Temperature
Measurement
and Control

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FOREWORD

THERE is an unceasing demand today for trained technologists as opposed to pure scientists and, whilst the universities, in spite of being heavily engaged, are effectively dealing with the latter, they are considerably handicapped by the lack of trained technologists as teaching staff. The Technical Colleges suffer from this same difficulty in staff recruitment.

Furthermore, the chemical industry is changing from what could be described as organized batch processing, to continuous automatic control of process variables such as temperature, flow, pH, conductivity, etc. In this transition the problem of automatic process control in turn shows how lacking are fundamental data on the physical aspects of many control problems and leads to another demand for specialized control techniques.

The major demand in the modern chemical industry is for trained personnel handling development, pilot production, and main production lines. Some of these executives arrive in responsible positions by virtue of the adoption of an apprenticeship scheme followed up with sandwich courses leading to Higher National Certificates or external University degrees. They might be described as approaching their goal as fully trained chemical engineers from the practical to the theoretical. Their opposite numbers leaving the universities with degrees in chemistry are of course progressing contrariwise.

Whichever the approach there is a lack of informed technological textbooks which can be used with advantage by both types of student. To some extent it is hoped that this series of books will help to fill this gap. Since the appeal is both to the graduate and non-graduate, an attempt has been made to introduce theoretical studies where necessary and to discuss physical processes in the chemical engineering industry with an emphasis upon modern industrial applications.

The overall approach has therefore been one of presenting a series of books on technology to enable both types of student to obtain practical information on the kind of problems which prevail in every chemical or metallurgical production with theoretical discussions necessary to bring understanding, thereby enabling the student 'to reason why'.

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PREFACE

THIS book is written as an introduction to the problems of temperature measurement and control for young people entering Industry. Much of the theoretical mathematics associated with temperature considerations has been omitted and the early chapters present basic physical data briefly, rather as a quick revision course prior to discussion of instruments and control methods.

Considerable use has been made of British Standard 1041 : 1943 entitled *Temperature Measurement* (Figures 50–53, 63–68, 108, 164–169, and Tables 2–16, 18, 19, 21, 23, 24, 26) which is essential since this forms a common link in terminology between the instrument manufacturer and user. The author wishes to acknowledge the co-operation he has received from the British Standards Institution.

When the author was teaching at King Edwards' Grammar School, Birmingham, and lecturing at the Polytechnic, Regent Street, he used extensively an excellent textbook, *Intermediate Physics* (1935 edn.), by C. J. Smith (Arnold, London) and he has made full use of this book and some of its illustrations in the introductory chapters referred to above (Figures 1–27). He wishes to thank Dr. Smith for his permission.

An excellent advanced treatise on automatic process control is entitled *An Introduction to Process Control Systems Design* by A. J. Young (Longmans, Green, London) and information from this book and some of the illustrations (Figures 153–162) have been used particularly in dealing with the frequency response of temperature systems. Again the author wishes to thank Dr. Young for permission to use his illustrations. The terminology on automatic process control has been taken from the third draft revision of British Standards 1523.

A considerable part of the book deals with typical measuring and control instruments as well as the use of these instruments and control equipment in practice. The author has drawn extensively upon the literature of Electroflo Meters Co. Ltd (Figures 41–49, 54–62, 69–87, 99–107, 119–122, 148–151, 190–195), Elliott Brothers (London) Ltd (Figure 113), and James Gordon Ltd (Figures 175–189) (all members of Elliott-Automation Ltd), while reference is made in the text where equipment manufactured by other firms is described.

The author wishes to acknowledge the help he has received from the Directors and staff of Electroflo Meters Co. Ltd and for the use

of their reports and technical information sheets, although, of course, the author accepts personal responsibility for the statements made and the opinions expressed.

The author also wishes to express his indebtedness to the following for the use of original illustrations: *Instrument Manual* (United Trade Press, London) (Figures 30–40, 109–112); Fielden Electronics Ltd (Figures 10–15, 88, 92); Evershed and Vignoles Ltd (Figures 93–98); Cambridge Instrument Co. Ltd (Figures 114–117); Harrison and Son, Stoke-on-Trent (Figure 118, Table 22); Land Pyrometer Ltd (Figures 171, 172); *The Steam Engineer* (Figures 173, 174); *The New Scientist* (Figures 197–199); *Metal Treatment* (Figure 196); and *Ceramic Data Book* (Figures 200–204).

