

Applied Finite Mathematics

T A N



F O U R T H
E D I T I O N



Applied Finite Mathematics

FOURTH EDITION

S. T. TAN

Stonehill College



PWS Publishing Company

B O S T O N



PWS PUBLISHING COMPANY
20 Park Plaza, Boston, MA 02116-4324

Copyright © 1994 by PWS Publishing Company
Copyright © 1990 by PWS-KENT Publishing Company
Copyright © 1987 by PWS Publishers

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transcribed in any form or by any means—electronic, mechanical, photocopying, recording, or otherwise—without the prior written permission of PWS Publishing Company.

PWS Publishing Company is a division of Wadsworth, Inc.



International Thomson Publishing
The trademark ITP is used under license.

Library of Congress Cataloging-in-Publication Data

Tan, Soo Tang.
Applied finite mathematics / Soo Tang Tan.—4th ed.
p. cm.
Includes index.
ISBN 0-534-93513-3
1. Mathematics. I. Title.
QA39.2.T34 1994 93-4401
510—dc20 CIP

Photo Credits

Chapter 1—© Henley and Savage/Tony Stone Worldwide (p. 2); *Chapter 2*—© William Rivelli/The Image Bank (p. 56); *Chapter 3*—© Poulides/Thatcher/Tony Stone Worldwide (p. 140); *Chapter 4*—© Brent Jones/Tony Stone Worldwide (p. 176); *Chapter 5*—© Patti McConville/The Image Bank (p. 232); *Chapter 6*—© John William Bangan/The Image Bank (p. 272); *Chapter 7*—© Alan Levenson/Tony Stone Worldwide (p. 316); *Chapter 8*—© Sobel/Klonsky/The Image Bank (p. 382); *Chapter 9*—© Grant V. Faint/The Image Bank (p. 452).

Sponsoring Editor: Steve Quigley

Production Coordinator: Robine Andrau

Assistant Editor: Kelle Flannery

Marketing Manager: Marianne C. P. Rutter

Manufacturing Coordinator: Ruth Graham

Editorial Assistant: John Ward

Interior/Cover Designer: Julia Gecha

Production: Del Mar Associates

Interior Illustrators: Kristi Mendola and Deborah Ivanoff

Cover Art: George Snyder, "Road Trip," 1990, acrylic on canvas. Used with permission of the artist.

