electronics designers' handbook

second edition

L. J. Giacoletto, editor

Electronics Designers' Handbook

Second Edition

First Edition by
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Preface

This is the Second Edition of a handbook that was considered a landmark in technical publishing when it was first issued in 1957. Owing to the remarkable developments in electronics technology over the past decades, this Second Edition is a completely new work and is more than twice the size of the First Edition.

Major goals for the Second Edition were: (1) the introduction of solid-state technology and design, (2) uniform use of SI (metric) units throughout, and (3) increased use of analytical design techniques to reflect the greater sophistication of today's electronics designers. The uniform employment of SI units simplifies the presentation of results, since there is no need to recite units associated with equations. All equations are dimensionally correct with the introduction of SI units. Also, there are a large number of conversion tables, which facilitates conversion to other units as may be required.

A handbook represents yet another step in the consolidation of professional knowledge. In view of the explosive growth of relevant literature during the past 20 years, careful selection was required to keep the size of the Second Edition within bounds. To cull, winnow, and select from the tremendous amount of available literature and then to organize the material in a systematic fashion was a major task.

Consistency in the use of symbols, letters, and designations was a frustrating and time-consuming problem. Conflicts in usage in the literature are frequent, and it is a struggle to resolve these conflicts. In a few cases the conflicts were resolved by introducing special symbols, but by and large the Handbook has hewed closely to established standards, particularly those of the electronics profession. Perhaps this Handbook will serve as a focus for more universal and uniform use of standard symbols, letters, and designations, as significant time is required to adjust to unfamiliar designations. Readers will find Section 1 particularly useful for resolving questions of units, symbols, and designations.

This Handbook has been organized with the basic developments first, followed by numerical tabulation of material properties. Next, components, circuit analysis, and circuit design are introduced, and one progresses from smaller to larger systems. Each section has been structured to be largely independent, with a separate table of contents, footnotes, exercises, and a bibliography of related material. The *Electronics Designers' Handbook* will be a valuable one-volume source book for a spectrum of electronics designers, from hands-on designers to analytical designers. Ease of usage has been enhanced by extensive tabular and graphical information. Circuits, many with component values included, are described, evaluated, and compared with alternatives. By deliberate intent, all figures have been extensively annotated to serve as additional information sources.

The First Edition was successfully used as a textbook, and it is to be hoped that the Second Edition will be too. A good basic education in electronic design is contained within the covers of this Handbook. The organization, annotation, and many worked-out exercises make it suitable for use as a textbook or for self-learning. I have used some sections in the classroom and would be pleased to learn about the experiences of others in this regard.

The value of a handbook is compromised if accuracy is doubtful. Considerable time and effort have been expended in ten separate proofreadings to eliminate errors, but some may exist. I would appreciate their being brought to my attention so that corrections can be made at the first opportunity.

L. J. Giacoletto

Contents

Section 1

Units, Symbols, Definitions, Formulas, and Conversion Factors

L. J. GIACOLETTO

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1.1. International System of Units. In 1964 the National Bureau of Standards (U.S.)[1] announced the policy of using the International System (SI) of independent units as defined and given official status by the 11th General Conference on Weights and Measures, Paris, October 1960. See Table 1.1.

TABLE 1.1 Base SI Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

[1] National Bureau of Standards (U.S.), Units of Weights and Measure, *NBS Misc. Publ.* 286, May 1967. See also *Int. Stand. ISO* 1000, SI Units and Recommendations for the Use of Their Multiples and of Certain Other Units, 1st ed., American National Standards Institute, New York, 1973.

The supplementary units shown in Table 1.2 are dimensionless. The following additional dimensionless units will be used: cycle = 2π radians = c; revolution = 2π radians = r; turn = 2π radians = n.

TABLE 1.2 Supplementary SI Units

Quantity	Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

The independent units as well as other derived units can be multiplied or submultiplied as shown in Table 1.3.