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WILFRED F. COXON
SHEPPERTON, FEBRUARY, 1959

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CONTENTS

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SIGNIFICANT PRELIMINARY DATA
ON HEAT

ALTHOUGH the reader is assumed to have a general background
knowledge of Physics, it nevertheless seems worth while to recapitu-
late those important theories in Physics, especially those of heat and
electricity which are indispensable to an understanding of the
working of the instruments used in the measurement and control
of temperature.

The popular conception of a hot or cold body is based upon the
sense of touch or feeling. In this way the terms 'hot' and 'cold' are
relative and, while serving to distinguish between a hot and a cold
body, they do not do so with any degree of accuracy. This is
particularly so if there is little difference in the relative temperatures
of the bodies. The first point to be considered is that heat is a form
of energy. There is obviously much more heat in the River Thames
as it passes under a bridge than there is in a kettle of boiling water
because, as will be discussed later, the mass of a material is related
to its heat content. Nevertheless the kettle of boiling water feels hot
and the river cold. Instruments which are capable of differentiating
between different 'degrees of heat' are called thermometers, and
the well known mercury-in-glass thermometer is one of the easiest
and most elementary examples.

It is common with all thermometric systems to consider the two
well known fixed points, namely the melting point of ice and the
boiling point of water. Between these two limits one can devise
different thermometric scales according to the number of divisions
one cares to place between the two points. On the Fahrenheit scale
the melting point of ice is 32° F and the boiling point of water is
considered to be 212° F. On the other hand, on the Centigrade or
Celsius scale the melting point of ice is considered to be zero and
the boiling point of water is 100° C. In the Réaumur scale, again
the melting point of ice is considered to be zero and the boiling
point of water 80° R. All solid or liquid expansion thermometers
depend upon a thermometric substance which will expand when
the temperature changes. The material used must have a relatively
large expansion, must be easily observed, have a large working

range, be easily obtainable, not easily contaminated and rapidly assume the temperature to which it is subjected. Mercury, although possessing most of these characteristics, suffers from the defect that it freezes at $-40°$ C, so that for temperatures below $-30°$ C an alcohol or pentane thermometer is used. Furthermore, mercury expands in jerks; this is due to the change in volume of the containing bulb when subjected to changes in pressure caused by variation in the angle of contact of the mercury. The density of mercury is high, so that the bulb of the thermometer is affected by the changes in the pressure produced by the variations in the length of the mercury column. In this respect alcohol and chloroform are preferred, but they cannot be used at high temperatures because there is a tendency for them to distil into the upper parts of the thermometer.

To increase the working range of the mercury thermometer the space above it may be filled with nitrogen. When the mercury expands, the pressure inside increases so that the boiling point of the mercury is raised. The thermometer may therefore be used to measure higher temperatures, but the variations of the volume of the bulb due to pressure are increased so that such a thermometer cannot be regarded as a reliable instrument. It has also been proposed to use an alloy of sodium and potassium which is liquid from $-8°$ C to $700°$ C; although it has also been found that, as a result of the chemical reduction of the glass by the alloy, a brown silica deposit is formed so that after a while the liquid cannot be seen.

Most substances expand when their temperature is raised, and there is a definite relationship between the amount of this expansion of the material and the temperature through which it is raised. *The change in length per unit length of a body for a rise in temperature of $1°$ C is called the Coefficient of Linear Expansion of the material.*

Let $l_1 =$ length of body at temperature t_1
and $l_2 =$ length of body at temperature t_2
where l_2 and $t_2 > l_1$ and t_1.

Then $\dfrac{l_2 - l_1}{l_1(t_2 - t_1)} =$ change in length/unit length/degree rise in temperature

= mean coefficient of linear expansion

= λ

If $l_0 =$ length at $0°$ C, then

$$\lambda = \frac{l - l_0}{l_0 t}$$

between $0°$ C and $t°$ C where $l =$ length at $t°$ C.

