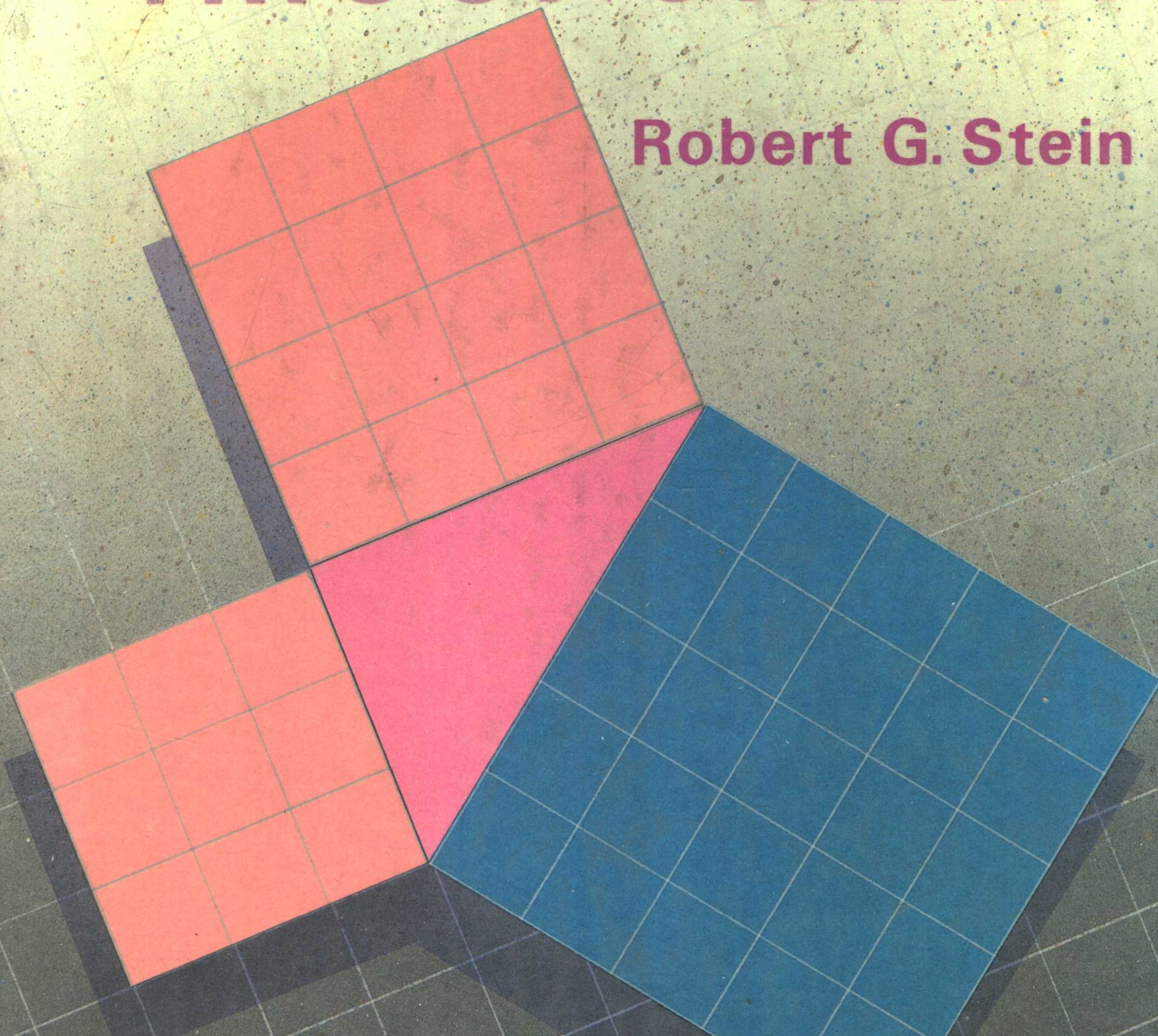
FUNDAMENTALS OF

COLLEGE ALGEBRA WITH TRIGONOMETRY



Robert G. Stein

Fundamentals of College Algebra with Trigonometry

Robert G. Stein
California State University, San Bernardino

Nelson-Hall
Chicago

Copy Editor: Ellen Credille
Designer: Claudia von Hendricks
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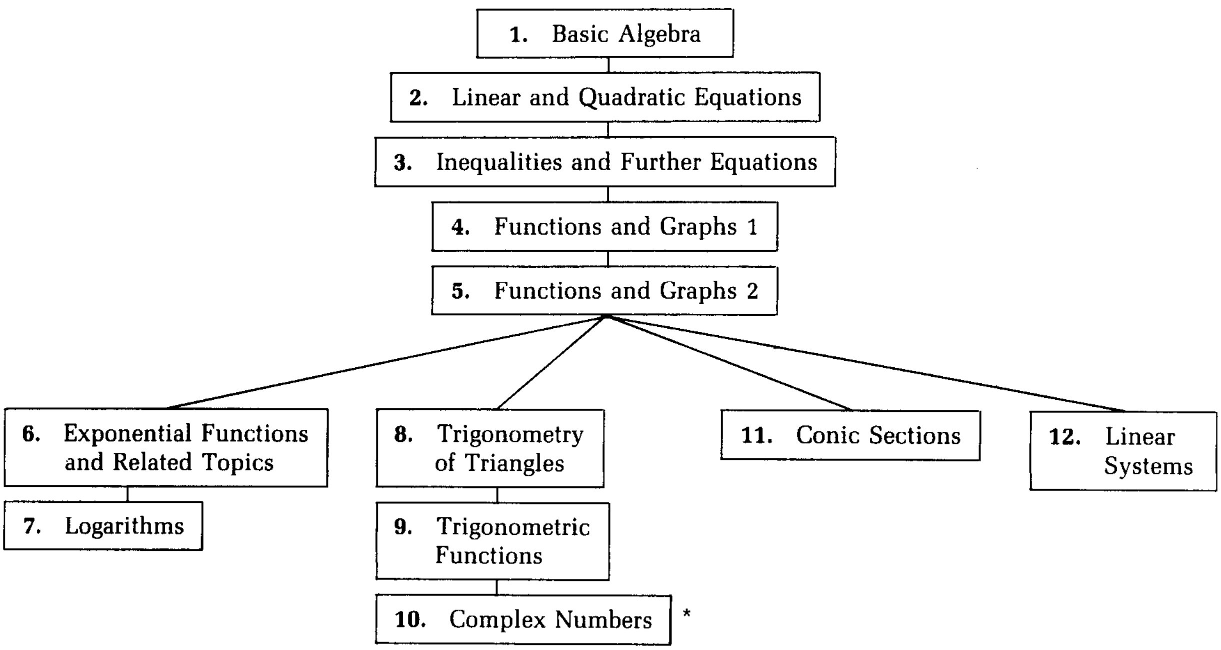
The paper in this book is pH neutral (acid-free).

Note to the Instructor

You can create a variety of courses from this book, depending on the time available and the needs and backgrounds of your students. The book is self-contained and can be used for a systematic and thorough study of algebra from the beginning, but with well-prepared students, you could treat much of the early material as review in order to concentrate on later chapters. The flowchart below shows that chapters 6–12 are largely independent of the first chapters and of one another, which allows you considerable flexibility in designing the later parts of the course.



*Sections 10.1 and 10.2 require only chapter 2, but sections 10.3, 10.4, and 10.5 require chapters 8 and 9.

In organizing this book, my guiding principle has been to introduce each topic when it would be most natural and useful. Thus, while the material is quite standard for today's algebra-trigonometry texts, there are some deviations from current fashion in the sequencing of the topics. For example, rational exponents are not included in chapter 1 because they are not needed until chapter 6. More significant is the inclusion of arithmetic and geometric progressions in the chapter on exponential functions. These progressions illuminate the relationship between their continuous analogues, linear and exponential functions—a relationship which underlies the theory of logarithms. Finally, the binomial theorem is introduced in chapter 1, partly to set the stage for its later uses with De Moivre's theorem (chapter 10), but mostly to ensure that this basic theorem is given the prominence it deserves and is not skipped. This theorem is especially important for students who will go on to calculus, where they will probably apply it to differentiate x^n by the delta process.

In planning your course, take advantage of the part B exercises, which are a source of stimulating enrichment involving discovery explorations, outlines of proofs, and non-routine applications. A detailed discussion of the part B exercises appears in the *Instructors Resource and Solutions Manual*, but the important point to bear in mind for planning purposes is that you can often use them to serve more than one end. For example, if you want to assign practice on division of polynomials, take advantage of problems in section 1.3 to combine division practice with discovery of the remainder theorem. That way the students can participate actively in the act of mathematical discovery, and section 3.6 (The Remainder and Factor Theorems) will be easier to teach because you prepared the students for it in advance.