TABLE 1.3 Official Decimal Multiples and Submultiples

Multiple or submultiple	Prefix	Symbol	Multiple or submultiple	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deka	da	10^{-18}	atto	a

The approved derived SI units are listed in Table 1.4.

TABLE 1.4 Derived SI Units

Quantity	Unit	Unit symbol	Dimension
Energy	joule	J	$kg\ m^2\ s^{-2}$
Power	watt	W	$kg\ m^2\ s^{-3} = J\ s^{-1}$
Force	newton	N	$kg\ m\ s^{-2} = J\ m^{-1}$
Electric charge	coulomb	C	$A\ s$
Voltage	volt	V	$kg\ m^2\ s^{-3} = J\ A^{-1}\ s^{-1}$
Electric field strength	volt per meter	V/m	$kg\ m\ s^{-3} = V\ m^{-1}$
Electric flux density	coulomb per meter squared	C/m²	$A\ s\ m^{-2}$
Electric capacitance	farad	F	$A^2\ s^4\ kg^{-1}\ m^{-2} = A\ s\ V^{-1}$
Magnetic flux	weber	Wb	$kg\ m^2\ s^{-2}\ A^{-1} = V\ s$
Magnetomotive force	ampere	A	A
Magnetic field strength	ampere per meter	A/m	$A\ m^{-1}$
Magnetic flux density	tesla	T	$kg\ s^{-2}\ A^{-1} = V\ s\ m^{-2}$
Inductance	henry	H	$kg\ m^2\ s^{-2}\ A^{-2} = V\ s\ A^{-1}$
Electrical resistance	ohm	Ω	$kg\ m^2\ s^{-3}\ A^{-2} = V\ A^{-1}$
Electrical conductance	siemens	S	$A^2\ s^3\ m^{-2}\ kg^{-1} = AV^{-1}$
Frequency	hertz	Hz	$s^{-1} = c\ s^{-1}$
Velocity	meter per second	m/s	$m\ s^{-1}$
Acceleration	meter per second squared	m/s²	$m\ s^{-2}$
Area	square meter	m²	m^2
Volume	cubic meter	m³	m^3
Density	kilogram per cubic meter	kg/m³	$kg\ m^{-3}$
Pressure	newton per square meter	Pa	$kg\ m^{-1}\ s^{-2} = N\ m^{-2}$
Angular velocity	radian per second	rad/s	$s^{-1} = rad\ s^{-1}$
Angular acceleration	radian per second squared	rad/s²	$s^{-2} = rad\ s^{-2}$
Luminance	candela per square meter	cd/m²	$cd\ m^{-2}$
Luminous flux	lumen	lm	$cd = cd\ sr$
Illumination	lux	lx	$cd\ m^{-2} = cd\ sr\ m^{-2}$

1.2. Universal Physical Constants. Physical constants in Table 1.5 are evaluated on the basis of SI units, but the values indicated are based on National Bureau of Standards 1969 realizations of the SI units.