The quantity $(l - l_0)/l_0$ is a number, so that the dimensions of λ are those of a temperature reciprocal, i.e. λ depends on the temperature scale employed. If λ_C and λ_F are the coefficients of expansion for Centigrade and Fahrenheit scales,

then
$$\lambda_C = \frac{l - l_0}{100 l_0}$$

and
$$\lambda_F = \frac{l - l_0}{180 l_0}$$

or
$$\lambda_F = \tfrac{5}{9}\lambda_C.$$

In a similar way the mean coefficient of volume expansion between $0°$ C and t is

$$\alpha = \frac{v - v_0}{v_0 t}$$

The coefficient of linear expansion of a rod of length l at a temperature t is

$$\frac{1}{l} \cdot \frac{dl}{dt}$$

and the coefficient of volume expansion is

$$\frac{1}{v} \cdot \frac{dv}{dt}$$

If a cube 1 cm edge is heated through $1°$ C, each edge becomes $(1 + \lambda)$ and the volume $(1 + \lambda)^3 = 1 + 3\lambda + 3\lambda^2 + \lambda^3$. Since λ is small, λ^2 and λ^3 can be ignored and the final volume $= 1 + 3\lambda$. In other words, *the coefficient of volume expansion is three times the coefficient of linear expansion.*

So far we have been considering the expansion of solids. There is a slightly different approach when liquids are considered. A liquid has no definite shape of its own and assumes the shape of the vessel in which it is contained. Thus we cannot speak of a linear expansion but only its volume expansion. However, when a liquid is placed in a graduated container and expands, the expansion observed is the expansion of the liquid together with that of the containing vessel—it is therefore termed the *apparent expansion* as distinct from the *real expansion* of the liquid itself.

The absolute coefficient of expansion of a liquid is determined from observations on the density of a liquid at different temperatures. If ρ is the density, v is the volume of 1 g (the specific volume) then $\rho = 1/v$. At $0°$ C $\rho_0 = 1/v_0$. At $t°$ C $\rho_t = 1/v_t$. But $v_t = v_0(1 + \alpha t)$,

where α is the mean absolute coefficient of cubical expansion over a temperature range from $0°$ C to $t°$ C. Hence $\rho_0/\rho_t = 1/v_0 \times v_0(1 + \alpha t) = 1 + \alpha t$, so that α can be determined if the density of a liquid at two temperatures is known.

There are many classical methods of determining this absolute coefficient of expansion of a liquid for which reference should be made to the standard textbooks on heat. In passing, reference might be made to the anomalous expansion of water, for this has a maximum density at about $4°$ C, a fact which shows that the statement 'the expansion of water increases with rise of temperature' is anomalous, i.e. water at $4°$ C actually expands when it cools.

It is known that the volume of a given mass of gas at constant temperature depends upon the pressure to which it is subjected, so that if we wish to investigate how its volume varies with temperature the pressure inside the apparatus can be maintained at a constant value. The 'coefficient of increase in volume at constant pressure' is defined as the fraction of the volume at the temperature of melting ice at which the volume of a given mass of gas increases for a rise in temperature of $1°$ C, the pressure remaining constant.

$$\text{Hence } \alpha_p = \frac{\text{increase in volume at constant pressure}}{(\text{volume at temperature of melting ice}) \times (\text{change in temperature})}$$

$$= \frac{v_t - v_0}{v_0 t}$$

It was Gay-Lussac and later Regnault who investigated the thermal expansion of gases at constant pressure. They found that for the so-called permanent gases, those gases at that time believed to be incapable of liquefaction, such as hydrogen, oxygen, nitrogen, helium, etc., this coefficient was equal to 0.00367 or $1/273°$ C (an expression of a law generally attributed to Charles).

On the other hand, if a gas heated under such conditions that its volume remains constant then its pressure will increase. *The coefficient of increase in pressure at constant volume is defined as the fraction of the pressure at the temperature of melting ice by which the pressure of a given mass of gas increases for a rise in temperature of $1°$ C, the volume remaining constant.* If the gas obeys the laws of Boyle and Charles, then these two coefficients of expansion, i.e. at constant volume and constant pressure, are equal, i.e. $\alpha_p = \alpha_v$.

From time to time reference is made to the absolute zero of temperature. A gas at constant pressure expands by $1/273$ of its volume at $0°$ C for every degree Centigrade rise in temperature

