WORLD METEOROLOGICAL ORGANIZATION

TECHNICAL NOTE No. 121

DISPERSION AND FORECASTING OF AIR POLLUTION

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(Report by a working group of the Commission for Atmospheric Sciences, prepared by R. E. Munn, chairman – A. E. J. Eggleton – L. Facy – D. H. Pack – F. H. Schmidt)

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NOTE

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FOREWORD

At this time when so much concern is being expressed both nationally and internationally about the quality of the environment, meteorologists are being called upon more and more to apply their knowledge to the understanding of problems of atmospheric pollution. It is appropriate therefore that the World Meteorological Organization should take a leading role in the encouragement of studies in this subject and facilitate the exchange of scientific information.

This Technical Note, the most recent of several dealing with various aspects of air pollution, is particularly welcome since it constitutes a comprehensive review of current theoretical knowledge and of applications of meteorology to the dispersion of pollutants in urban and non-urban situations. It will serve as excellent technical guidance in these aspects to scientific experts, consultants, civic planners and managers of industrial and utility services.

This work has been produced by the members of the Commission for Atmospheric Sciences' Working Group on Atmospheric Pollution and Atmospheric Chemistry, consisting of R. E. Munn (Canada), chairman, A. E. J. Eggleton (U.K.), L. Facy (France), D. H. Pack (U.S.A.) and F. H. Schmidt (Netherlands). Valuable contributions were also received from M. E. Berlyand (U.S.S.R.), A. G. Forsdyke (U.K.) and F. A. Gifford (U.S.A.).

It gives me great pleasure to express my appreciation on behalf of the World Meteorological Organization to these scientists for the excellent work they have produced.

D. A. Davies

D. A. DAVIES
Secretary-General

SUMMARY

The general discussion of Chapter 1 acquaints the reader with the progress in the state-of-the-art of dispersion theory and practice.

Chapter 2 is concerned with the height to which a buoyant plume will rise. The theoretical considerations are briefly outlined and a number of empirical formulae presented. The chapter concludes with a discussion of how to use the formulae successfully.

The classical expression for diffusion is discussed in Chapter 3, as it relates to transport and diffusion in non-urban environments. Simplifications of this expression as it applies to moderate travel distances during steady-state conditions are then presented. Complications caused by mesoscale or synoptic-scale systems and travel over long distances are then considered. Some pollutants undergo chemical reactions or are removed by precipitation — these processes are reviewed as far as known. The chapter concludes by showing how the surface of the Earth acts as a sink and sometimes (i.e. during duststorms) as a source.

The effect of cities on the transport and dispersion of pollutants is the subject of Chapter 4. After a review of the urban microclimate, models for various types of sources are presented. Finally chemical reactions and removal processes over cities are discussed.

Chapter 5 treats the forecast aspect of meteorological services for air-pollution abatement. The observations required and the forecast parameters are reviewed. Forecasting of air-pollution potential is then described, followed by a discussion of forecast verification. Finally the more difficult problem of forecasting the concentration of the pollution itself is examined.

The Appendix contains a detailed description of atmospheric diffusion investigations in the U.S.S.R.

It should be noted that, except in the Appendix, detailed descriptions and discussions have been avoided throughout this Note. The practice has been to refer the reader to the relevant literature. Over 180 references are cited. An exception has been made in the Appendix where Soviet experiences are described fully as these do not normally appear in English-language literature.

RESUMÉ

Le chapitre 1 consiste en un exposé de caractère général destiné à informer le lecteur de la situation actuelle quant à la théorie de la dispersion et à ses applications pratiques.

Le chapitre 2 porte sur l'altitude atteinte par un nuage d'effluents sous l'effet de la poussée hydrostatique. Après un bref rappel des considérations théoriques, un certain nombre de formules empiriques sont fournies. Ce chapitre s'achève par des conseils quant à la manière d'utiliser judicieusement les formules proposées.