Cover Printer: Henry N. Sawyer Company

Typesetter: Jonathan Peck Typographers

Printer and Binder: R. R. Donnelley & Sons/Willard

Printed and bound in the United States of America

93 94 95 96 97 98 — 10 9 8 7 6 5 4 3 2 1

2p44/17

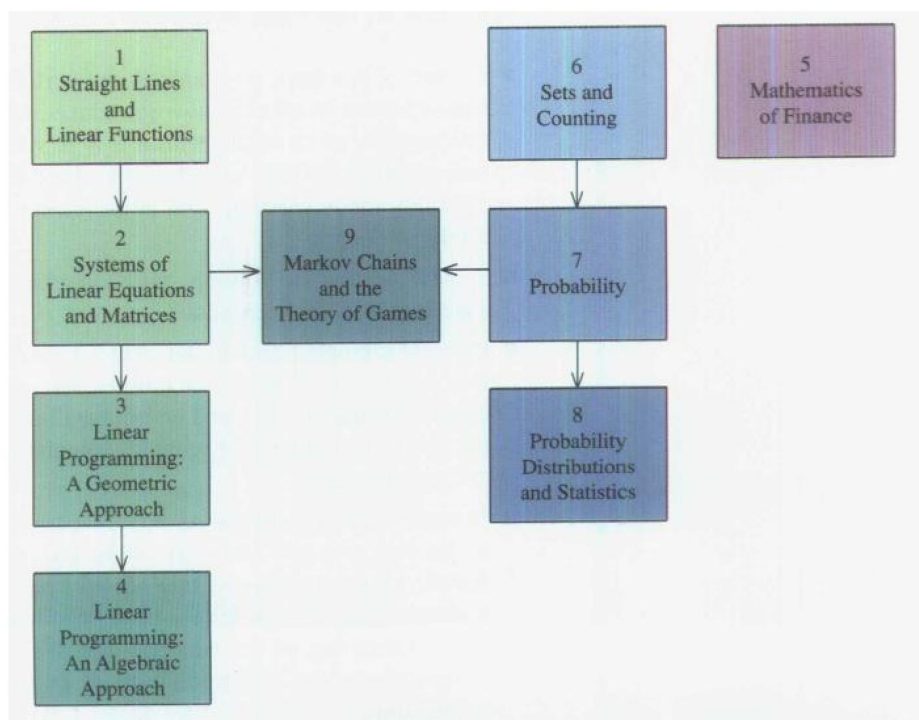
PREFACE

Applied Finite Mathematics, Fourth Edition is directed toward students in the managerial, life, and social sciences. The only prerequisite for understanding this book, which treats the standard topics in finite mathematics, is a year of high school algebra. The objective of *Applied Finite Mathematics* is twofold: (1) to provide students with background in the quantitative techniques necessary to better understand and appreciate the courses normally taken in undergraduate training and (2) to lay the foundation for more advanced courses, such as statistics and operations research. We hope to accomplish this by striking a careful balance between theory and applications. This edition incorporates many valuable comments and suggestions by users of the third edition and reviewers of the fourth edition.

Features

The following list includes some of the many important features of the book:

- **Coverage of Topics** Since the book contains more than ample material for a one-semester or two-quarter course, the instructor may be flexible in choosing the topics most suitable for his or her course. The following chart on chapter dependency is provided to help the instructor design a course that is most suitable for the intended audience.



- **Approach** The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of the material. Figures are used extensively to help students visualize the concepts and ideas being presented.
- **Level of Presentation** Our approach is intuitive and we state the results informally. However, we have taken special care to ensure that this approach does not compromise the mathematical content and accuracy.
- **Applications** The text is application oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets.
- **Exercises** Each section of the text is accompanied by an extensive set of exercises containing an ample set of problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics. Each chapter of the text also contains a set of review exercises. Answers to all odd-numbered exercises appear in the back of the book.
- **Portfolios** These interviews are designed to convey to the student the real-world experiences of professionals who have a background in mathematics and use it in their professions.

Changes in the Fourth Edition

- Chapter 2 has been reorganized and certain parts have been rewritten. The Gauss-Jordan method is now introduced with a 2×2 system of equations followed by an example with a 3×3 system (Section 2.2). Unit columns and pivoting are now introduced in this chapter. Systems of equations with infinitely many solutions and those having no solution are considered separately in Section 2.3.
- A new section on solving nonstandard problems by the simplex method (Section 4.3) has been added.
- Computer/graphics calculator exercises have been added. Textual examples that make use of graphing utilities are marked with a computer/graphics calculator margin icon, and end-of-section exercises meant to be solved with the use of a computer or graphics calculator are identified by a small boxed "C" in the margin.
- In many sections, easier examples have been added to illustrate a new concept or the use of a new technique when the concept or technique is first introduced. More explanatory side remarks have also been included to help the student. Many new problems have been added, including practice problems at the beginning of the exercise sets to help students gain confidence and computational skill before they move on to the more difficult problems and applications.

- Chapter overviews with photographs relating to the applications to be covered have been added.
- Exercise sets have been restructured. Exercises are now paired (even and odd) where appropriate. The review exercises for each chapter have also been expanded.
- Common errors and pitfalls have been highlighted by the caution symbol.

Supplements

- *Student's Solutions Manual*, available to both students and instructors, includes the solutions to odd-numbered exercises.
- *Instructor's Solutions Manual* includes solutions to all exercises.
- *Test Bank with Chapter Tests*, free to adopters of the book, contains sample tests for each chapter.
- *EXPTest* and *ExamBuilder* With these computerized testing systems for IBM PCs and Macintosh, respectively, instructors can select or modify questions from prepared test banks or add their own test items to create any number of tests. These tests can be viewed on screen and printed with typeset-quality mathematical symbols, notation, graphs, and diagrams.
- *Graphing Calculator Supplement*, written by Robert E. Seaver of Lorain County Community College, available to both students and instructors, further develops selected examples and exercises from the text as well as including additional problems for reinforcement. It is specifically geared for use with the TI-line of programmable graphics calculators.
- *Derive* and *Theorist Notebooks*, from *The PWS Notebook Series*™, which are packaged with the text at additional nominal cost to students, provide more than 1000 book-specific and manipulatable electronic problems to be used in conjunction with popularly used and distributed computer algebra systems.

