MATHEMATICAL APPLICATIONS

SECOND EDITION

HARSHBARGER / REYNOLDS

MATHEMATICAL APPLICATIONS

FOR MANAGEMENT, LIFE, AND SOCIAL SCIENCES

SECOND EDITION

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Preface

To paraphrase Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. This desirable goal is not always an easy one to realize with students whose primary interest is in an area other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to areas that are important to students in the management, life, and social sciences. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years, or the equivalent, of high school algebra, this text begins with a brief review of algebra, which if covered will aid in preparing students for the work ahead.

Important pedagogical features that have been retained in this new edition are the following:

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Each topic is carefully explained, and examples illustrate the techniques involved. Exercises stress computation and drill, but there are enough challenging problems to stimulate students.

Flexibility. At different colleges or universities the coverage and sequencing of topics may vary according to the purpose of this course. To accommodate this, the text has a great deal of flexibility in the order of topics. At the beginning of each chapter the Chapter Warmup identifies which sections are prerequisite to the material covered in the chapter. Instructors may find this useful in creating a syllabus.

Applications. We have found that offering applied topics such as cost, revenue, and profit functions in a separate section brings the preceding mathematical discussions into clear and concise focus. There are 16 such sections in this book. Beyond this there are 1200 applied exercises and hundreds of applied examples throughout the text.

New to this second edition are the following features:

Previous Chapters 1 and 2 have been condensed into one chapter on linear equations and functions. This material may now be covered more quickly at the beginning of the course.

Section 3.4 on the simplex method has been rewritten with new explanations to further clarify this difficult concept.

Chapter 6, Mathematics of Finance, has been reorganized and improved. Applications are used to present the concepts of sequences, series, and sigma notation in Sections 6.1 through 6.3. Section 6.7 has been added to deal with the current rules for depreciation of property.

Section 7.8 has been added to discuss Markov chains.

Previous Chapters 10 and 11 have been combined to present limits and derivatives so that students may move more quickly into differentiation.

Integration is now covered in two chapters. Chapter 12, Indefinite Integrals, includes a new section on differential equations and their applications to drug absorption rate, carbon-14 dating, and Gompertz curves. Chapter 13, Definite Integrals, has a new section on improper integrals and their applications as well as new applications in probability and in finance.

With the increased coverage of calculus, this text may be used in both one- and two-semester courses.

The number of exercises has been enlarged by nearly 50%. The book now has 3500 exercises ranging in difficulty from routine to challenging.

Student Solutions Guide. In addition to an answer section at the end of the text, the solutions to all odd-numbered exercises are included in this supplementary booklet.

Acknowledgments. We would like to thank the many people who have helped us at various stages of this project. The encouragement, criticism, and suggestions that have been offered have been invaluable to us. We are especially indebted to Samuel Laposata, Virginia Electric Power Company, who provided ideas and encouragement, and to Frank Kocher, The Pennsylvania State University, who provided support and reviews throughout the first edition's evolution. Our special thanks are due William E. Beatty, Rochester Institute of Technology; James R. Hickey, Baylor University; Roseanne Hofmann, Montgomery County Community College; John Hourlland, The Pennsylvania State University; Barbara Pettler, The Pennsylvania State University; Rosemary Schmalz, SP, University of Scranton; Albert G. White, St. Bonaventure University; and Benjamin W. Volker, Bucks County Community College, who reviewed the entire manuscript or parts of it and made many helpful comments. Survey respondents offered many valuable suggestions; our thanks to: Mark Ciancutti, Robert Morris College; Charles R. Diminnie, St. Bonaventure University; Douglas Lonnstrom, Siena College; Andris Niedra, Robert Morris College; K. Thanigasalam, The Pennsylvania State University; and Ramon J. Voltz, Grove City College. We would also like to express our appreciation to the editorial staff at D. C. Heath for its continued enthusiasm and support.

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Contents

PART ONE
ALGEBRA REVIEW

0

Algebra Concepts

This chapter provides a brief review of the algebraic concepts that will be used throughout the text. You should already be familiar with its topics, but it will be to your advantage to spend some time reviewing them. You will also find this chapter useful as a reference as you study related topics in later chapters.