Le chapitre 3 analyse l'expression classique de la diffusion, pour ce qui est du transport et de la diffusion en milieu rural. Des expressions simplifiées de cette expression, valables dans le cas où le déplacement s'effectue sur une distance modérée et où les conditions demeurent stables, sont présentées. Les complications engendrées par les systèmes de l'échelle moyenne et de l'échelle synoptique et par les déplacements sur de longues distances sont examinées ensuite. Les connaissances actuelles quant aux réactions chimiques auxquelles se trouvent soumis certains polluants et quant au lessivage de ceux-ci par les précipitations sont également exposées. Pour conclure, ce chapitre montre comment la surface du globe joue le rôle de lieu de dépôt pour les polluants mais parfois aussi (par exemple lors des tempêtes de poussière) de source de polluants.

Le chapitre 4 a pour sujet l'effet qu'exercent les villes sur le transport et la dispersion des polluants. Il examine d'abord le microclimat urbain, puis présente des modèles pour divers types de sources. Enfin, il analyse la question des réactions chimiques et des processus d'élimination des polluants au-dessus des villes.

Le chapitre 5 traite de la question de la prévision par les services météorologiques dans le cadre de la lutte contre la pollution atmosphérique. Il passe en revue les observations nécessaires et les paramètres à considérer. Il expose ensuite la prévision du niveau potentiel de pollution de l'air et analyse ensuite les méthodes de vérification des prévisions. Finalement, il examine le problème qui soulève le plus de difficultés, à savoir la prévision de la concentration des polluants.

L'appendice expose en détail les recherches effectuées en U.R.S.S. sur la diffusion atmosphérique.

Il convient de noter qu'en dehors de l'appendice on a évité de faire figurer dans cette Note des descriptions et des analyses détaillées. On a préféré renvoyer le lecteur à la documentation pertinente. A cet effet, plus de 180 références bibliographiques sont citées. L'appendice fait exception à cette règle et les expériences soviétiques y sont décrites en détail étant donné qu'on ne les trouve pas normalement dans la documentation publiée en langue anglaise.

РЕЗЮМЕ

Глава 1 знакомит читателя с успехами в области теории и практики дисперсии.

Глава 2 касается высоты, на которую поднимается восходящий шлейф. Кратко описываются теоретические положения и приводится ряд эмпирических формул. В конце главы обсуждаются методы успешного применения этих формул.

В главе 3 обсуждается классическое выражение для диффузии и его связь с переносом и диффузией во внегородском окружении. Затем представляются упрощенные варианты этого выражения в применении к переносу на умеренные расстояния в условиях устойчивого состояния. Затем рассматриваются сложные случаи, вызванные мезомасштабными системами или системами синоптического масштаба, и перенос на длинные расстояния. Некоторые загрязняющие вещества подвергаются химическим реакциям или удаляются под воздействием осадков — эти процессы рассматриваются в той мере, в которой они известны. В конце главы указывается, каким образом поверхность земли выступает в качестве места осаждения и иногда (например, во время пыльных бурь) в качестве источника.

Глава 4 посвящена вопросу влияния городов на перенос и дисперсию загрязняющих веществ. После рассмотрения городского микроклимата приводятся модели для источников различных типов. В заключение рассматриваются химические реакции и процессы удаления, происходящие над городами.

В главе 5 рассматриваются вопросы прогноза снижения уровня загрязнения воздуха. Рассматриваются вопросы необходимых наблюдений и прогностических параметров. Далее описывается прогнозирование уровня загрязнения воздуха, и рассматриваются вопросы оценки оправдываемости прогнозов. В заключение рассматривается более трудная проблема, а именно, проблема прогнозирования концентрации самого загрязняющего вещества.

В приложении содержится подробное описание исследований по атмосферной диффузии в СССР.

Следует отметить, что во всей технической записке, за исключением приложения, автор избегал приводить подробные описания и обсуждения. Как правило, делались ссылки на соответствующую литературу. В технической записке содержится более 180 библиографических ссылок. Исключением из этого правила явилось только приложение, в котором опыт Советского Союза описывается полностью, так как эти материалы, как правило, не появляются в литературе на английском языке.

