

MATHEMATICAL APPLICATIONS

SECOND EDITION

HARSHBARGER / REYNOLDS

MATHEMATICAL APPLICATIONS

**FOR MANAGEMENT, LIFE, AND
SOCIAL SCIENCES**

SECOND EDITION

RONALD J. HARSHBARGER

The Pennsylvania State University

JAMES J. REYNOLDS

The Pennsylvania State University

Cover photograph: Sculpture at the World Trade Center, New York.
Douglas Faulkner/Photo Researchers, Inc.

Copyright © 1985 by D. C. Heath and Company.

Previous edition copyright © 1981 by D. C. Heath and Company.

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage or retrieval system, without permission in writing from the publisher.

Published simultaneously in Canada.

Printed in the United States of America.

International Standard Book Number: 0-669-07337-7

Library of Congress Catalog Card Number: 84-80480

Preface

To paraphrase Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. This desirable goal is not always an easy one to realize with students whose primary interest is in an area other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to areas that are important to students in the management, life, and social sciences. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years, or the equivalent, of high school algebra, this text begins with a brief review of algebra, which if covered will aid in preparing students for the work ahead.

Important pedagogical features that have been retained in this new edition are the following:

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Each topic is carefully explained, and examples illustrate the techniques involved. Exercises stress computation and drill, but there are enough challenging problems to stimulate students.

Flexibility. At different colleges or universities the coverage and sequencing of topics may vary according to the purpose of this course. To accommodate this, the text has a great deal of flexibility in the order of topics. At the beginning of each chapter the Chapter Warmup identifies which sections are prerequisite to the material covered in the chapter. Instructors may find this useful in creating a syllabus.

Applications. We have found that offering applied topics such as cost, revenue, and profit functions in a separate section brings the preceding mathematical discussions into clear and concise focus. There are 16 such sections in this book. Beyond this there are 1200 applied exercises and hundreds of applied examples throughout the text.

New to this second edition are the following features:

Previous Chapters 1 and 2 have been condensed into one chapter on linear equations and functions. This material may now be covered more quickly at the beginning of the course.

Section 3.4 on the simplex method has been rewritten with new explanations to further clarify this difficult concept.

Chapter 6, Mathematics of Finance, has been reorganized and improved. Applications are used to present the concepts of sequences, series, and sigma notation in Sections 6.1 through 6.3. Section 6.7 has been added to deal with the current rules for depreciation of property.

Section 7.8 has been added to discuss Markov chains.

Previous Chapters 10 and 11 have been combined to present limits and derivatives so that students may move more quickly into differentiation.

Integration is now covered in two chapters. Chapter 12, Indefinite Integrals, includes a new section on differential equations and their applications to drug absorption rate, carbon-14 dating, and Gompertz curves. Chapter 13, Definite Integrals, has a new section on improper integrals and their applications as well as new applications in probability and in finance.

With the increased coverage of calculus, this text may be used in both one- and two-semester courses.

The number of exercises has been enlarged by nearly 50%. The book now has 3500 exercises ranging in difficulty from routine to challenging.

Student Solutions Guide. In addition to an answer section at the end of the text, the solutions to all odd-numbered exercises are included in this supplementary booklet.

Acknowledgments. We would like to thank the many people who have helped us at various stages of this project. The encouragement, criticism, and suggestions that have been offered have been invaluable to us. We are especially indebted to Samuel Laposata, Virginia Electric Power Company, who provided ideas and encouragement, and to Frank Kocher, The Pennsylvania State University, who provided support and reviews throughout the first edition's evolution. Our special thanks are due William E. Beatty, Rochester Institute of Technology; James R. Hickey, Baylor University; Roseanne Hoffmann, Montgomery County Community College; John Hourlland, The Pennsylvania State University; Barbara Pettler, The Pennsylvania State University; Rosemary Schmalz, SP, University of Scranton; Albert G. White, St. Bonaventure University; and Benjamin W. Volker, Bucks County Community College, who reviewed the entire manuscript or parts of it and made many helpful comments. Survey respondents offered many valuable suggestions; our thanks to: Mark Ciancutti, Robert Morris College; Charles R. Diminnie, St. Bonaventure University; Douglas Lonnstrom, Siena College; Andris Niedra, Robert Morris College; K. Thanigasalam, The Pennsylvania State University; and Ramon J. Voltz, Grove City College. We would also like to express our appreciation to the editorial staff at D. C. Heath for its continued enthusiasm and support.