Acknowledgments

I wish to thank Arthur J. Rosenthal, Salem State College, for his excellent job of error checking the text and exercise answers for this edition. I also wish to express my personal appreciation to each of the following reviewers, whose many suggestions have helped make a much improved book.

Reviewers of the Previous Editions:

Ronald D. Baker <i>University of Delaware</i>	Patricia Hickey <i>Baylor University</i>
Frank E. Bennett <i>Mount Saint Vincent University</i>	Harry C. Hutchins <i>Southern Illinois University</i>
Leslie S. Cobar <i>University of New Orleans</i>	Martin Kotler <i>Pace University</i>
Jerry Davis <i>Johnson State College</i>	Paul E. Long <i>University of Arkansas</i>
Sharon S. Hewlett <i>University of New Orleans</i>	James D. Nelson <i>Western Michigan University</i>

Lavon B. Page
*North Carolina State
University*
James Perkins
*Piedmont Virginia Community
College*
Richard D. Porter
Northeastern University

Robert H. Rodine
Northern Illinois University
Arnold Schroeder
Long Beach City College

Reviewers of the Fourth Edition:

Teresa L. Bittner
Canada College
Frederick J. Carter
St. Mary's University
Robert B. Eicken
Illinois Central College
Bruce Johnson
University of Victoria
Martin Kotler
Pace University
Larry Luck
*Anoka-Ramsey Community
College*
Gary MacGillivray
University of Victoria

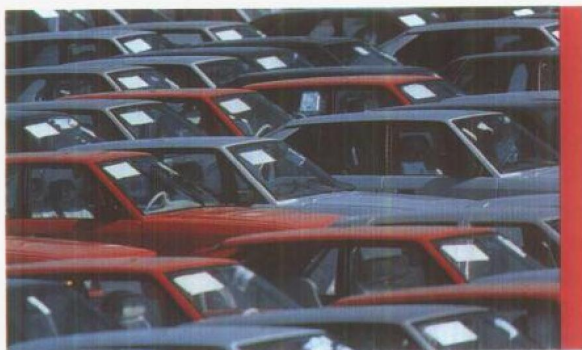
Gary A. Martin
University of Massachusetts—Dartmouth
Norman R. Martin
Northern Arizona University
Ruth Mikkelsen
University of Wisconsin—Stout
John A. Muzzey
Lyndon State College
Sandra Pryor Clarkson
Hunter College—SUNY
C. Rao
University of Wisconsin
Ron Smit
University of Portland

A special thanks also goes to Bruce R. Johnson, University of Victoria. I wish to thank the Reverend Robert J. Kruse, vice-president of Stonehill College, for his enthusiastic support of this project. My thanks also go to the editorial and production staffs of PWS Publishing Company, Steve Quigley, Kelle Flannery, and Robine Andrau, for their thoughtful contributions and patient assistance and cooperation during the development and production of this book. Finally, I wish to thank Nancy Sjoberg of Del Mar Associates and Andrea Olshevsky for doing an excellent job ensuring the accuracy and readability of this fourth edition.