RESUMEN

La exposición de carácter general que se hace en el Capítulo 1 permite al lector darse cuenta de los progresos últimamente realizados con referencia a la teoría y práctica de la dispersión.

El Capítulo 2 trata de la altura que puede alcanzar el penacho de una nube. Se citan de manera breve ciertas consideraciones teóricas junto con algunas fórmulas empíricas. Este capítulo termina exponiendo la manera de utilizar acertadamente las fórmulas.

En el Capítulo 3 se discute la expresión clásica de la difusión, en el sentido relativo al transporte y difusión en ambientes no urbanos. A continuación se especifican ciertas simplificaciones de esta expresión que se aplican a distancias moderadas de transporte durante condiciones conservativas. Se exponen luego las complicaciones originadas por los sistemas mesoescalares o de escala sinóptica de transporte a largas distancias. Se estudia también el hecho de que algunos agentes contaminadores experimentan reacciones químicas o son eliminados por las precipitaciones, y se revisan los procesos que a este respecto se conocen. El capítulo termina mostrando la manera en que la superficie de la tierra actúa como foco de disipación y algunas veces como fuente de contaminación como, por ejemplo, durante las tempestades de polvo.

En el Capítulo 4 se estudia el efecto que ejercen las ciudades en el transporte y dispersión de los agentes contaminadores. Después de exponer el microclima urbano, se describen modelos correspondientes a los distintos tipos de fuentes de contaminación. Finalmente se estudian las reacciones químicas y los procesos de eliminación de la contaminación sobre las ciudades.

El Capítulo 5 trata de las tareas de predicción de los Servicios Meteorológicos, referentes a la eliminación de la contaminación del aire. Se especifican las observaciones que se necesitan para esta labor así como los parámetros utilizados en la predicción. A continuación se describe la contaminación potencial y se citan algunos razonamientos referente a la verificación de las predicciones. Finalmente se estudian los más difíciles problemas referentes a la predicción de la concentración de los agentes contaminadores.

El Apéndice contiene una descripción detallada de las investigaciones sobre difusión atmosférica que han sido realizadas en la U.R.S.S.

Se observará que, excepto en el Apéndice, se han eliminado las descripciones y exposiciones detalladas en toda la presente publicación. Se ha seguido la norma de hacer referencia a las correspondientes publicaciones en cada caso, para que el lector las consulte. Se citan más de ciento ochenta referencias. En el Apéndice se ha hecho una excepción a este respecto ya que se describen completamente las experiencias soviéticas en vista de que éstas no figuran habitualmente en las publicaciones hechas en inglés.

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CHAPTER 1

INTRODUCTION

The wind and its turbulent components play an important role in determining the quality of the air. Sometimes pollution is diluted very quickly, particularly when winds are strong or when the intensity of turbulence is high; sometimes, however, the atmospheric transport and diffusion processes are damped, and air quality deteriorates.

Air contaminants do not remain in the atmosphere indefinitely. Fortunately, there are a number of cleansing mechanisms such as chemical decay, precipitation scavenging and ground absorption. Thus the air quality depends not only on the emission rates and on the efficiency of the transport and turbulent diffusion processes but also on the depletion rates. This implies that the meteorologist and the atmospheric chemist both have important contributions to make in the solution of air-pollution problems.

This Technical Note is concerned with dispersion in the lower troposphere, mainly on the meso- and macro-scales. The emphasis is on time scales of less than a day or so. Thus the topic of air-pollution climatology is excluded, with the exception of Professor M. E. Berlyand's account of recent work in the U.S.S.R. (see Appendix): in that case it seemed appropriate to include a unified survey of Soviet methods because they are relatively unknown to English-speaking readers.