RONALD J. HARSHBARGER
JAMES J. REYNOLDS

Contents

PART ONE ALGEBRA REVIEW

0	Algebra Concepts	3
0.1	Sets	3
0.2	The Real Numbers	9
0.3	Exponents and Radicals	13
0.4	Operations with Algebraic Expressions	18
0.5	Factoring	23
0.6	Algebraic Fractions	28
	Review Exercises	33

PART TWO LINEAR MODELS

1	Linear Equations and Functions	37
1.1	Solution of Linear Equations in One Variable	38
1.2	Graphing Linear Equations	43
1.3	Functions	49
1.4	Applications of Functions in Business and Economics	59
	The Consumption Function 60 • Supply and Demand Functions 60 • Total Cost, Total Revenue, and Profit Functions 64	
1.5	Slope of a Line; Writing Equations of Lines	70

- 1.6 Solution of Linear Equations in Two Variables 79
 1.7 Business Applications of Linear Equations in Two Variables 86
 National Consumption 86 • Market Equilibrium 87 • Break-Even Analysis 91
 Review Exercises 98

2 Matrices 103

- 2.1 Matrices 104
 2.2 Multiplication of Matrices 110
 2.3 Gauss-Jordan Elimination: Solving Systems of Equations 119
 Systems with Unique Solutions 119 • Systems with Nonunique Solutions 126
 2.4 Inverse of a Square Matrix 135
 2.5 Applications of Matrices: Leontief Input-Output Models 142
 Review Exercises 153

3 Inequalities and Linear Programming 157

- 3.1 Linear Inequalities in One Variable 158
 3.2 Linear Inequalities in Two Variables 161
 3.3 Linear Programming: Graphical Methods 166
 3.4 The Simplex Method: Maximization 176
 3.5 The Simplex Method: Minimization 190
 Review Exercises 198

PART THREE NONLINEAR MODELS

4 Quadratic Functions 203

- 4.1 Quadratic Equations 204
 4.2 Quadratic Functions: Parabolas 208
 4.3 Business Applications of Quadratic Functions 217
 Supply, Demand, and Market Equilibrium 217 • Break-Even Points and Profit
 Maximization 219
 Review Exercises 224

5 Exponential and Logarithmic Functions 227

- 5.1 Exponential Functions 228
- 5.2 Logarithmic Functions and Their Properties 234
- 5.3 Applications of Exponential and Logarithmic Functions 243
 - Growth and Decay 243 • Economic and Management Applications 247
- Review Exercises 252

6 Mathematics of Finance 255

- 6.1 Simple Interest; Sequences 256
- 6.2 Compound Interest; Geometric Sequences 261
- 6.3 Sigma Notation; Sums of Terms of Sequences 269
- 6.4 Annuities 278
- 6.5 Loans; Amortization; Sinking Funds 284
 - Loans 284 • Amortization 286 • Sinking Funds 288
- 6.6 Depreciation of Property Purchased Before 1981 (Optional) 291
 - Straight-Line Methods 292 • Double-Declining-Balance Method 293 • Sum-of-the-Years-Digits Method 296
- 6.7 Depreciation of Property Purchased After 1980 (Optional) 300
 - Investment Tax Credit 304 • Expensing 304
- Review Exercises 306

PART FOUR PROBABILISTIC MODELS

7 Introduction to Probability 311

- 7.1 Probability 312
- 7.2 Independent and Dependent Events 322
- 7.3 Mutually Exclusive Events and Nonmutually Exclusive Events 329
- 7.4 Conditional Probability; Bayes' Formula 336
- 7.5 Counting; Permutations 345
- 7.6 Combinations 350