(or contracts for falling temperatures), so that it follows that at 273° below 0° C the volume of the gas would become zero if the gas retained its normal properties, i.e. if it did not liquefy or solidify. For mathematical reasons it is desirable to introduce the concept of such a temperature and this is referred to as the Absolute Zero. In Gay-Lussac's law we find that the volume of a given mass of gas at constant pressure is directly proportional to its absolute temperature.
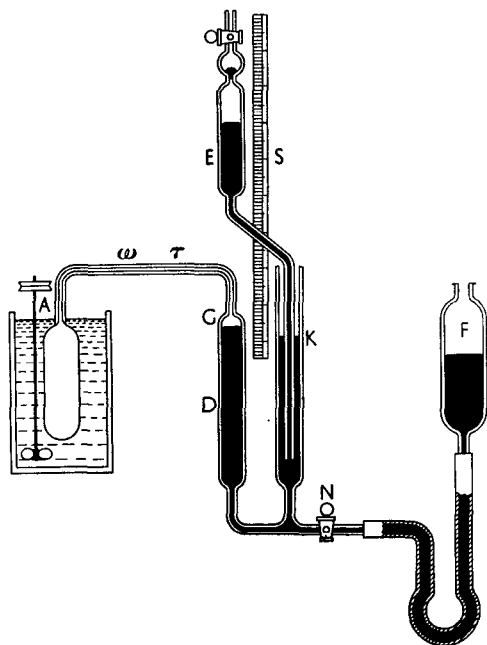

Figure 1. Typical constant volume gas thermometer

A simple form of constant volume gas thermometer is shown in Figure 1. Here a bulb A is connected by means of a capillary tube to a manometer DE, the space above the mercury being exhausted so that observations on a second barometer are unnecessary. If at any time any gas should find its way into the space above E it may be forced into the small bulb above the constriction shown by raising F. When F is restored to its normal position a small pellet of mercury remains in the constriction, the gas being trapped above it. The reservoir F containing mercury may be raised or lowered by means of a pulley. In order to use this apparatus to measure an

unknown steady temperature, it must first be used to determine the reading on the absolute scale of temperature corresponding to $0°$ C, the temperature of steam under standard conditions being defined as $100°$ C. To do this the bulb A is first immersed in melting ice and then in steam. In either instance the pressure inside the bulb A is determined when the mercury in the manometer is in contact with the extremity of a piece of black glass G which serves as an arbitrary mark.

Neglecting the volume of the dead space and the expansion of the bulb A and assuming $\bar{V}$ to be the constant volume of A, let $T_0 =$ absolute temperature corresponding to zero on the Centigrade scale and $t =$ steam temperature at time of experiment and the absolute temperature $= (T_0 + t)$.

Let p_0 and $p_t =$ pressures in A at temperatures $0°$ C and $t°$ C.

$$\frac{p_0 \bar{V}}{R T_0} = \text{mass of gas in A} = \frac{p_t \bar{V}}{R(T_0 + t)}$$

therefore,

$$\frac{p_0}{T_0} = \frac{p_t}{(T_0 + t)}$$

from which T_0 may be calculated. If $\theta =$ the temperature on the Centigrade scale to be measured, and $p_\theta =$ pressure in A when its temperature is θ, then

$$\frac{p_0}{T_0} = \frac{p_0}{T_0 + \theta}$$

so that θ may be determined. In practice, however, this simplification is likely to introduce errors. Let $p =$ pressure and $V =$ volume of A with suffixes denoting these variables at corresponding temperatures. Let $\omega =$ volume of dead space and L its mean temperature as measured by calibrated mercury thermometers placed near it (these thermometers may be used since terms containing ω and L only appear as small quantities in the final equations).

$L_0 =$ mean temperature of ω when A is at $0°$ C. ω_0 is the particular value of ω under these conditions, etc. Then

$$\frac{p_0 V_0}{R T_0} + \frac{p_0 \omega_0}{R(T_0 + L_0)} = \text{mass of gas enclosed}$$

$$= \frac{p_t V_t}{R(T_0 + t)} + \frac{p_t \omega_t}{R(T_0 + L_t)}$$

$$= \frac{p_t V_0[1 + \gamma t]}{R(T_0 + t)} + \frac{p_t \omega_t[1 + \gamma(L_t - L_0)]}{R(T_0 + L_t)}$$

where $\gamma =$ coefficient of cubical expansion of glass.

To solve this equation for T_0 we must omit all terms containing ω—the 'correction terms'—and use the resulting equation to get an approximate value of T_0. This value is then inserted in the correction terms of the more exact equation and the equation so obtained solved for T_0.