TABLE 1.5 Universal Physical Constants

Name of constant	Symbol	Value	Remarks
Speed of propagation of electromagnetic waves in vacuum	c_0	$2.99792458(1.2) \times 10^{8}$ m/s	NBS 1974 value
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ kg m A^{-2} s^{-2} (or H/m)	*Note 2*
Permittivity of vacuum	ϵ_0	$8.85418782(7) \times 10^{-12}$ A^2 s^4 kg^{-1} m^{-3} (or F/m)	$\epsilon_0 = (\mu_0 c_0^2)^{-1} \approx 10^{-9}/36\pi$ (*Notes* 1, 3)
Impedance of vacuum	Z_0	$376.73031(2)$ kg m^2 A^{-2} s^{-3} (or V/A)	$Z_0 = (\mu_0/\epsilon_0)^{1/2}$
Photon charge	q_γ	0	
Photon rest mass	m_γ	0	
Photon magnetic moment	μ_γ	0	
Electron charge	q_e	$-1.6021892(46) \times 10^{-19}$ A s (or C)	*Notes* 1, 4
Electron rest mass	m_e	$9.109534(47) \times 10^{-31}$ kg	*Note* 1
Electron Compton wavelength	λ_e	$2.4263089(40) \times 10^{-12}$ m	$\lambda_e = h/m_e c_0$ (*Note* 1)
Electron classical radius	r_e	$2.8179380(70) \times 10^{-15}$ m	$r_e = q_e^2/4\pi\epsilon_0 m_e c_0^2$ (*Note* 1)
Electron magnetic moment	μ_e	$9.284832(36) \times 10^{-24}$ A m^2	$\mu_e/\mu_B = 1.0011596567(35)$ (*Note* 1)
Proton charge	q_p	$-q_e$	
Proton rest mass	m_p	$1.6726485(86) \times 10^{-27}$ kg	$m_p/m_e = 1836.15152(70)$ (*Note* 1)
Proton Compton wavelength	$\lambda_{C,p}$	$1.3214099(22) \times 10^{-15}$ m	$\lambda_{C,p} = h/m_p c_0$ (*Note* 1)
Proton radius	r_p	$\approx 3 \times 10^{-15}$ m	
Proton magnetic moment	μ_p	$1.4106171(55) \times 10^{-26}$ A m^2	$\mu_p/\mu_B = 1.521032209(16) \times 10^{-3}$ (*Note* 1)
Neutron charge	q_n	0	
Neutron rest mass	m_n	$1.6749543(86) \times 10^{-27}$ kg	$m_n/m_e = 1838.68$ (*Note* 1)
Neutron Compton wavelength	$\lambda_{C,n}$	$1.3195909(22) \times 10^{-15}$ m	$\lambda_{C,n} = h/m_n c_0$ (*Note* 1)
Neutron radius	r_n	$\approx 3 \times 10^{-15}$ m	
Neutron magnetic moment	μ_n	$-9.66322(10) \times 10^{-27}$ A m^2	$\mu_n/\mu_N = -1.91315$
Atomic mass unit	μ	$1.6605518(17) \times 10^{-27}$ kg	$\mu = 10^{-3}/N_A$ NBS 1974 value
Avogadro constant	N_A	$6.0220943(63) \times 10^{23}$ $mole^{-1}$	$N_A = 10^{-3}/\mu$ NBS 1974 value
Faraday constant	F	$9.648456(27) \times 10^{7}$ C $mole^{-1}$	$F = -q_e N_A$ (*Note* 1)
Planck constant	h	$6.626176(36) \times 10^{-34}$ kg m^2 s^{-1} (or J s)	*Note* 1
Angular Planck constant	$\hbar$	$1.0545887(57) \times 10^{-34}$ kg m^2 s^{-1} (or J s)	$\hbar = h/2\pi$ (*Note* 1)
Boltzmann constant	k	$1.380662(44) \times 10^{-23}$ kg m^2 K^{-1} s^{-2} (or J/K)	$k = R/N_A$ (*Note* 1)
Gas constant	R	$8.31441(26)$ J K^{-1} $mole^{-1}$	$R = kN_A$ (*Note* 1)
Rydberg constant	R_∞	$1.097373177(83) \times 10^{7}$ m^{-1}	$R_\infty = m_e q_e^4/8\epsilon_0^2 c_0 h^3$ (*Note* 1)
Electron charge/mass ratio	$-q_e/m_e$	$1.7588047(49) \times 10^{11}$ A s kg^{-1}	*Note* 1
Magnetic flux quantum	Φ_0	$2.0678506(54) \times 10^{-15}$ kg m^2 A^{-1} s^{-2} (or V s)	$\Phi_0 = -h/2q_e$ (*Note* 1)
Quantum of circulation	$h/2m_e$	$3.6369455(60) \times 10^{-4}$ m^2 s^{-1}	
Bohr radius	a_0	$5.2917706(44) \times 10^{-11}$ m	$a_0 = \epsilon_0 h^2/\pi m_e q_e^2$ (*Note* 1)
Bohr magneton	μ_B	$9.274078(36) \times 10^{-24}$ A m^2	$\mu_B = -\hbar q_e/2m_e$ (*Note* 1)
Nuclear magneton	μ_N	$5.050824(20) \times 10^{-27}$ A m^2	$\mu_N = -\hbar q_e/2m_p$ (*Note* 1)
Stefan-Boltzmann constant	σ	$5.67032(71) \times 10^{-8}$ kg s^{-3} K^{-4} (or W/m^2 K^4)	$\sigma = \pi^2 k^4/60\hbar^3 c_0^2$ (*Note* 1)
First radiation constant	c_1	$3.741832(20) \times 10^{-16}$ kg m^4 s^{-3} (or W m^2)	$c_1 = 2\pi h c_0^2$ (*Note* 1)
Second radiation constant	c_2	$1.438786(45) \times 10^{-2}$ K m	$c_2 = hc_0/k$