7.7	Expected Value: Decision Making	354
7.8	Markov Chains	359
	Review Exercises	368

8 Introduction to Statistics 373

8.1	Frequency Histograms	374
8.2	Measures of Central Tendency	382
8.3	Measures of Variation	391
8.4	Binomial Trials	399
8.5	The Binomial Distribution	404
8.6	The Normal Distribution	409
8.7	Samples from Normal Populations; Testing Hypotheses	416
8.8	Linear Regressions and Correlation	422
	Review Exercises	429

PART FIVE **CALCULUS**

9 Derivatives 435

9.1	Limits: Polynomial and Rational Functions	436
9.2	Continuous Functions	450
9.3	The Derivative: Rates of Change; Tangent to a Curve	456
9.4	Derivative Formulas	468
9.5	Product and Quotient Rules	477
9.6	The Chain Rule and Power Rule	483
9.7	Using Derivative Formulas	489
9.8	Applications of Derivatives in Business and Economics	493
	Review Exercises	498

10 Applications of Derivatives 501

10.1	Relative Maxima and Minima; Curve Sketching	502
10.2	More Maxima and Minima; Undefined Derivatives	510

10.3 Applications of Maxima and Minima to Business and Economics 516

Maximizing Revenue 517 • Minimizing Average Cost 518 • Maximizing Profit 519

10.4 Applications of Maxima and Minima 524

Review Exercises 530

11 Derivatives Continued 533

11.1 Higher-Order Derivatives 534**11.2 Concavity; Points of Inflection 537****11.3 Implicit Differentiation 542****11.4 Derivatives of Logarithmic Functions 548****11.5 Derivatives of Exponential Functions 533****11.6 Applications in Business and Economics 557**

Profit Maximization in a Competitive Market 557 • Profit Maximization in a Monopolistic Market 558 • Taxation in a Competitive Market 559 • Elasticity of Demand 561

Review Exercises 566

12 Indefinite Integrals 569

12.1 The Indefinite Integral 570**12.2 The Power Rule 574****12.3 Integrals Involving Logarithmic and Exponential Functions 579****12.4 Applications of the Indefinite Integral in Business and Economics 583**

Total Cost and Profit 584 • National Consumption and Savings 587

12.5 Differential Equations 590

Review Exercises 599

13 Definite Integrals 603

13.1 Area Under a Curve 604**13.2 The Definite Integral; The Fundamental Theorem of Calculus 611****13.3 Applications of Definite Integrals in Business and Economics; Consumer's Surplus, Producer's Surplus, and Present Value 618**

Consumer's Surplus 619 • Producer's Surplus 622 • Present Value of an Annuity 624

13.4	Using Tables of Integrals	627
13.5	Integration by Parts	631
13.6	Improper Integrals and Their Applications	635
	Review Exercises	644

14 Functions of Two or More Variables 647

14.1	Functions of Two or More Variables	648
14.2	Partial Differentiation	653
14.3	Applications of Functions of Two Variables in Business and Economics	661
	Joint Cost and Marginal Cost	661 • Production Functions
	Functions	662 • Demand Functions
		663
14.4	Higher-Order Partial Derivatives	668
14.5	Maxima and Minima	671
14.6	Maxima and Minima of Functions Subject to Constraints; Lagrange Multipliers	678
	Review Exercises	684

Appendix A1

Table I	Exponential Functions	A1
Table II	Selected Values of $\log_e x$	A2
Table III	Areas Under the Standard Normal Curve	A2
Table IV	Amount of \$1 at Compound Interest $\left(1 + \frac{i}{k}\right)^{kn}$	A5
Table V	Amount of an Ordinary Annuity of \$1 ($s_{\overline{n} i}$)	A8
Table VI	Present Value of an Annuity of \$1 ($a_{\overline{n} i}$)	A10
Table VII	Periodic Payment to Amortize \$1 at the End of n Periods ($1/a_{\overline{n} i}$)	A12
Table VIII	Periodic Payment for a Sinking Fund of \$1 at the End of n Periods ($1/s_{\overline{n} i}$)	A14

Answers to Selected Exercises A17

Index A67

PART ONE

ALGEBRA REVIEW

Algebra Concepts

This chapter provides a brief review of the algebraic concepts that will be used throughout the text. You should already be familiar with its topics, but it will be to your advantage to spend some time reviewing them. You will also find this chapter useful as a reference as you study related topics in later chapters.