Until the last decade or so, the air-pollution meteorologist was mainly interested in the classical problems of: (*a*) diffusion from a point source over a homogeneous surface; and (*b*) the rise of a buoyant plume. Steady-state conditions were assumed, and mixing was assumed to take place either in the surface boundary layer (about 100 metres in thickness) where the shearing stress remained constant with height, or in a region where the mean wind itself was invariant with position. Perfect reflection of the plume at the ground surface and no atmospheric chemical reactions were included in the models. For averaging times of from ten minutes to an hour or so, and for downwind distances of from 100 metres to a kilometre, it was fair to say that most of the practical problems had been solved. Because the meteorologist felt that diffusion processes in large cities were too complex to be treated by the classical methods, he used regression and multivariate analysis techniques to seek relations between urban pollution values and airport meteorological observations. Meanwhile, large-scale atmospheric chemists and dynamic meteorologists were trying to understand the global budgets of pollution and were undertaking studies of the various sink mechanisms. There was little dialogue, however, between these investigators and the micrometeorologists.

Beginning about 1950, but particularly after 1960, the situation began to change. In the first place, chimneys have become taller and taller, exceeding 300 metres in height in a number of cases today, and the entire Ekman layer (up to about one kilometre) must be included in investigations of plume rise. Secondly, downwind distances of practical significance have lengthened, presently exceeding 100 kilometres. No longer is the assumption of steady-state, homogeneous flow at all realistic; in addition, various sink mechanisms are important at these distances. Thirdly, the subject of urban air pollution has become of more than scientific interest. In many countries, modern air-pollution legislation is based on air-quality criteria rather than on emission standards as in the past when the Ringelmann chart was the principal tool of the control officer. This change of emphasis requires the evaluation of the separate contributions of many sources to the air quality at a point, a task that can only be done using meteorological techniques. In some countries, in fact, urban numerical diffusion models are employed regularly for regional planning purposes. Finally, an operational requirement is developing or already exists in many countries for daily pollution or pollution-potential forecasts.

In conclusion, it should be mentioned that diffusion is defined in this Note as a process that dilutes atmospheric contaminants by turbulent mixing. The result is *dispersion*. Dispersion also occurs as a result of depletion by such processes as chemical decay, precipitation scavenging and ground absorption.

CHAPTER 2

PLUME RISE

2.1 Introduction

A particularly difficult aspect of the problem of diffusion of atmospheric pollution is the determination of the height to which a buoyant plume, with an initial exit velocity, will rise. For diffusion from a continuous point source, the problem is equivalent to determining the difference, Δh, between the ultimate height h of the pollution plume — if such an ultimate height exists — and the height h_1 of the source, as a rule the nozzle of a chimney stack. (Values of various quantities at the point of emission are henceforth indicated by the suffix 1.)

In principle the height difference, Δh, may be a consequence *inter alia* of the *exit velocity*, w_1, with which the plume leaves the chimney and of the *density difference*, $\Delta\rho_1$, between the material contained in the plume and the surrounding air. With respect to this density difference, the plume may usually be considered to consist of air of the same composition as the surrounding air, the amounts of pollutants such as SO_2, oxides of nitrogen, and dust being so small that they hardly affect the plume's density. An exception occurs when the plume contains water; a change of state from steam to water droplets and/or to ice crystals will add latent heat to the plume, while evaporation will have the opposite effect. In general, however, a density difference between the plume and the surrounding air — always to be related to the same level inside and outside the plume, i.e., to the same pressure — can be completely ascribed to the temperature difference between the two. This *temperature difference*, ΔT, can be considered directly or it may be related to the *heat emission rate*, Q_H, of the stack. Although in principle ΔT may be positive or negative, a plume is nearly always initially warmer than its surroundings, i.e., ΔT_1 is positive in practical applications. The plume will then tend to rise. We shall therefore restrict our attention to this latter situation, although we should note in passing the possibility that a heated plume may overshoot the level at which $\Delta T = O$, exhibiting thereafter damped vertical oscillations.

Turning next to the meteorological variables of importance, the mean wind speed $\bar{u}$ must certainly be included. In this connexion the air flow is usually assumed to be horizontal but the flow may be tilted due to the local geometry of the building (and adjacent buildings) or due to topographic influences.

