambridge, in August 1975; and in part by other leading workers in the theory of fuzzy sets and its applications, both in the United States and abroad.

The wide ranging nature of the theory of fuzzy sets, the diversity of its applications and the geographical dispersion of contributors made the task of organizing and editing this volume a rather difficult undertaking. As editors, we have attempted to provide the reader with a broad exposure both to the basic theory of fuzzy sets and a representative selection of its applications. To this end, the volume presents a review of fuzzy set theory, including expositions of fuzzy algebra, fuzzy measures and fuzzy integrals; surveys applications of the theory to decision processes, control systems, fuzzy reasoning, fuzzy algorithms, medical diagnosis and related fields; and provides an up-to-date annotated bibliography covering the period 1965 to the present.

The material in this volume is of particular relevance to those fields in which human judgment, perception and reasoning play an important role. This includes systems analysis, especially of socioeconomic systems, psychology, sociology, law, management science, operations research, medicine, linguistics, artificial intelligence and related areas.

In the years ahead, the theory of fuzzy sets is likely to gain increasing recognition as an effective tool for the analysis of systems too complex or too ill-defined to be susceptible to analysis by conventional quantitative techniques. We hope that this volume will serve to introduce the reader to the basic concepts and techniques of fuzzy set theory and acquaint him/her with some of its more important applications.

The task of organizing and editing a collection of papers on a subject as new and as diverse as the theory of fuzzy sets has not been an easy one to accomplish. We are deeply appreciative of the spirit of cooperation and understanding manifested by all of the contributors to this volume and especially those who participated in the round table discussion at the IFAC World Congress. In particular, R. G. Lex of the IFAC Advisory

Committee deserves special mention for his interest and support during and after the IFAC World Congress.

Finally, We wish to express my heartfelt appreciation to Elsevier North-Holland for undertaking the publication of this volume.

MADAN M. GUPTA
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PART ONE
INTRODUCTION

FUZZY SET THEORY—A PERSPECTIVE 1

L. A. Zadeh

Fuzziness, vagueness and imprecision are terms with pejorative connotations. We accord respect to what is precise, logical and clear and we look with disdain upon reasoning that is fuzzy or lacking in mathematical discipline. And yet, as we learn more about human cognition, we may well arrive at the realization that man's ability to manipulate fuzzy concepts is a major asset rather than a liability, and it is this ability, above all, that constitutes a key to the understanding of the profound difference between human intelligence, on the one hand, and machine intelligence, on the other.

To some, fuzziness is a disguised form of randomness. This is a misconception—a deep-seated misconception that has retarded the development of a conceptual framework for dealing with fuzziness as a basic and distinct facet of reality. Indeed, fuzziness is more than a facet of reality; it is one of its most pervasive characteristics—a characteristic rooted in the bounded capacity of the human mind to process and store information.

More specifically, fuzziness relates not to the uncertainty concerning the membership of a point in a set, but to the graduality of progression from membership to nonmembership. Thus, the pervasiveness of fuzziness derives from the fact that, in most of the classes of objects that we form in our perception of reality, the transition from membership to nonmembership is gradual rather than abrupt. This is true of the classes of tall men, beautiful women and large numbers. And it is true of the meaning of such concepts as meaning, intelligence, truth, democracy, and love. In fact, the only domain of human knowledge in which nonfuzzy concepts play the dominant role is that of classical mathematics. On the one hand, this endows mathematics with a beauty, power and universality unmatched by any other field. On the other, it severely restricts its applicability in fields in which fuzziness is pervasive—as is true, in particular, of humanistic systems, that is, systems in which human judgment, perceptions and emotions play a central role.

Since its inception about a dozen years ago, the theory of fuzzy sets has evolved in various directions and is finding applications in a wide variety of fields—as is evidenced by the papers appearing in this volume. What is important to recognize, however, is that there are two distinct directions in which the evolution of the theory of fuzzy sets is likely to progress in the years ahead. In one, fuzzy sets are treated as precisely defined mathematical objects subject to the rules of classical logic. In another and more recent development associated with the linguistic approach, the underlying logic is not the classical two-valued logic, but a fuzzy logic in which the truth-values themselves are fuzzy sets and the rules of inference are approximate rather than exact. In this case, it is not only the assertions about fuzzy sets that are fuzzy in nature, but also their truth-values and the rules of inference by which the consequent assertions are derived.

It is my belief that, in the years to come, approximate reasoning and fuzzy logic will evolve into an important field in its own right, providing a basis for new approaches to problems in philosophy, linguistics, psychology, sociology, management science, medical diagnosis, decision analysis and other fields. At the same time, we shall also witness many important developments in the mathematical theory of fuzzy sets based on classical logic—developments that will rank as significant contributions to pure as well as applied mathematics. Needless to say, what will happen during the next decade can be foreseen only dimly at this early stage of the development of the theory of fuzzy sets. But what is certain is that, with many new and talented investigators joining the ranks of fuzzy set theorists and users, the theory of fuzzy sets will grow rapidly in importance, influence and applicability and, eventually, will be accorded recognition as one of the basic areas of human knowledge and scientific methodolgy.

"FUZZY-ISM", THE FIRST DECADE 2

Madan M. Gupta

"Fuzzy-ism" is a body of concepts and techniques aimed at providing a systematic framework for dealing with the vagueness and imprecision inherent in human thought processes. In particular, it enables one to give a precise mathematical description of what are normally vague statements. Thus, it is an attempt to remove "linguistic" barriers between humans, who think in fuzzy terms, and machines that accept only precise instructions.