0.1 Sets

A **set** is a well-defined collection of objects. We may talk about a set of books, a set of dishes, or a set of students. We shall be concerned with sets of numbers. There are two ways to tell what a given set contains. One way is by listing the **elements** (or **members**) of the set (usually between braces). We may say that a set A contains 1, 2, 3, and 4 by writing $A = \{1, 2, 3, 4\}$. To say that 4 is a member of set A , we write $4 \in A$.

If all the members of the set can be listed, the set is said to be a **finite set**. $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$ are examples of finite sets. Although we cannot list all the elements of an infinite set, we can use three dots to indicate the unlisted members of such a set. For example, $N = \{1, 2, 3, 4, \dots\}$ is an infinite set. This set N is called the set of **natural numbers**. Although they

are not all listed, we know $10 \in N$, $1121 \in N$, and $15,331 \in N$, but $\frac{1}{2}$ is not a member of N (that is, $\frac{1}{2} \notin N$) because $\frac{1}{2}$ is not a natural number.

Another way to specify the elements of a given set is by description. For example, we may write $D = \{x: x \text{ is a Ford automobile}\}$ to describe the set of all Ford automobiles. $F = \{y: y \text{ is an odd natural number}\}$ is read “ F is the set of all y such that y is an odd natural number.” Thus $3 \in F$, $5 \in F$, and $7 \in F$ because they are odd natural numbers, and $6 \notin F$ because 6 is not an odd natural number.

- EXAMPLE 1 Write the following sets in two ways:
- The set A of natural numbers less than 6.
 - The set B of natural numbers greater than 10.
 - The set C containing only 3.

Solution

- $A = \{1, 2, 3, 4, 5\}$ or $A = \{x: x \text{ is a natural number less than 6}\}$
- $B = \{11, 12, 13, 14, \dots\}$ or $B = \{x: x \text{ is a natural number greater than 10}\}$
- $C = \{3\}$ or $C = \{x: x = 3\}$ ■

Note that set C of Example 1 contains one member, 3; set A contains five members; and set B contains an infinite number of members. It is possible for a set to contain no members. Such a set is called the **empty set** or the **null set**, and it is denoted by \emptyset or by $\{ \}$. The set of living veterans of the War of 1812 is empty because there are no living veterans of that war. Thus

$$\{x: x \text{ is a living veteran of the War of 1812}\} = \emptyset.$$

Special relations that may exist between two sets are defined as follows.

Relations with Sets

DEFINITION	EXAMPLE
1. Sets X and Y are equal if they contain the same elements.	1. If $X = \{1, 2, 3, 4\}$ and $Y = \{4, 3, 2, 1\}$, then $X = Y$.
2. $A \subseteq B$ if every element of A is an element of B . A is called a subset of B . The empty set is a subset of every set.	2. If $A = \{1, 2, c, f\}$ and $B = \{1, 2, 3, a, b, c, f\}$, then $A \subseteq B$.
3. If C and D have no elements in common, they are called disjoint .	3. If $C = \{1, 2, a, b\}$ and $D = \{3, e, 5, c\}$, C and D are disjoint.

In the discussion of particular sets, the assumption is always made that the sets under discussion are all subsets of some larger set, called the **universal set** U . The choice of the universal set depends upon the prob-

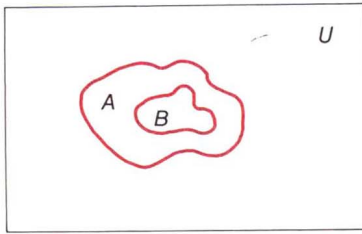


Figure 0.1

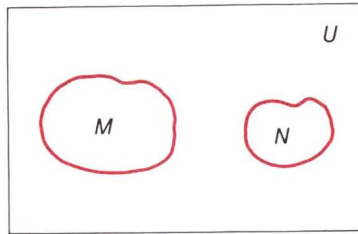


Figure 0.2

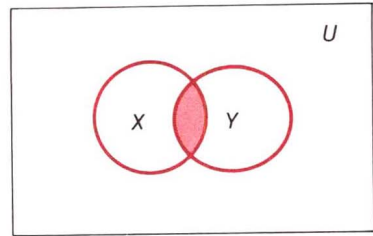


Figure 0.3

lem under consideration. For example, in discussing the set of all students and the set of all female students, we may use the set of all humans as the universal set.