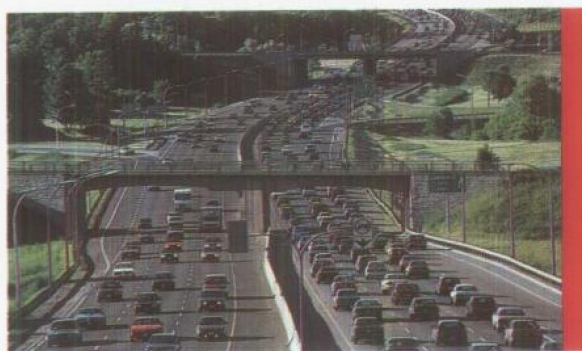
S. T. Tan
Stonehill College

APPLICATIONS

*In **Applied Finite Mathematics** we attempt to solve a wide variety of problems arising from many diverse fields of study. A small sample of the types of practical problems we will consider follows.*



ALLOCATION OF FUNDS Madison Finance has a total of \$20 million earmarked for homeowner and auto loans. On the average, homeowner loans will have a 10 percent annual rate of return, whereas auto loans will yield a 12 percent annual rate of return. Management has also stipulated that the total amount of homeowner loans should be greater than or equal to four times the total amount of automobile loans. Determine the total amount of loans of each type Madison should extend to each category in order to maximize its returns.



AUTOMOBILE SAFETY In an experiment conducted to study the effectiveness of an eye-level third brake light in the prevention of rear-end collisions, 250 of a state's 500 highway patrol cars were equipped with such lights. At the end of a one-year trial period, the record revealed that of the cars equipped with a third brake light, 14 were involved in rear-end collisions. There were 22 such incidents involving cars not equipped with the accessory. Based on these data, what is the probability that a highway patrol car equipped with a third brake light will be rear-ended within a one-year period?



SERUM CHOLESTEROL LEVELS The serum cholesterol levels in mg/dl in a current Mediterranean population are found to be normally distributed with a mean of 160 and a standard deviation of 50. Scientists at the National Heart, Lung, and Blood Institute consider this pattern ideal for a minimal risk of heart attacks. Find the percentage of the population having blood cholesterol levels between 160 and 180 mg/dl.



URBAN-SUBURBAN POPULATION FLOW Because of the continued successful implementation of an urban renewal program, it is expected that each year 3 percent of the population currently residing in the city will move to the suburbs, and 6 percent of the population currently residing in the suburbs will move to the city. At present, 65 percent of the total population of the metropolitan area live in the city itself, whereas the remaining 35 percent live in the suburbs. If we assume that the total population of the metropolitan area remains constant, what will the distribution of the population be like one year from now?

POLITICAL POLLS Six months before an election, the Morris Polling Group conducted a poll in a state in which a Democrat and a Republican were running for governor and found that 60 percent of the voters intended to vote for the Democrat and 40 percent intended to vote for the Republican. A poll conducted three months later found that 70 percent of those who had earlier stated a preference for the Democratic candidate still maintained that preference, whereas 30 percent of these voters now preferred the Republican candidate. Of those who had earlier stated a preference for the Republican, 80 percent still maintained that preference, whereas 20 percent now preferred the Democratic candidate. If the election were held at this time, who would win?

In developing the tools for solving these and many other problems, we will delve into many areas of mathematics, including linear algebra, linear programming, probability, statistics, and logic.

Applications Photo Credits

Allocation of Funds—© Ken Cooper/The Image Bank;
Automobile Safety—© Cralle/The Image Bank; *Serum*
Cholesterol Levels—© Larry Dale Gordon/The Image Bank;
Urban-Suburban Population Flow—© William Taufic/
 Stockphotos; *Political Polls*—© Daemmrich/Stock Boston

CONTENTS

CHAPTER 1

Straight Lines and Linear Functions 2

- 1.1 The Cartesian Coordinate System 4
- 1.2 Straight Lines 10
- 1.3 Linear Functions and Mathematical Models 24
- 1.4 Intersection of Straight Lines 35
- *1.5 The Method of Least Squares 44

Chapter 1 Review Exercises 54

CHAPTER 2

Systems of Linear Equations and Matrices 56

- 2.1 Systems of Linear Equations—Introduction 58
- 2.2 Solving Systems of Linear Equations I 66
- 2.3 Solving Systems of Linear Equations II 81
- 2.4 Matrices 91
- 2.5 Multiplication of Matrices 100
- 2.6 The Inverse of a Square Matrix 113
- *2.7 Leontief Input-Output Model 128