"Fuzzy-ism" is young [44] and has just entered into the second decade of its existence, but its doctrine is having a profound impact on the development of the theory of decision-making. In recent years, fuzzy set theory has been applied to a wide range of problems. In many applications, conventional quantitative mathematics has been replaced by fuzzy mathematics. At the same time, researchers have developed many new mathematical concepts in fuzzy theory applicable to humanistic processes [1,27,35,47].

"FUZZY-ISM" AND DECISION-MAKING

The stimulus for advances in fuzzy set theory may be summarized by a principle—Zadeh [46] calls it "the principle of incompatibility",—which may be stated as follows:

> The closer one looks at a 'real world' problem, the fuzzier becomes its solution. Stated informally, the essence of this principle is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.

Advances in science and technology have made our modern society very complex, and with this our decision processes have become increasingly fuzzy and hard to analyze. The human brain possesses some special characteristics that enable it to learn and reason in a vague and fuzzy environment. It has the *ability* to arrive at decisions based on imprecise, qualitative data in contrast to formal mathematics and formal logic which demand precise and quantitative data. Modern computers possess capacity but lack the human ability. Undoubtedly, in many areas of cognition, human intelligence far excells the computer 'intelligence' of today, and the development of fuzzy concepts is a step forward toward the development of tools capable of handling humanistic types of problems [15–19].

We do have sufficient mathematical tools and computer-based techniques for analyzing and solving the problems embodied in deterministic and uncertain (probabilistic) environment [4–6,36,37,41–43]. Here uncertainity may arise from the probabilistic behavior of certain physical phenomena in mechanistic systems.[1] We knew the important role that vagueness and inexactitude play in human decision-making, but we did not know until 1965 how the vagueness arising from subjectivity (which is inherent in human thought processes) can be modeled or analyzed [44].

In 1965, Professor Zadeh laid the foundation of 'fuzzy-ism' by introducing what he called "fuzzy Sets." In effect, fuzzy set theory is a body of concepts and techniques that laid a form of mathematical precision to human thought processes that in many ways are imprecise and ambiguious by the standards of classical mathematics. Today, these concepts are gaining a growing acceptability among engineers, scientists, mathematicians, linguists, and philosophers.

EXPOSURE TO 'FUZZY-ISM'

I was, and still am a member of the school of 'determinism' and 'stochasticism.' It was in the summer of 1968 that I had an opportunity to listen to Professor Zadeh, the founder of 'Fuzzy-ism,' at the IFAC Symposium on Adaptivity and Sensitivity held at Dubrovnic, Yugoslavia. There I heard his lucid exposition of his ideas and was impressed by his break with the traditional modes of thinking. These concepts were just three years old at that time and it was difficult to assign any certainty to the growth and acceptibility of 'fuzzy mathematics' in mathematical and technological circles.

I continued a casual interest in the field by occasionally reading the literature but without much excitement. It was in 1972 that I came across some very interesting and convincing papers [12,14,31,45] which reawakened my interest in the field.

I was invited to organize a special Round-Table Discussion session on 'Estimation and Control in a Fuzzy Environment' at the Third IFAC Symposium on Identification and Parameter Estimation held at The Hague in June 1973. Panel members of international repute were invited to

[1]Mechanistic systems are those which, in the main, are governed by the laws of mechanics, electromagnetism, and thermodynamics.

present their views on the subject, followed by a long discussion. Although most of the work presented there was theoretical in nature, the discussion did help to remove some misconceptions and open challenging opportunities for further theoretical development and applied research in the field [21,23]. The discussion inspired the interest of many more researchers. In particular, it was found that there was a great deal of interest in Japan and that Japanese researchers were contributing significantly to the field.

Following this, a very successful U.S.—Japan Seminar on 'Fuzzy Set Theory' was held at the University of California, Berkeley, in July 1974. The seminar was marked by many interesting applications of Fuzzy Set Theory to Cognitive and Decision Processes. The important papers have appeared in a volume edited by Zadeh, Fu, Tanaka, and Shimura [47].

Following the success of the discussion session at The Hague, I was invited to organize and chair the Second IFAC Round-Table Discussion session on 'Fuzzy Automata and Decision Processes' at MIT during the Sixth Trienniel World IFAC Congress, Boston/Cambridge, August 24–30, 1975 [24]. A panel of researchers from various institutions were invited to present their work. The presentation had an integrated and balanced mixture of theory and applications. A detailed report appears in the IFAC Proceedings as well as in *Automatica* [32], and a number of selected papers appear in this volume.

GROWTH OF FUZZY-ISM

The first decade of 'Fuzzy-ism' is exhibiting an exponential growth: from two publications in 1965, the year it was founded, to about 100 by 1973, the year the first IFAC Round-Table Discussion session was held, to about 450 by the time of the second IFAC Round-Table Diccussion, to over 600 by the end of May 1976 (see the annotated bibliography at the end of this volume).

Aizerman [1] has rightly pointed out: "The boldness of this (fuzzification) idea combined with not only a scientific talent but also a missionary zest and great energy of Professor Zadeh, have led to the adoption of his ideas by many scientists."

Undoubtedly, impressive progress has been achieved. In the first place, we have a better understanding of the need to abandon many of the traditional conceptions associated with the construction of mathematical models of real-world phenomena. Actually, we were confronted with this type of problem all along in the past, but now the problem has become more pressing because of the increasing complexity of our social and scientific environment and the inability of conventional computers to model the behavior of ill-defined, large-scale systems.