0.1 Sets

A **set** is a well-defined collection of objects. We may talk about a set of books, a set of dishes, or a set of students. We shall be concerned with sets of numbers. There are two ways to tell what a given set contains. One way is by listing the **elements** (or **members**) of the set (usually between braces). We may say that a set A contains 1, 2, 3, and 4 by writing $A = \{1, 2, 3, 4\}$. To say that 4 is a member of set A, we write $4 \in A$.

If all the members of the set can be listed, the set is said to be a **finite set.** $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$ are examples of finite sets. Although we cannot list all the elements of an infinite set, we can use three dots to indicate the unlisted members of such a set. For example, $N = \{1, 2, 3, 4, \ldots\}$ is an infinite set. This set N is called the set of **natural numbers.** Although they are not all listed, we know $10 \in N$, $1121 \in N$, and $15{,}331 \in N$, but $\frac{1}{2}$ is not a member of N (that is, $\frac{1}{2} \notin N$) because $\frac{1}{2}$ is not a natural number.

Another way to specify the elements of a given set is by description. For example, we may write $D = \{x: x \text{ is a Ford automobile}\}$ to describe the set of all Ford automobiles. $F = \{y: y \text{ is an odd natural number}\}$ is read "F is the set of all y such that y is an odd natural number." Thus $3 \in F$, $5 \in F$, and $7 \in F$ because they are odd natural numbers, and $6 \notin F$ because 6 is not an odd natural number.

EXAMPLE 1 Write the following sets in two ways:

(a) The set A of natural numbers less than 6.
(b) The set B of natural numbers greater than 10.
(c) The set C containing only 3.

Solution
(a) $A = \{1, 2, 3, 4, 5\}$ or $A = \{x: x \text{ is a natural number less than } 6\}$
(b) $B = \{11, 12, 13, 14 \ldots\}$ or $B = \{x: x \text{ is a natural number greater than } 10\}$
(c) $C = \{3\}$ or $C = \{x: x = 3\}$ ■

Note that set C of Example 1 contains one member, 3; set A contains five members; and set B contains an infinite number of members. It is possible for a set to contain no members. Such a set is called the **empty set** or the **null set**, and it is denoted by $\varnothing$ or by $\{\ \}$. The set of living veterans of the War of 1812 is empty because there are no living veterans of that war. Thus

$$\{x: x \text{ is a living veteran of the War of 1812}\} = \varnothing.$$

Special relations that may exist between two sets are defined as follows.

Relations with Sets

DEFINITION	EXAMPLE
1. Sets X and Y are **equal** if they contain the same elements.	1. If $X = \{1, 2, 3, 4\}$ and $Y = \{4, 3, 2, 1\}$, then $X = Y$.
2. $A \subseteq B$ if every element of A is an element of B. A is called a **subset** of B. The empty set is a subset of every set.	2. If $A = \{1, 2, c, f\}$ and $B = \{1, 2, 3, a, b, c, f\}$, then $A \subseteq B$.
3. If C and D have no elements in common, they are called **disjoint.**	3. If $C = \{1, 2, a, b\}$ and $D = \{3, e, 5, c\}$, C and D are disjoint.

In the discussion of particular sets, the assumption is always made that the sets under discussion are all subsets of some larger set, called the **universal set** U. The choice of the universal set depends upon the prob-

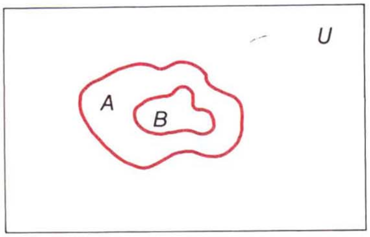

Figure 0.1

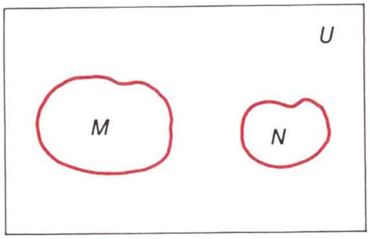

Figure 0.2

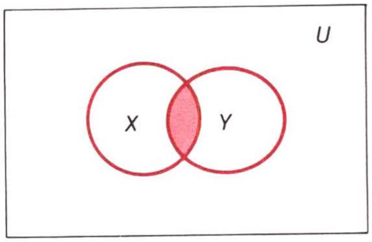

Figure 0.3

lem under consideration. For example, in discussing the set of all students and the set of all female students, we may use the set of all humans as the universal set.