Second, the *intensity of turbulence* given, e.g., by $\overline{(u')^2}$, $\overline{(v')^2}$ and $\overline{(w')^2}$, influences the behaviour of a plume; turbulence is the mechanism that causes mixing of plume air with the surrounding air. In this connexion, the turbulence naturally present in the atmosphere and that generated by the plume's own upward motion must both be considered. Furthermore, not only the intensity but also the spectrum of turbulence is probably important, the size of eddy contributing most to entrainment being a function of plume diameter: for increasing downwind distance, therefore, lower and lower frequencies must be examined separately.

Third, the *vertical temperature gradient in the environmental atmosphere* should be taken into account because the height change in ΔT during the buoyant plume rise is influenced by this gradient. Introduction of *potential temperature*, θ_e, given as a function of height, z, and not necessarily of pressure, is obviously advantageous here.

Finally, one must consider the geometry of the area, which affects the tilt of the wind and may cause flow separation. For aerodynamic downwash directly in the wake of the chimney or of adjacent buildings, the following parameters are important: the strength of the undisturbed wind at the top of the boundary layer, the height of the chimney, and the dimensions and spacing of local obstructions to the wind. When downwash does occur, an adiabatic lapse rate can safely be assumed because the turbulent mixing is so vigorous. Thus the possibility of wind-tunnel modelling is very attractive.

For orographic effects (tilt and flow separation) at distances of up to a few kilometres from the source, non-linear theory indicates that the following parameters are significant: the height of the chimney, the bulk Richardson number and the ratio of the height to the width of hills (Dobryshman, 1971, personal communication).

Summarizing, one finds that at least the following parameters must be taken into account when describing the behaviour of a plume in an environment with different density.

Parameters to be considered in the plume-rise problem

Plume:
- exit velocity w_1.
- temperature difference in the nozzle : ΔT_1 or heat emission rate Q_H. (Q_H can be determined from ΔT_1 and w_1, provided the dimensions of the nozzle are known, e.g. the *diameter* D_1.)
- water-vapour content, particularly at low temperatures.

Atmosphere:
- mean wind speed $\bar{u}$. (Strictly speaking the change in wind speed as well as in direction in both the vertical and the horizontal should be taken into account. This has sometimes been done for speed by introducing a power law for $\bar{u}$ but neglecting the change in wind direction). Henceforth the bar over the symbol u will be omitted.
- intensity of turbulence, given by $\overline{(u')^2}$, $\overline{(v')^2}$ and $\overline{(w')^2}$ or preferably by the turbulence spectrum.
- vertical temperature gradient in the environment given $\frac{\partial \theta_e}{\partial z}$ or an equivalent quantity.

Geometry:
- dimensions and spacing of local and regional bluff surface features.

It is clear that the three atmospheric variables mentioned above are more or less interrelated. The exact form of the interrelationship depends on local circumstances such as terrain roughness etc., and cannot easily be determined. This is one of the reasons why the problem of plume rise is so difficult to solve.

2.2 General philosophy

All plume-rise formulae, even when formulated from dimensional considerations, contain at least one dimensionless constant that must be evaluated experimentally: in many cases, furthermore, the formulae are largely empirical, and the values of the "constants" vary from place to place. It is in fact very difficult to develop an exact theory for determining Δh, taking into account all the parameters listed in section 2.1. In many cases, therefore, plume-rise formulae are of an empirical or semi-empirical nature, making extrapolation dangerous.

Experimental data have been of variable quality, although the situation has improved in recent years. Some of the difficulties are as follows:

(*a*) Early observations were sometimes made too close to the source, before the plume has become truly horizontal.

(*b*) Not all of the relevant meteorological variables were always measured, particularly the vertical gradients of wind and temperature.

(*c*) Experimental measurements could only be made at places where chimneys existed, so that local geometry and topographic effects could not be excluded.

(*d*) Early experiments were restricted to studies of the behaviour of plumes emitted from relatively short stacks ($h \sim 50$ m), the gases having only small initial excess temperatures.