Chapter 2 Review Exercises 137

CHAPTER 3

Linear Programming: A Geometric Approach 140

- 3.1 Graphing Systems of Linear Inequalities in Two Variables 142
- 3.2 Linear Programming Problems 150
- 3.3 Graphical Solution of Linear Programming Problems 161

Chapter 3 Review Exercises 174

CHAPTER 4

Linear Programming: An Algebraic Approach 176

- 4.1 The Simplex Method: Standard Maximization Problems 178
- 4.2 The Simplex Method: Standard Minimization Problems 199
- *4.3 The Simplex Method: Nonstandard Problems 213

Chapter 4 Review Exercises 229

*Sections marked with an asterisk are not prerequisites for later material.

CHAPTER 5	Mathematics of Finance 232
	5.1 Compound Interest 234
	5.2 Annuities 243
	5.3 Amortization and Sinking Funds 251
	*5.4 Arithmetic and Geometric Progressions 259
	<i>Chapter 5 Review Exercises 269</i>
CHAPTER 6	Sets and Counting 272
	6.1 Sets and Set Operations 274
	6.2 The Number of Elements in a Finite Set 284
	6.3 The Multiplication Principle 291
	6.4 Permutations and Combinations 297
	<i>Chapter 6 Review Exercises 312</i>
CHAPTER 7	Probability 316
	7.1 Experiments, Sample Spaces, and Events 318
	7.2 Definition of Probability 327
	7.3 Rules of Probability 337
	7.4 Use of Counting Techniques in Probability 345
	7.5 Conditional Probability and Independent Events 353
	7.6 Bayes' Theorem 368
	<i>Chapter 7 Review Exercises 378</i>
CHAPTER 8	Probability Distributions and Statistics 382
	8.1 Distributions of Random Variables 384
	8.2 Expected Value 393
	8.3 Variance and Standard Deviation 407
	8.4 The Binomial Distribution 418
	8.5 The Normal Distribution 429
	8.6 Applications of the Normal Distribution 439
	<i>Chapter 8 Review Exercises 450</i>
CHAPTER 9	Markov Chains and the Theory of Games 452
	9.1 Markov Chains 454
	9.2 Regular Markov Chains 464
	9.3 Absorbing Markov Chains 475
	9.4 Game Theory and Strictly Determined Games 483

9.5 Games with Mixed Strategies 495*Chapter 9 Review Exercises 508***Appendix A Introduction to Logic A1**

A.1 Propositions and Connectives A2

A.2 Truth Tables A7

A.3 The Conditional and the Biconditional Connectives A9

A.4 Laws of Logic A15

A.5 Arguments A19

A.6 Applications of Logic to Switching Networks A25

Appendix B The System of Real Numbers A31**Appendix C Tables A33**

Table 1 Compound Amount, Present Value, Annuity A34

Table 2 Binomial Probabilities A50

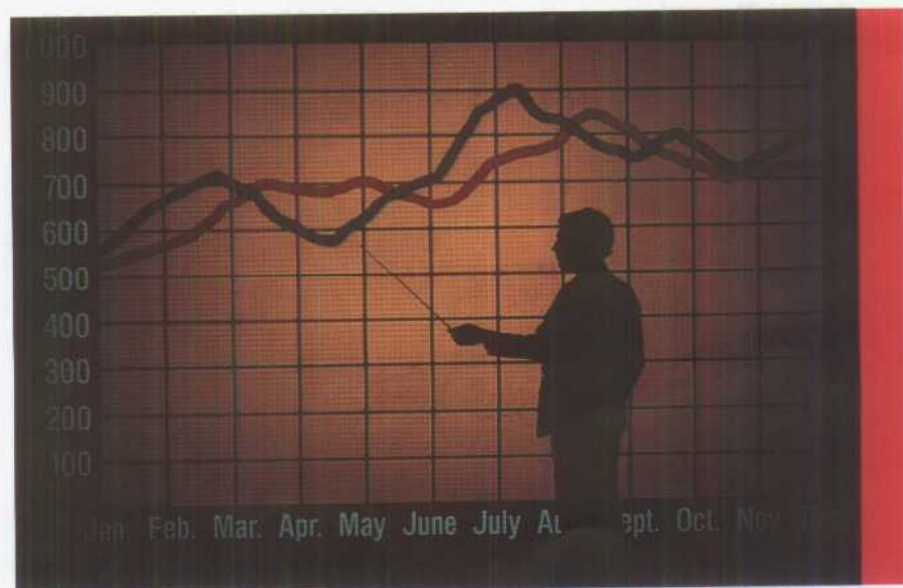
Table 3 The Standard Normal Distribution A53