When A is at $\theta°$ C we have

$$\frac{p_0 V_0}{RT_0} + \frac{p_0 \omega_0}{R(T_0 + L_0)} = \frac{p_\theta V_\theta}{R(T_0 + \theta)} + \frac{p_0 \omega_\theta}{R(T_0 + L_\theta)}$$

$$= \frac{p_\theta V_0[1 + \gamma\theta]}{R(T_0 + \theta)} + \frac{p_\theta \omega_\theta[1 + \gamma(L_\theta - L_0)]}{R(T_0 + L_\theta)}$$
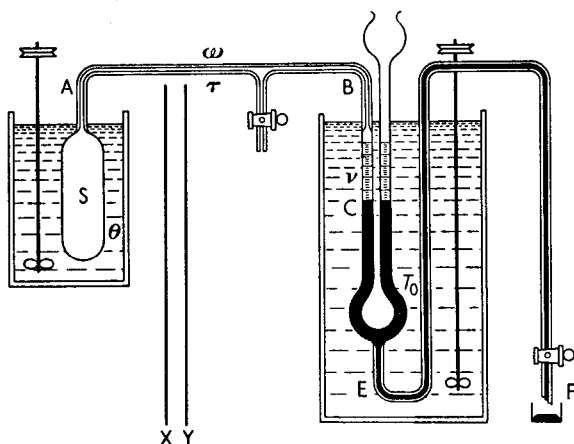

Figure 2. Typical constant pressure gas thermometer

A constant pressure gas thermometer is shown in Figure 2. Although the apparatus shown in Figure 1 may be used as a constant pressure gas thermometer, such thermometers are not capable of giving accurate results because gas tends to leak past the mercury pellet. A more modern form is therefore shown in Figure 2. Here a bulb S is connected to a mercury manometer by means of a narrow tube AB of 1-mm diameter. The amount of mercury in the manometer, and hence the pressure of the apparatus, may be controlled by the siphon EF. The bulb S and the manometer CE are immersed in baths, the temperature of the former being varied while that of the latter is kept at T_0. Stirrers placed in these baths help to keep the temperatures uniform. Polished metal screens X and Y diminish the exchange of heat between the two sides of the

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apparatus. To use this thermometer to measure a steady temperature the instrument is standardized with its bulb in ice and then in steam as mentioned previously.

Let V = volume of bulb S,

ω = volume of connecting tubes,

v = volume above mercury column in manometer,

L = mean temperature of ω as measured with calibrated thermometers,

π = pressure of gas (constant throughout experiment),

$$\frac{\pi V_0}{T_0} + \frac{\pi \omega_0}{(T_0 + L_0)} + \frac{\pi v_0}{T_0} = \frac{\pi V_t}{(T_0 + t)} + \frac{\pi \omega_t}{(T_0 + L_t)} + \frac{\pi v_t}{T_0}$$

where v_t = volume above mercury column when bulb S is at $t°$ C. Let γ = coefficient of cubical expansion of glass, therefore

$$\frac{V_0}{T_0} + \frac{\omega_0}{(T_0 + L_0)} + \frac{v_0}{T_0}$$

$$= \frac{V_0(1 + \gamma t)}{(T_0 + t)} + \frac{\omega_0[1 + \gamma(L_t - L_0)]}{(T_0 + L_t)} + \frac{v_t}{T_0}$$

Therefore T_0 may be calculated since v_t is known from the position of the mercury in the manometer. In a similar way

$$\frac{V_0}{T_0} + \frac{\omega_0}{(T_0 + L_0)} + \frac{v_0}{T_0} = \frac{V_0(1 + \gamma\theta)}{(T_0 + \theta)} + \frac{\omega_0[1 + \gamma(L_\theta - L_0)]}{(T_0 + L_\theta)} + \frac{v_\theta}{T_0}$$

so that θ may be determined since T_0 is known.

Callendar's Compensated Gas Thermometer, shown in Figure 3, overcomes the errors of the instrument described in Figure 2 chiefly due to the determination of v since the mercury column is viewed through a water bath. In eliminating this error, and also the dead space correction, this gas thermometer was devised. It consists of a glass or silica bulb V attached by a 1-mm diameter capillary tubing to another bulb M containing mercury which is kept in melting ice. Mercury may be withdrawn from it by means of the siphon A. The compensating bulb S is maintained at 0° C throughout the determination. (The apparatus was bent at X and Y to enable the capillary tubes to lie side by side at the same mean temperature.) Initially all bulbs are 0° C and the pressure, π, and mass of gas, m, on each side are equal.

Let V, S, and M be the volumes of gas in V, S, and above the mercury in M.