TABLE 1.5 Universal Physical Constants *(continued)*

Name of constant	Symbol	Value	Remarks
Molar volume of ideal gas	V_m	22.41383(70) m^3 $mole^{-1}$ at 273.15 K and 1.01325 $\times 10^5$ N m^{-2}	*Note* 1
Gravitational constant	G	6.6720(41) $\times 10^{-11}$ m^3 kg^{-1} s^{-2}	*Note* 1
Standard acceleration of free fall....................................	g_n	9.80665 m s^{-2}	*Note* 5

Note 1. These values are from E. Richard Cohen and B. N. Taylor, The 1973 Least-Square Adjustment of the Fundamental Constants, *J. Phy. Chem. Ref. Data,* vol. 2, no. 4, pp. 663–734, 1973. The numbers in parentheses are the standard deviation uncertainties in the last digits of the quoted value on the basis of internal consistency. See also *Nat. Bur. Stand. (U.S.) Tech. News Bull.*, vol. 47, no. 10, p. 175, October 1963; also B. N. Taylor et al., "The Fundamental Constants and Quantum Electrodynamics," Academic Press, Inc., New York, 1969.

Note 2. ANSI Y10.5-1968 indicates Γ_m as the symbol instead of μ_0, and refers to it as a magnetic constant.

Note 3. ANSI Y10.5-1968 indicates Γ_e as the symbol instead of ϵ_0, and refers to it as an electric constant.

Note 4. ANSI Y10.5-1968 indicates e as the symbol instead of q_e, and refers to it as an elementary charge.

Note 5. This value is as defined by the Conférence Générale des Poids et Mesures (CGPM) in 1901.

1.3. Definition of Mathematical Symbols

TABLE 1.6 General Algebra

(1) π	Circumference/diameter ratio = 3.141592654 . . .
(2) e	Natural base = 2.718281828 . . .
(3) j	Quadrature (90° rotative) operator = $\sqrt{-1}$, $j^2 = -1$, $j^3 = -\sqrt{-1}$, $j^4 = 1$
(4) a	120° rotative operator = $e^{j2\pi/3}$
(5) x, y, z, x_i, y_i, z_i	Real variables
(6) $w = u + jv$; $s = \sigma + j\omega$; $\gamma = \alpha + j\beta$	Complex variables
(7) $w^* = u - jv$	Conjugate complex variable
(8) $z = \text{Re } z + \text{Im } z = \|z\|e^{j\phi}$	Phasor variable
(9) $\|z\| = \text{Mag } z$	Magnitude (absolute value) variable
(10) $\phi = \text{Arg } z = \tan^{-1} \frac{\text{Im } z}{\text{Re } z}$	Argument variable
(11) Re z	Real part of z. Re $(\alpha + j\beta) = \alpha$
(12) Im z	Imaginary (quadrature) part of z. Im $(\alpha + j\beta) = \beta$
(13) $\equiv$	Identically equals (by definition)
(14) $=$	Equals
(15) $\approx$	Approximately equals
(16) $\doteq$	Equals in the limit
(17) $\sim$	Proportional to; varies as
(18) $\neq$	Does not equal
(19) $\rightarrow$	Approaches
(20) $\Rightarrow$	Implies
(21) $\geqslant$	Greater than or equal to
(22) $\leqslant$	Less than or equal to
(23) $>$	Greater than (perhaps 1 to 10 times)
(24) $<$	Less than (perhaps 1 to 0.1 times)
(25) $\gg$	Much greater than (perhaps more than 10 times)
(26) $\ll$	Much less than (perhaps less than 0.1 times)
(27) $+, -$	Algebraic addition or subtraction
(28) $\pm, \mp$	Plus or minus; minus or plus
(29) $\times, \div$	Algebraic multiplication or division
(30) $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$	Summation