We may use **Venn diagrams** to illustrate the relationships among sets. We use a rectangle to represent the universal set and closed figures inside the rectangle to represent the sets under consideration. Figure 0.1 shows such a Venn diagram.

Figure 0.1 shows that B is a subset of A; that is, $B \subseteq A$. In Figure 0.2, M and N are disjoint sets. In Figure 0.3, sets X and Y overlap; that is, they are not disjoint.

The shaded portion of the diagram indicates where the two sets overlap. The set containing the members that are common to two sets is said to be in the **intersection** of the two sets.

Set Intersection The intersection of A and B, written $A \cap B$, is

$$A \cap B = \{x: x \in A \text{ and } x \in B\}.$$

EXAMPLE 2 If $A = \{2, 3, 4, 5\}$ and $B = \{3, 5, 7, 9, 11\}$, find $A \cap B$.

Solution $A \cap B = \{3, 5\}$ because 3 and 5 are in both A and B.

Figure 0.4 shows the sets and their intersection. ■

The **union** of two sets is the set that contains all members of the two sets.

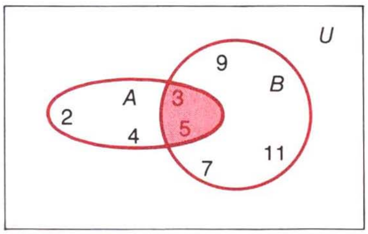

Figure 0.4

Set Union The union of A and B, written $A \cup B$, is

$$A \cup B = \{x: x \in A \text{ or } x \in B \text{ (or both)}\}.$$

EXAMPLE 3 If $X = \{a, b, c, f\}$ and $Y = \{e, f, a, b\}$, find $X \cup Y$.

Solution $X \cup Y = \{a, b, c, e, f\}$ ■

EXAMPLE 4 Let $A = \{x: x$ is a natural number less than $6\}$ and $B = \{1, 3, 5, 7, 9, 11\}$.
(a) Find $A \cap B$. (b) Find $A \cup B$.

Solution (a) $A \cap B = \{1, 3, 5\}$
(b) $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11\}$ ■

We can illustrate the intersection and union of two sets by the use of Venn diagrams. The shaded region in Figure 0.5 represents $A \cap B$, the intersection of A and B, while the shaded region in Figure 0.6 represents $A \cup B$.

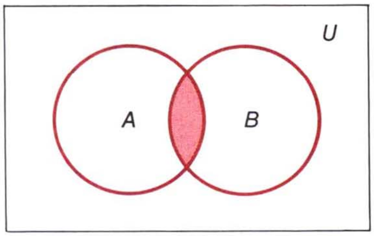

Figure 0.5

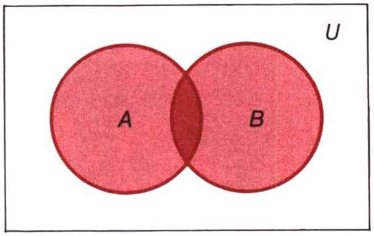

Figure 0.6

All elements of the universal set that are not contained in a set A form a set called the **complement** of A.

Set Complement The complement of A, written A', is

$$A' = \{x: x \in U \quad \text{and} \quad x \notin A\}.$$

EXAMPLE 5 If $U = \{x \in N: x < 10\}$, $A = \{1, 3, 6\}$, and $B = \{1, 6, 8, 9\}$, find
(a) A' (b) B' (c) $(A \cap B)'$ (d) $A' \cup B'$

Solution (a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so $A' = \{2, 4, 5, 7, 8, 9\}$
(b) $B' = \{2, 3, 4, 5, 7\}$
(c) $A \cap B = \{1, 6\}$, so $(A \cap B)' = \{2, 3, 4, 5, 7, 8, 9\}$
(d) $A' \cup B' = \{2, 4, 5, 7, 8, 9,\} \cup \{2, 3, 4, 5, 7\}$
$= \{2, 3, 4, 5, 7, 8, 9\}$ ■