A brief word about calculators is in order. Most of the examples and problems in this book are designed to illustrate mathematical concepts, and experience suggests that students grasp these most readily when distractions, including lengthy numerical or algebraic calculation, are minimized. Therefore most of the problems can readily be done without a calculator, and tables of square roots, common logarithms, and trigonometric functions are provided at the back. Those few problems for which a calculator would be a significant advantage are clearly marked.

Your comments, especially suggestions for improving this book, will be most welcome.

Acknowledgments

If mathematics teachers and writers were confined to the material they thought up, I and most of my colleagues would be unemployed and this book would never have been written. Therefore it is fitting to acknowledge my debt to the mathematicians, writers, and teachers who have taught me. Outstanding in this regard are Harold Jayson, Evelyn Rosenthal, George Casey, Hans Hollstein, Michael Artin, W. W. Sawyer, Richard Courant, Otto Toeplitz, Tom Apostol, John Lamperti, Ernst Snapper, and Ray Carry.

More immediately, it is a pleasure to acknowledge the invaluable help I received from those who participated, one way or the other, in the writing and production of this book. Dorinda Thurman and Cathy Podrasky typed preliminary drafts; Marsha Shanteler and

Nancy Pennington typed revisions and material that was put in later. John Hafstrom, Art Grigory, Fred Keene, and Bill McClung used preliminary versions in their classes, and they, along with their students, suggested many helpful changes. Steve Meyering went through an early version thoroughly, and Jolene Kraushar did the same for the final version, working *all* the problems and pointing out dozens of possible improvements. The faculty and administration at California State University, San Bernardino encouraged and supported my efforts in this project. The people at Nelson-Hall, notably Ron Warncke, Dick Epler, and Claudia von Hendricks, have been delightful to work with. Above all, I would like to thank my family. Our cats, Mousa and Persia, personally inspected as many pages as they could by walking or sitting on them, and the kids were remarkably understanding. Above all, my wife Roni put up with me throughout, even though the project sometimes seemed endless. I happily dedicate this book to her, Joey, and Lucy.

Robert G. Stein

Note to the Student

"The great book of nature lies ever open before our eyes . . . it is written in mathematical language . . ."—Galileo

Galileo's words were prophetic. Soon after he wrote them, the laws of planetary motion and gravitation were discovered; they were mathematical in nature. Since that beginning, mathematics has turned out to be the key to understanding more and more of the world around us. Today mathematics is used in many ways previously undreamed of, such as solving scheduling problems, allocating resources efficiently, and deciding where to locate factories and warehouses. Automatic landing systems for aircraft and the CAT scanners used so successfully in modern medicine are both based directly on mathematics. So are error-correcting codes, which were first developed to bring information back from space quickly and accurately and are now also used to make beautiful digital recordings. The possibilities for future uses of mathematics appear at least as dramatic as the applications we have seen to date. More than ever before, mathematics appears to be the key to success in the decades to come. Evidently, mathematics is a subject worth knowing; it merits and requires serious study. Therefore, a couple of hints on how to study mathematics are in order.

1. **Read the book.** That doesn't guarantee success, but it helps. Try to read each section *before* it is covered in class. That way any questions that arise from the reading can be cleared up in class. Read actively, checking the arithmetic and algebra in the worked examples.
2. **Do plenty of exercises.** Like sports or music, mathematics involves skills, and the only way to become good at them is to practice. That's why this book has plenty of problems. Many of these may be solved much like the worked examples in the preceding section of text, but others, like the miscellaneous exercises at the end of each chapter and the exercises marked with a B, go beyond routine practice to show you some of the power and beauty of mathematics. Use the answers to odd-numbered problems at the back of the book to check your work.

In spite of your best efforts, you may encounter the frustration of not being able to see how to deal with a problem. This is natural and happens even to outstanding mathematicians. (One of them, George Polya, once said that anything you can solve in five minutes should not be considered a problem. That's true in most of life; why should mathematics be different?) If you get stuck, try asking someone for help, or else put the problem aside for a while and return to it later, much as you might with a difficult crossword or picture puzzle. Of course there is still no guarantee that you will solve the problem, but, paradoxically, you will usually learn more from struggling with a tough problem—even one you never solve—than from a dozen easy ones.