TABLE 1.6 General Algebra *(continued)*

(31) $\prod_{i=1}^{n} a_i = a_1 a_2 a_3 \ldots a_n$	Product
(32) $n! = n(n-1)(n-2) \cdots 3 \times 2 \times 1$	Factorial of n (an integer)
(33) (a,b)	Open interval extending from a to b
(34) $[a,b]$	Closed interval including a and b end points
(35) $(a,b]$; $[a,b)$	Half-closed interval including b or a end point
(36) $f(x)$	Function of x
(37) $f(x)\|_a^b$	Function difference $= f(b) - f(a)$
(38) $\lim_{x \to a} f(x)$	Limit of $f(x)$ as x approaches a
(39) $\ln x$	Natural logarithm of x. Logarithm to the base e
(40) $\log x$	Common logarithm of x. Logarithm to the base 10
(41) $\log_r x$	Logarithm of x to the base (radix) r. $\log_{r1} x = \log_{r2} x \log_{r1} r2$
(42) $2.7\dot{7}$	Repeated 7 integer
(43) $2.1\dot{3}84\dot{2}$	Repeated 3842 sequence of integers
(44) 8.9625 . . .	Continuation of integers (continuing fraction)

TABLE 1.7 Calculus

(1) $\exists$	There exists
(2) $\ni$	Such that
(3) $\forall$	For all
(4) $\in$	Belongs to
(5) $\notin$	Does not belong to
(6) d	Total differential operator
(7) ∂_x	Partial differential operator with respect to x
(8) d/dx	Total derivative with respect to x operator
(9) $\partial/\partial x$	Partial derivative with respect to x operator
(10) d^n/dx^n	Total nth-order derivative with respect to x operator
(11) $\partial^n/\partial x^n$	Partial nth-order derivative with respect to x operator
(12) Δ	Increment operator
(13) $\int$	Indefinite integration operator. $\int df(x) = f(x)$
(14) $\int_x$	Partial integration operator. $\int_x \partial_x f(x,y) = f(x,y)$
(15) $\int_a^b$	Definite integration operator. $\int_a^b df(x) = f(b) - f(a)$
(16) $\int_L$	Line integral along a specified curve
(17) $\oint_L$	Line integral around a closed curve
(18) $\int_A$	Area integral over a specified surface
(19) $\oint_A$	Area integral over a closed surface
(20) $\int_v$	Volume integral throughout a specified volume
(21) $\iint$; $\iiint$; $\int n \int$	Second-, third-, and nth-order successive indefinite integration operators

TABLE 1.8 Vector Analysis

(1) $\hat{i}, \hat{j}, \hat{k}$	Unit x, y, and z vectors (nondimensional)
(2) x, y, z	Cartesian coordinates
(3) $\hat{\rho}, \hat{\phi}, \hat{z}$	Unit ρ, φ, and z vectors (nondimensional)
(4) ρ, ϕ, z	Cylindrical coordinates
(5) $\hat{r}, \hat{\theta}, \hat{\phi}$	Unit r, θ, and φ vectors (nondimensional)
(6) r, θ, ϕ	Spherical coordinates
(7) $\hat{n}$	Unit vector normal to a surface (nondimensional)
(8) $\hat{t}$	Unit vector tangent to a surface (nondimensional)
(9) $\cdot$	Scalar product (dot product, inner product)
(10) $\times$	Vector product (cross product, outer product)
(11) ∇	Gradient operator
(12) $\nabla \cdot$	Divergence operator
(13) $\nabla \times$	Curl operator
(14) $\nabla \cdot \nabla = \nabla^2$	Laplacian operator
(15) $\Box^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$	D'Alembertian operator