*Answers to Odd-Numbered Exercises A55**Index A79*

Applied Finite Mathematics

-
- 1.1 The Cartesian Coordinate System
 - 1.2 Straight Lines
 - 1.3 Linear Functions and Mathematical Models
 - 1.4 Intersection of Straight Lines
 - 1.5 The Method of Least Squares (Optional)

Which process should the company use? The Robertson Controls Company must decide between two manufacturing processes for its Model C electronic thermostats. In Example 4, page 38, you will see how to determine which process will be more profitable.





Straight Lines and Linear Functions



This chapter introduces the Cartesian coordinate system, a system that allows us to represent points in the plane in terms of ordered pairs of real numbers. This, in turn, enables us to compute the distance between two points algebraically. We will also study straight lines. *Linear functions*, whose graphs are straight lines, can be used to describe many relationships between two quantities. These relationships can be found in fields of study as diverse as business, economics, the social sciences, physics, and medicine. We will also see how some practical problems can be solved by finding the point(s) of intersection of two straight lines. Finally, we will learn how to find an algebraic representation of the straight line that “best” fits a set of data points that are scattered about a straight line.



1.1

The Cartesian Coordinate System

The Cartesian Coordinate System

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We will assume that you are familiar with the rules governing these algebraic operations (see Appendix B).

Real numbers may be represented geometrically by points on a line. Such a line is called the *real number line*, or *coordinate line*. We can construct the real number line as follows: Arbitrarily select a point on a straight line to represent the number zero. This point is called the *origin*. If the line is horizontal, then choose a point at a convenient distance to the right of the origin to represent the number 1. This determines the scale for the number line. Each positive real number x lies x units to the right of zero, and each negative real number $-x$ lies x units to the left of zero.

In this manner, a one-to-one correspondence is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (Figure 1.1).

FIGURE 1.1
The real number line

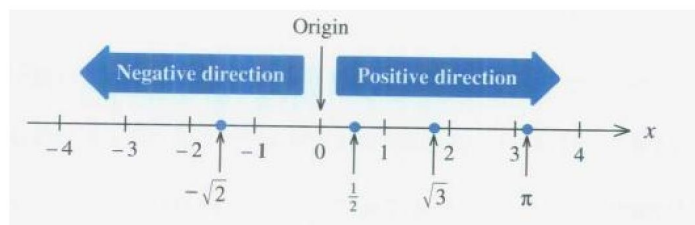
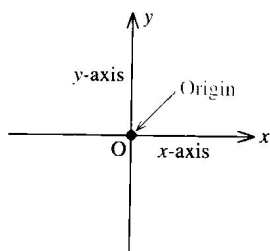


FIGURE 1.2
The Cartesian coordinate system

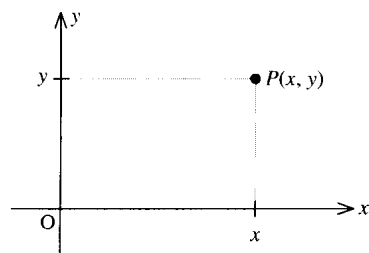


In a similar manner, we can represent points in a plane (a two-dimensional space) by using the Cartesian coordinate system, which we construct as follows: Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O , called the *origin* (Figure 1.2). The horizontal line is called the *x-axis*, and the vertical line is called the *y-axis*. A number scale is set up along the *x-axis* with the positive numbers lying to the right of the origin and the negative numbers lying to the left of it. Similarly, a number scale is set up along the *y-axis* with the positive numbers lying above the origin and the negative numbers lying below it.