As you can see, nobody can honestly promise you an easy road to success in mathematics, but I can assure you that the subject will richly reward your efforts. I wish you success in them.

Robert G. Stein

Contents

1 Basic Algebra

Introduction

This chapter lays the foundation for the entire book. It presents in detail every skill you need to work with algebraic expressions. Tailor your study of this material to your own mathematical background. If you encounter topics which you already know well, study them lightly, reviewing the main ideas. Then test yourself by working some of the more difficult odd-numbered exercises and checking your work. (Answers to the odd-numbered problems are given in the back of the book, starting on page 419). If you consistently succeed in solving these problems, then move ahead quickly, but move slowly and methodically through topics which are new to you or on which you are rusty. Pay particular attention to the worked examples. Try to work each one by yourself before studying the solution. Your success will deepen your understanding of both the problems and the techniques used to solve them. Furthermore, it will make you an active participant in the learning process. This is important; in learning mathematics, as in learning any skill, such as skiing or playing the piano, active involvement is the key.

1.1 Number Systems

The invention of numbers is lost in the distant past, but it undoubtedly began with counting. Even today the numbers 1, 2, 3, 4, 5, ... are called the **counting numbers**. Other operations, such as addition, subtraction, multiplication, division, measurement, raising to powers, and extracting roots, grew out of counting. Over the centuries these new operations led to generalizations in the concept of number. Historically, these generalizations evolved haltingly and unevenly, but a brief review of them from a modern perspective reveals underlying patterns.

Subtraction questions lead to expanding the system of counting numbers to form the set of **integers**: ... −4, −3, −2, −1, 0, 1, 2, 3, 4, ...

Division questions lead to a further expansion of the number system to include all quotients of integers $\frac{m}{n}$ $(n \neq 0)$. The result is the system of **rational numbers.** The word "rational" comes from "ratio." A number is **rational** if it can be expressed as a quotient of integers.

We often picture numbers as points on a **number line,** with the integers at regularly spaced intervals, and other numbers, a few of which are shown in figure 1, in between.

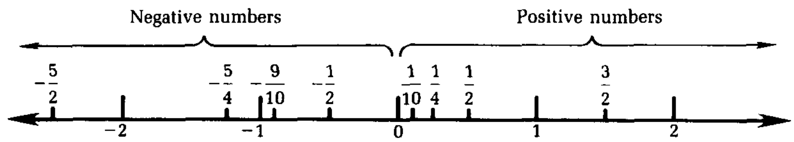


Figure 1.

Numbers on the same side of zero as 1 are called **positive,** and those on the opposite side are called **negative;** zero itself is neither positive nor negative. The positive integers are the counting numbers. The counting numbers together with zero are sometimes called the **nonnegative integers.**

The **opposite** of n, written $-n$, is the same distance from zero as n, but in the opposite direction. Similarly, $-(-n)$ is the opposite of the opposite of n, namely, n itself.

Every rational number corresponds to a point on the number line. Does every point on the number line correspond to a rational number? At first you might think so, but ancient Greeks found, to their surprise, that certain points on the number line do not correspond exactly to rational numbers. The basis for this discovery was the Pythagorean theorem:*

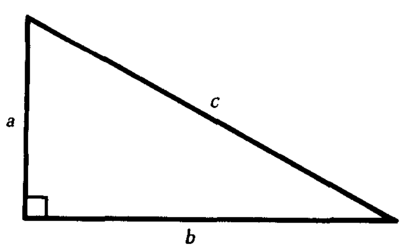


Figure 2.

> ***Pythagorean Theorem***
> Let a, b, and c be the lengths of the sides of a right triangle, with c the largest as in figure 2. The longest side of a right triangle, called the **hypotenuse,** is opposite the right angle. Then $c = \sqrt{a^2 + b^2}$.

The radical sign, $\sqrt{\ }$, denotes the nonnegative square root of

*Credit for discovery of this theorem is usually given to Pythagoras, a Greek mystic of the sixth century B.C. who travelled widely throughout the Middle East. In what is now Italy he founded a school devoted mainly to the study of numbers and their role in the universe. The Pythagoreans, a secret brotherhood of students, were eventually viewed as a threat to the state, and Pythagoras was exiled.

It is not clear to what extent Pythagoras deserves credit for this theorem. Historical records show it was known in ancient Babylon and China.

the number inside. For example, if x is a nonnegative number, then $\sqrt{x}$ is the nonnegative number whose square is x. A simple proof of the Pythagorean theorem is given in problem 29, Exercises 1.1.