TABLE 1.9 Matrix Algebra

(1) $\{a_i\} = (a_i)^T$	Column matrix
(2) $(a_i) = \{a_i\}^T$	Row matrix
(3) $[a_{ij}]$	Matrix with element a_{ij} situated in the ith row and in the jth column
(4) $[a_{ij}]^T = [a_{ji}]$	Transpose matrix
(5) $[a_{ij}]^* = [a_{ij}^*]$	Conjugate matrix
(6) $[a_{ij}]^\dagger = [a_{ij}^*]^T$	Hermitian conjugate (tranjugate) matrix
(7) $U_n = \delta_{ij}$	Identity matrix of order n
(8) 0_{ij}	Null matrix
(9) $[a_{ij}]^{-1}$	Inverse matrix of $[a_{ij}]$
(10) adj $[a_{ij}]$	Adjoint matrix of $[a_{ij}]$
(11) det $[a_{ij}] = \lvert a_{ij}\rvert$	Determinant of $[a_{ij}]$
(12) cof $[a_{ij}]$	Cofactor of $[a_{ij}]$
(13) tr $[a_{ij}]$	Trace of $[a_{ij}]$
(14) $[a_{ij}][b_{kl}]$	Matrix multiplication. Note that columns of $[a_{ij}]$ must equal the rows of $[b_{kl}]$
(15) $(\{a_i\}\cdot(b_i)) = \sum_{i=1}^{n} a_i^* b_i$	Scalar (inner) product of column matrix $\{a_i\}$ and row matrix (b_i)
(16) $\lVert a\rVert = (\{a_i\}; (a_i))^{1/2}$	Euclidean (Frobenius) norm
(17) $[a_{ij}] \otimes [b_{kl}]$	Direct (outer) product of $[a_{ij}]$ and $[b_{kl}]$ matrices
(18) $\lvert J\rvert$	Jacobian (functional) determinant
(19) $\lvert H\rvert$	Hessian determinant

TABLE 1.10 Logic and Boolean Algebra

(1) $a \in A$	a is contained in set A.
(2) $A \cap B, A \cdot B$	Logical multiplication. Intersection of set A and set B. A AND B.
(3) $A \cup B, A + B$	Logical addition. Union of set A and set B. A OR B.
(4) $A \oplus B$	Exclusive OR.
(5) $A \supset B$	Logical inclusion. Inclusion of set B in set A.
(6) $A \ominus B$	Complement of set B in set A.
(7) $\bar{A}, \tilde{A}$	Logical complementation. NOT set A. Negation.
(8) $\emptyset, 0$	Logical impossibility. *Empty (null) set. Zero state.*
(9) $I, 1$	Logical certainty. Universal set. One state.