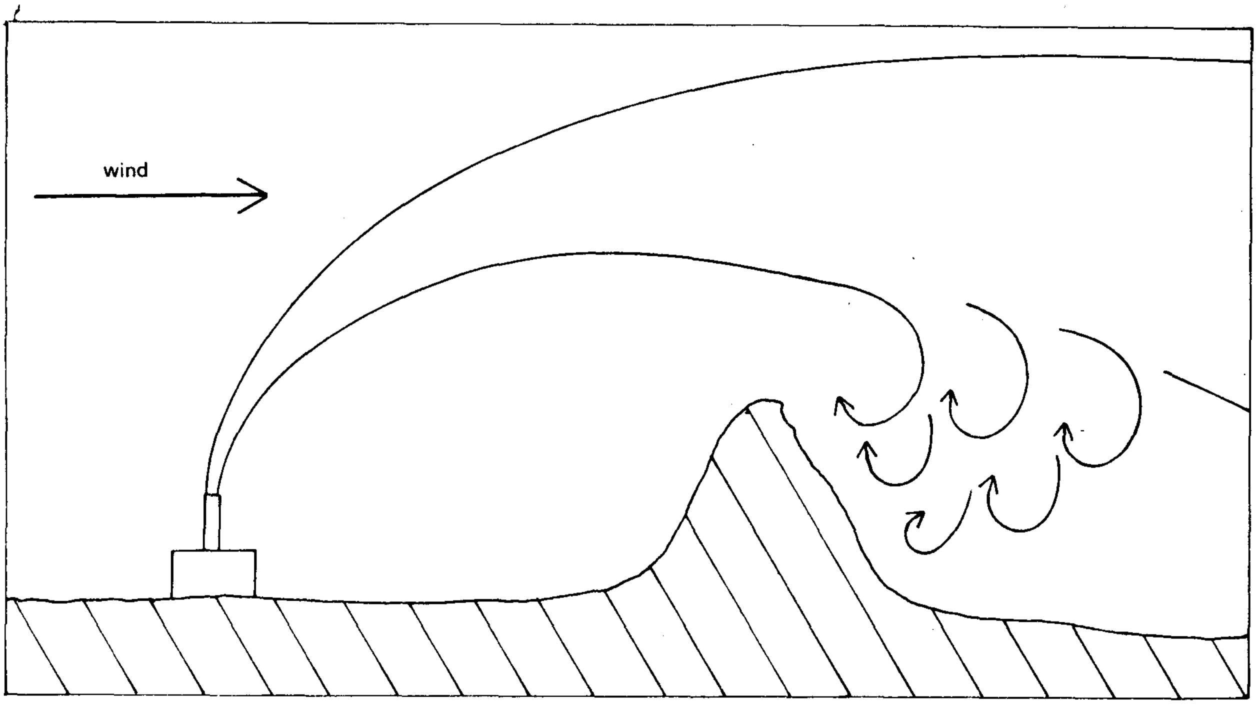

Figure 2.1 — Schematic representation of plume tilt and of separation of the mesoscale

At one time there were almost as many plume-rise formulae as there were sets of data. This situation is changing but there is still insufficient information on the wind shear and turbulence in the Ekman layer where many of the plumes from the new tall chimneys are to be found. In addition, smelters and thermal generating stations are often found in complex topography, and each installation is essentially a new problem. An example is given schematically in Figure 2.1, which shows the possible behaviour of a plume as it crosses a ridge. *A priori*, one might have expected that high plume concentrations and vegetation damage would occur on the upwind side of the hill: in fact, however, the major problem may be associated with the leeward slope. Coastlines are also preferred locations for industrial installations but even when the terrain is relatively flat, there may be horizontal gradients in the values of the meteorological elements, which may have a major influence on plume rise; information on atmospheric conditions at the shore-line itself may therefore be completely inadequate to predict the height of the plume a few kilometres downwind from the chimney. Another special case is plume rise in the Arctic, where environmental conditions are quite different from those at locations at which most of the experimental measurements have been taken. Finally, mention should be made of large and very hot sources, which may under certain circumstances be sufficiently powerful to influence the wind structure, creating mesoscale flow patterns (see Figure 3.3, section 3.1).