Extensions of the Rationals

The Pythagorean theorem raises the issue of computing square roots. It turns out that $\sqrt{2}$, $\sqrt{3}$, (and in general $\sqrt{n}$ if the integer n is not a perfect square) cannot be expressed as rational numbers. These were the first numbers to be identified as **irrational** (not rational). Today it is known that many other numbers, including most roots of rational numbers, are irrational.

The rational and irrational numbers together constitute the set of **real numbers.** Every real number corresponds to a point on the number line, and every point on the number line corresponds to a real number. Consequently, the number line is sometimes called the **real line**

The real number system is used for most of the mathematics in this book and in basic calculus as well. However, for some purposes it is convenient to expand the concept of number to the system of **complex numbers.** We shall first encounter these in chapter 2, when we deal with quadratic equations whose solutions involve square roots of negative numbers. In chapter 10 we shall study the system of complex numbers in more detail.

Field Properties

As the number concept expanded from the counting numbers to the more versatile real number system, the original meanings of basic operations were lost. Although addition of counting numbers grows directly out of counting itself, addition of real numbers such as $\sqrt{2} + 11\sqrt{3}$ does not. Multiplication of counting numbers is repeated addition, but products of real numbers such as

$$\sqrt{3}(1 + \sqrt{2})$$

cannot be so interpreted. Evidently, the basic operations must be re-examined in the larger system of real numbers.

One approach to this would be a thorough study of the real numbers, but that would lead us far from our goals. Instead we simply give a short list of properties of real numbers which we will assume without attempting to justify. The properties listed here are known as the **field properties.** They may be used to prove many, but not all, properties of real numbers. (Those properties which cannot be proved using the field properties alone involve additional assumptions about order or "completeness." Order is studied briefly in sections 3.2 and 3.3. Completeness is studied in calculus but will not be studied in this course.)

Field Properties of Real Numbers	*Addition*	*Multiplication*
Commutative properties. Order does not affect a sum or a product.	$a + b = b + a$	$ab = ba$
Associative properties. Grouping does not affect a sum or a product.	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity elements. Adding 0 or multiplying by 1 leaves every number unchanged.	$0 + a = a$	$1 \cdot a = a$
Inverse elements. Every number a has an **additive inverse** $-a$ for which $-a + a = 0$. Every number a except 0 has a **multiplicative inverse** $\frac{1}{a}$ for which $\frac{1}{a} \cdot a = 1$.	$-a + a = 0$	$\frac{1}{a} \cdot a = 1$ (for $a \neq 0$)
Distributive property. This property relates addition to multiplication.	$a(b + c) = ab + ac$	

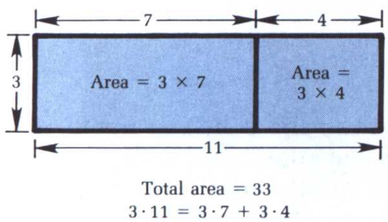


Figure 3. The Distributive Property.

These properties may seem obvious to you; you often use them informally. For example, if you add 43, 17, and 83 by first observing that $17 + 83 = 100$ and then adding 43, you are using the associative and commutative properties of addition. The distributive property is listed in between the addition and multiplication properties because it relates addition to multiplication. For example, if $a = 3$, $b = 7$, and $c = 4$, it states that $3(7 + 4)$ may be evaluated either as $3 \cdot 11$ or as $3 \cdot 7 + 3 \cdot 4$. Since the area of a rectangle is the product of its length and width, this may be pictured as in figure 3.

You may find it curious that the field properties do not mention subtraction and division, which are the inverse (undoing) operations for addition and multiplication. We define subtraction of any number b as addition of its additive inverse $-b$:

$a - b$ means $a + (-b)$

Similarly, we define division by any real number c (except zero) as multiplication by the multiplicative inverse $\frac{1}{c}$:

$a \div c$ means $a \cdot \dfrac{1}{c}$

These definitions allow us to manipulate real numbers by the usual rules of arithmetic. Some of these rules are reviewed in the exercises.

Order of Operations

A spoken phrase such as "three plus five times two" is ambiguous. Does it mean to add $3 + 5$ first, then multiply by 2, or to multiply $5 \cdot 2$ first, then add 3? The question is important because the dif-