TABLE 1.11 Statistics and Probability

(1) $p(x) = dP(x)/dx$	Differential probability (density) function of random variable x. Univariate frequency function
(2) $P(x) = \int_{-\infty}^{x} p(x')\,dx'$	Cumulative probability function of random variable x. Univariate distribution function
(3) $P(A < x < B)$	Cumulative probability that x is between A and B
(4) $P(E \cap F)$	Probability of simultaneous (joint) occurrence of E and F
(5) $P(E \cup F)$	Probability of occurrence of E or F or both
(6) $P(E\|F) = P(E \cap F)/P(F)$	Conditional probability. Probability of occurrence of E provided F has occurred
(7) $E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)\,dx$	Expected value of function of a random variable x
(8) $E(x) = \bar{x} = \int_{-\infty}^{\infty} xp(x)\,dx$	Expected (mean) value of random variable x
(9) $\alpha_r = E(x^r)$	rth moment of random variable x. rth moment about the origin
(10) $\mu_r = E(x - \bar{x})^r$	rth moment of random variable x from mean value. rth central moment
(11) $\text{Var}\,x = E[(x-\bar{x})^2] = \overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2$	Variance value of random variable x
(12) $\sigma = (\text{Var}\,x)^{1/2}$	Standard deviation of random variable x
(13) $M_x(s) = E(e^{sx})$	Moment generating function associated with random variable x
(14) $\psi_x(q) = E(e^{jqx})$	Characteristic function associated with random variable x
(15) $\psi_g(q) = E[e^{jqg(x)}]$	Characteristic function of $g(x)$ with random variable x
(16) $p(x,y) = d^2P(x,y)/dx\,dy$	Differential probability (density) function of random variables x and y. Bivariate frequency function
(17) $P(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} p(x',y')\,dx'dy'$	Cumulative probability function of random variables x and y. Bivariate distribution function
(18) $P(A < x < B, C < y < D)$	Cumulative probability that x is between A and B and that also y is between C and D. Cumulative joint probability
(19) $\text{Cov}\,(x,y) = E[(x-\bar{x})(y-\bar{y})] = \overline{(x-\bar{x})(y-\bar{y})}$	Covariance value of random variables x and y
(20) $\rho(x,y) = \text{Cov}\,(x,y)/\sigma_x\sigma_y$	Correlation coefficient of random variables x and y

TABLE 1.12 Functions, Transforms, and Miscellaneous

(1) $\exp z = e^z$	Exponential function of z
(2) $\ln x = \int_0^x \frac{du}{u}$	Logarithm of x to the base e
(3) $\log x$	Logarithm of x to the base 10
(4) $\sin x, \sin^{-1} x$	Direct and inverse sine operation on x
(5) $\cos x, \cos^{-1} x$	Direct and inverse cosine operation on x
(6) $\sec x, \sec^{-1} x$	Direct and inverse secant operation on x
(7) $\csc x, \csc^{-1} x$	Direct and inverse cosecant operation on x
(8) $\tan x, \tan^{-1} x$	Direct and inverse tangent operation on x
(9) $\cot x, \cot^{-1} x$	Direct and inverse cotangent operation on x
(10) $\sinh x, \sinh^{-1} x$	Direct and inverse hyperbolic sine operation on x
(11) $\cosh x, \cosh^{-1} x$	Direct and inverse hyperbolic cosine operation on x
(12) $\text{sech}\, x, \text{sech}^{-1} x$	Direct and inverse hyperbolic secant operation on x
(13) $\text{csch}\, x, \text{csch}^{-1} x$	Direct and inverse hyperbolic cosecant operation on x
(14) $\tanh x, \tanh^{-1} x$	Direct and inverse hyperbolic tangent operation on x
(15) $\coth x, \coth^{-1} x$	Direct and inverse hyperbolic cotangent operation on x
(16) $\text{Ei}\, x = \int_{-\infty}^{x} \frac{e^u}{u}\,du$	Exponential integral. $x > 0$
(17) $\text{Si}\, x = \int_0^x \frac{\sin u}{u}\,du$	Sine integral
(18) $\text{Ci}\, x = \int_{\infty}^{x} \frac{\cos u}{u}\,du$	Cosine integral

TABLE 1.12 Functions, Transforms, and Miscellaneous (*continued*)