For these and other reasons, the recommendation is made that except in the very simplest terrain situation, the advice of an experienced air-pollution meteorologist should be sought. Extrapolation of engineering handbook formulae for plume-rise estimation is generally a questionable procedure. If it must be done, however, several formulae should be applied to obtain a feeling for the range of uncertainty in the estimates. We should mention, finally, that even under the best of conditions, an uncertainty by a factor of two in estimates of Δh is likely on any one occasion because of the natural variability of the atmosphere.

2.3 Theoretical considerations

Slawson and Csanady (1967) have divided buoyant plume rise into three phases:

(*a*) Initial phase

Near the chimney, the atmospheric temperature gradient and turbulence have little effect on the rise of the plume, and a dimensional analysis leads to the result that

$$\Delta h \propto x^{2/3} \qquad (2.1)$$

where x is the downwind distance.*

Briggs (1971) notes that this relation has been satisfactorily verified experimentally and that the régime occurs up to downwind distances typically of the order of three to ten stack heights.

Csanady (1971a) has suggested that if gustiness and stability (or instability) are moderate, eq. (2.1) can be extrapolated, as a rough approximation, to distances of several kilometres.

(*b*) Intermediate phase

Atmospheric eddies of about the same dimensions as that of the plume cause pronounced mixing, and the plume rapidly diffuses its surplus heat and momentum to the environment. Slawson and Csanady predict that in this phase

$$\Delta h \propto \text{const.} - \frac{\text{const.}}{x} \qquad (2.2)$$

Thus the plume ceases to rise so quickly and in some cases may become quasi-horizontal.

* Unless otherwise noted, units used in equations are in m-sec-cal-°C.