When all bulbs are in melting ice

$$\frac{\pi V_0}{RT_0} + \frac{\pi \omega_0}{R(T_0 + L_0)} + \frac{\pi M_0}{RT_0} = \frac{\pi \omega_0}{R(T_0 + L_0)} + \frac{\pi S_0}{RT_0} = m$$

i.e.
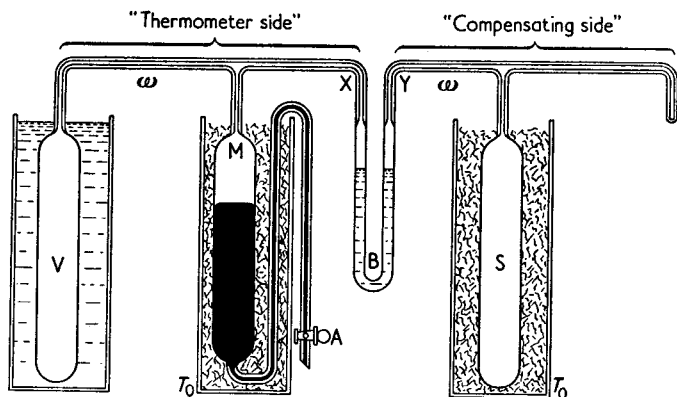$$\frac{V_0}{T_0} + \frac{M_0}{T_0} = \frac{S_0}{T_0}$$

Figure 3. Callendar's compensated gas thermometer

In a similar manner when V is at $t°$ C

$$\frac{\pi V_t}{R(T_0 + t)} + \frac{\pi \omega_t}{R(T_0 + L_t)} + \frac{\pi M_t}{RT_0} = \frac{\pi \omega_t}{R(T_0 + L_t)} + \frac{\pi S_0}{RT_0} = m$$

i.e.
$$\frac{V_t}{T_0 + t} + \frac{M_t}{T_0} = \frac{S_0}{T_0}$$

therefore,
$$\frac{V_0}{T_0} + \frac{M_0}{T_0} = \frac{V_t}{(T_0 + t)} + \frac{M_t}{T_0}$$

or
$$\frac{V_0(1 + \gamma t)}{(T_0 + t)} = \frac{V_0 + M_0 - M_t}{T_0}$$

Thus T_0 may be determined.
If V is at $\theta°$ C

$$\frac{V_0(1 + \gamma \theta)}{(T_0 + \theta)} = \frac{V_0 + M_0 - M_\theta}{T_0}$$

so that θ may be determined.

If, however, a silica bulb is used, γ may be neglected.

Therefore,
$$\frac{T_0 + \theta}{T_0} = \frac{V_0}{V_0 + M_0 - M_\theta}$$

or
$$\theta = T_0\left[\frac{(M_\theta - M_0)}{(V_0 + M_0 - M_\theta)}\right]$$

Another conception of extreme importance is the quantity of heat. The fact that two or more equal masses of different materials are at the same temperature does not mean that they contain the same quantity of heat. This is because different bodies contain different quantities of thermal energy. The unit of quantity of heat is called the calorie and is defined as *the quantity of heat required to raise the temperature of 1 g of water through 1° C*. This particular unit of heat is sometimes called the small or gram-calorie to distinguish it from the large or kilogram-calorie, which is defined as the amount of heat necessary to raise the temperature of 1 kg of water 1° C. Engineers also use another unit of heat which is known as the British Thermal Unit (B.t.u.). This is the amount of heat required to raise the temperature of 1 lb of water through 1° F. On the other hand, gas engineers find that this unit is too small so they have adopted a unit of heat called the Therm, which is equal to 100,000 B.t.u.

If equal masses of different materials (solid or liquid) are dropped into the same mass of water at the same temperature, generally the rise in temperature is different in each case. The bodies are said to have different specific heats. *The specific heat of a body is equal to the amount of heat necessary to raise the temperature of unit mass of the material through 1° C*. On the other hand, the thermal capacity of a body is the product of its mass and its specific heat.

Again, consideration of the specific heats of gases brings forward different considerations. When heat is imparted to a gas the resulting change in temperature depends upon the manner in which the gas is permitted to expand, because during expansion the gas will do work against the external pressure; the amount of this may be a very considerable fraction of the whole energy imparted to the gas. Thus if we are to define the specific heat of a gas the conditions under which the heating takes place must be stated. To each possible kind of expansion there is a corresponding specific heat of the gas. The two specific heats usually considered are the specific heat at constant volume, C_v, and at constant pressure, C_p.

To illustrate the significance of the difference between these two specific heats, consider 1 g of a gas contained in a cylinder fitted