We may use **Venn diagrams** to illustrate the relationships among sets. We use a rectangle to represent the universal set and closed figures inside the rectangle to represent the sets under consideration. Figure 0.1 shows such a Venn diagram.

Figure 0.1 shows that B is a subset of A ; that is, $B \subseteq A$. In Figure 0.2, M and N are disjoint sets. In Figure 0.3, sets X and Y overlap; that is, they are not disjoint.

The shaded portion of the diagram indicates where the two sets overlap. The set containing the members that are common to two sets is said to be in the **intersection** of the two sets.

Set Intersection The intersection of A and B , written $A \cap B$, is

$$A \cap B = \{x: x \in A \text{ and } x \in B\}.$$

EXAMPLE 2 If $A = \{2, 3, 4, 5\}$ and $B = \{3, 5, 7, 9, 11\}$, find $A \cap B$.

Solution $A \cap B = \{3, 5\}$ because 3 and 5 are in both A and B .

Figure 0.4 shows the sets and their intersection. ■

The **union** of two sets is the set that contains all members of the two sets.

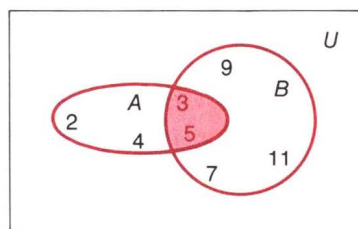


Figure 0.4

Set Union The union of A and B , written $A \cup B$, is
 $A \cup B = \{x: x \in A \text{ or } x \in B \text{ (or both)}\}.$

EXAMPLE 3 If $X = \{a, b, c, f\}$ and $Y = \{e, f, a, b\}$, find $X \cup Y$.

Solution $X \cup Y = \{a, b, c, e, f\}$ ■

EXAMPLE 4 Let $A = \{x: x \text{ is a natural number less than } 6\}$ and $B = \{1, 3, 5, 7, 9, 11\}$.
 (a) Find $A \cap B$. (b) Find $A \cup B$.

Solution (a) $A \cap B = \{1, 3, 5\}$
 (b) $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11\}$ ■

We can illustrate the intersection and union of two sets by the use of Venn diagrams. The shaded region in Figure 0.5 represents $A \cap B$, the intersection of A and B , while the shaded region in Figure 0.6 represents $A \cup B$.

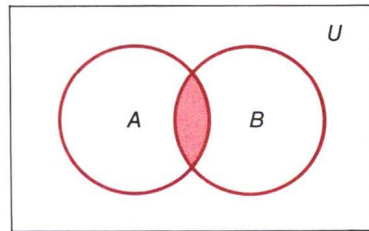


Figure 0.5

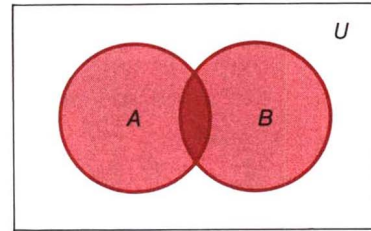


Figure 0.6

All elements of the universal set that are not contained in a set A form a set called the **complement** of A .

Set Complement The complement of A , written A' , is
 $A' = \{x: x \in U \text{ and } x \notin A\}.$

EXAMPLE 5 If $U = \{x \in \mathbb{N}: x < 10\}$, $A = \{1, 3, 6\}$, and $B = \{1, 6, 8, 9\}$, find
 (a) A' (b) B' (c) $(A \cap B)'$ (d) $A' \cup B'$

Solution (a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so $A' = \{2, 4, 5, 7, 8, 9\}$
 (b) $B' = \{2, 3, 4, 5, 7\}$
 (c) $A \cap B = \{1, 6\}$, so $(A \cap B)' = \{2, 3, 4, 5, 7, 8, 9\}$
 (d) $A' \cup B' = \{2, 4, 5, 7, 8, 9\} \cup \{2, 3, 4, 5, 7\}$
 $= \{2, 3, 4, 5, 7, 8, 9\}$ ■