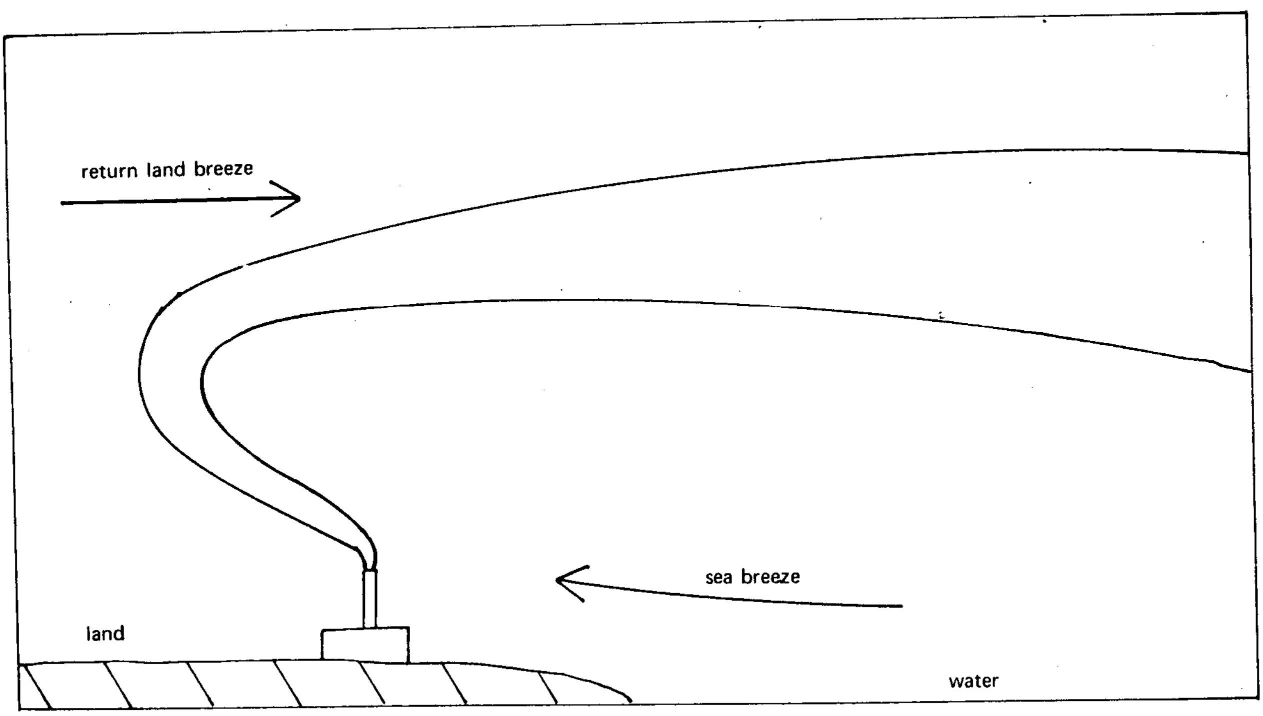

Figure 2.2 — Schematic representation of plume behaviour during a sea-breeze

(*c*) Final phase

The major energy-containing eddies in the atmosphere are now of a scale much smaller than that of the plume. The mixing is thus similar to that of molecular diffusion although of a completely different scale. This leads to the prediction that

$$\Delta h \propto x \tag{2.3}$$

The implication contained in eq. (2.3) is that the plume never becomes quite horizontal. Slawson and Csanady found experimentally in a neutral atmosphere with low to moderate turbulence that the slope in the final phase was of the order of 0.10. With higher atmospheric turbulence, the final slope becomes very low, while in the presence of stability/instability, the plume is modified, leading to an asymptote in the stable case and to a breakup of the plume into distinct parcels during superadiabatic conditions. A slope of 0.10 corresponding to a plume rise of 100 m/km may be important when predicting the dispersion of pollutants over long distances, but in the work of most other investigators, the plume is assumed to become horizontal at a downwind distance of a few kilometres.

Other factors are of equal importance in determining plume rise. There is general agreement, for example, that Δh varies inversely as wind speed, although special cases of large wind shear have not been extensively investigated. Behaviour in a sea breeze is illustrated schematically in Figure 2.2, and, clearly, none of the available formulae is capable of modelling such a régime.

Another important parameter is the quantity Q_H. Employing the plume-rise model of Priestley (Priestley, 1956), with some simplifications, the prediction may be obtained that in the intermediate phase in a neutral atmosphere,

$$\Delta h \propto Q_H^{1/3}/u \tag{2.4}$$

The more complete formulation containing some additional terms (that are usually of second order) is given by Briggs (1969) as his eq. (4.3.4). Briggs suggests as a good working approximation for fossil-fuel plants with heat emission of 20 MW or more:

$$\begin{aligned} \Delta h &= 1.6\ (3.7\times10^{-5}\, Q_H)^{1/3} u^{-1} (10\, h)^{2/3} \\ &= 0.25\, Q_H^{1/3} u^{-1} h^{2/3} \end{aligned} \tag{2.5}$$

However, there is as yet little experimental evidence to verify this equation.

For tall chimneys, unstable lapse rates occur rarely over the entire layer through which the plume is mixing, superadiabatic conditions usually being limited to the lowest 100 m above the ground except in desert terrain. When such a condition does exist, however, the plume behaviour is rather chaotic, and it is difficult experimentally to determine a mean plume height. Briggs suggests that, in the absence of any other models, eq. (2.5) should be used for unstable conditions, noting that larger fluctuations about the mean will occur and that the hot plume may induce a mesoscale circulation.

For stable lapse rates, eq. (2.4) must be modified. The limiting cases in the second phase turn out to be of the form (Briggs, 1969):

(*a*) Calm winds

$$\begin{aligned} \Delta h &= 5\,(3.7\times10^{-5}\, Q_H)^{1/4}((g/T)\,\partial\theta_e/\partial z)^{-3/8} \\ &= 0.39\, Q_H^{1/4}((g/T)\,\partial\theta_e/\partial z)^{-3/8} \\ &= 1.38\, Q_H^{1/4}(\partial\theta_e/\partial z)^{-3/8} \end{aligned} \tag{2.6}$$

where the potential temperature gradient is averaged over the expected Δh. Experimental verification of this relation is shown in Figure 3.3 (Briggs, 1971) and is excellent although the data are still limited in number.