REMARK The number scales on the two axes need not be the same. Indeed, in many applications different quantities are represented by x and y . For example, x may represent the number of typewriters sold and y the total revenue resulting from the sales. In such cases it is often desirable to choose different number scales to represent the different quantities. Note, however, that the zeros of both number scales coincide at the origin of the two-dimensional coordinate system.

We can represent a point in the plane uniquely in this coordinate system by an ordered pair of numbers; that is, a pair (x, y) where x is the first number and y the second. To see this, let P be any point in the plane (Figure 1.3). Draw perpendiculars from P to the *x-axis* and *y-axis*, respectively. Then the

FIGURE 1.3
An ordered pair in the
coordinate plane



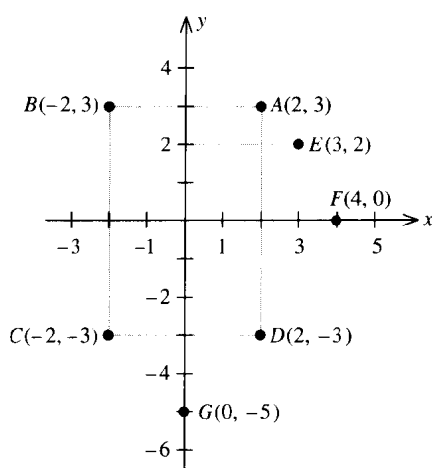
number x is precisely the number that corresponds to the point on the x -axis at which the perpendicular through P hits the x -axis. Similarly, y is the number that corresponds to the point on the y -axis at which the perpendicular through P crosses the y -axis.

Conversely, given an ordered pair (x, y) with x as the first number and y the second, a point P in the plane is uniquely determined as follows: Locate the point on the x -axis represented by the number x and draw a line through that point parallel to the y -axis. Next, locate the point on the y -axis represented by the number y and draw a line through that point parallel to the x -axis. The point of intersection of these two lines is the point P (Figure 1.3).

In the ordered pair (x, y) , x is called the *abscissa*, or x -coordinate, y is called the *ordinate*, or y -coordinate, and x and y together are referred to as the *coordinates* of the point P . The point P with x -coordinate equal to a and y -coordinate equal to b is often written $P(a, b)$.

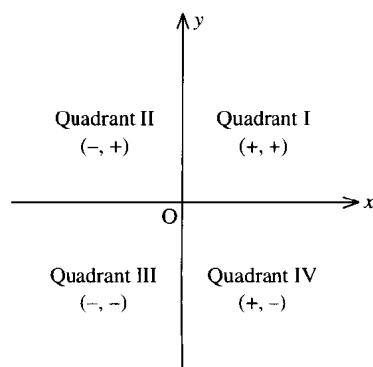
The points $A(2, 3)$, $B(-2, 3)$, $C(-2, -3)$, $D(2, -3)$, $E(3, 2)$, $F(4, 0)$, and $G(0, -5)$ are plotted in Figure 1.4.

FIGURE 1.4
Several points in the coordinate
plane



REMARK In general, $(x, y) \neq (y, x)$. This is illustrated by the points A and E in Figure 1.4.

FIGURE 1.5
The four quadrants in the
coordinate plane



The axes divide the plane into four quadrants. Quadrant I consists of the points P with coordinates x and y , denoted by $P(x, y)$, satisfying $x > 0$ and $y > 0$; Quadrant II, the points $P(x, y)$ where $x < 0$ and $y > 0$; Quadrant III, the points $P(x, y)$ where $x < 0$ and $y < 0$; and Quadrant IV, the points $P(x, y)$ where $x > 0$ and $y < 0$ (Figure 1.5).

The Distance Formula

One immediate benefit that arises from using the Cartesian coordinate system is that the distance between any two points in the plane may be expressed solely in terms of the coordinates of the points. Suppose, for example, that (x_1, y_1) and (x_2, y_2) are any two points in the plane (Figure 1.6). Then the distance d between these two points is, by the Pythagorean Theorem,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

For a proof of this result see Exercise 35, page 9.