(19) $\text{Shi } x = \int_0^x \frac{\sinh u}{u}\, du$	Hyperbolic sine integral
(20) $\text{Chi } x = \int_\infty^x \frac{\cosh u}{u}\, du$	Hyperbolic cosine integral
(21) $\Gamma(z) = \int_0^\infty u^{z-1} e^{-u}\, du$	Gamma (factorial) function. Re $z > 0$
(22) $\psi^{(n)}(z) = \frac{d^{n+1}\Gamma(z)}{dz^{n+1}}$	Polygamma function. Re $z > 0$, $n = 1, 2, 3, \ldots$
(23) $\Gamma(z,x) = \int_x^\infty u^{z-1} e^{-u}\, du$	Incomplete gamma function. Re $z > 0$
(24) $\text{erf } z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2}\, du$	Error function
(25) $\text{erfc } z = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-u^2}\, du$	Complementary error function
(26) $C(z) = \int_0^z \cos\left(\frac{\pi}{2} u^2\right) du$	Fresnel cosine integral
(27) $S(z) = \int_0^z \sin\left(\frac{\pi}{2} u^2\right) du$	Fresnel sine integral
(28) $P_n(z) = \frac{1}{2^n n!} \frac{d^n (z^2-1)^n}{dz^n}$	Legendre polynomial of first kind and integer degree n
(29) $P_\nu(z)$	Legendre function of first kind and degree ν
(30) $P_\nu^\mu(z)$	Associated Legendre function of first kind of degree ν and order μ
(31) $Q_\nu^\mu(z)$	Associated Legendre function of second kind of degree ν and order μ
(32) $J_\nu(z)$	Bessel function of first kind and order ν
(33) $Y_\nu(z) = N_\nu(z)$	Bessel function of second kind and order ν. Weber's function
(34) $H_\nu^{(1)}(z)$, $H_\nu^{(2)}(z)$	Bessel functions of third kind and order ν. Hankel functions
(35) $I_\nu(z)$	Modified Bessel function of first kind and order ν
(36) $K_\nu(z)$	Modified Bessel function of second kind and order ν
(37) $\text{ber}_\nu\, x + j\, \text{bei}_\nu\, x$	Kelvin function of order ν
(38) $\text{ker}_\nu\, x + j\, \text{kei}_\nu\, x$	Modified Kelvin function of order ν
(39) $\text{Ai } z$	Airy function of first kind
(40) $\text{Bi } z$	Airy function of second kind
(41) $\mathbf{H}_\nu(z)$	Struve function of order ν
(42) $M(a,b,z)$	Regular confluent hypergeometric (Kummer) function
(43) $U(a,b,z)$	Irregular (logarithmic) confluent hypergeometric (Kummer) function
(44) $F(a,b,c,z)$	Hypergeometric function
(45) $F_L(\eta,\rho)$	Regular Coulomb wave function
(46) $G_L(\eta,\rho)$	Irregular (logarithmic) Coulomb wave function
(47) sn u, cn u, dn u	Jacobi elliptic functions with pole on the imaginary axis
(48) cd u, sd u, nd u	Jacobi elliptic functions with complex pole
(49) dc u, nc u, sc u	Jacobi elliptic functions with pole on the real axis
(50) ns u, ds u, cs u	Jacobi elliptic functions with pole at the origin
(51) $F(\varphi/\alpha)$	Incomplete elliptic integral of the first kind
(52) $E(\varphi/\alpha)$	Incomplete elliptic integral of the second kind
(53) $ce_r(z,q)$, $se_r(z,q)$	Even and odd Mathieu functions of order r
(54) $T_n(x)$	Chebyshev polynomial of the first kind of degree n
(55) $U_n(x)$	Chebyshev polynomial of the second kind of degree n
(56) $L_n(x)$	Laguerre polynomial of degree n
(57) $H_n(x)$	Hermite polynomial of degree n
(58) $B_n(x)$	Bernoulli polynomial of degree n
(59) $E_n(x)$	Euler polynomial of degree n
(60) $\zeta(s) = \sum_{k=1}^\infty k^{-s}$	Riemann zeta function. Re $s > 1$

TABLE 1.12 Functions, Transforms, and Miscellaneous (continued)

(61) $_nP_k = P_k^n = \frac{n!}{(n-k)!}$	Permutations of n different elements in groups of k with recognition of order
(62) $_nC_k = C_k^n = \frac{n}{k} = \frac{n!}{k!(n-k)!}$	Combinations of n different elements in groups of k without recognition of order. Binomial coefficient
(63) $\delta(x) = \lim_{a \to 0} \frac{e^{-\pi^2 x^2 / a^2}}{a}$	Impulse (Dirac) function
(64) $U(x) = \int_{-\infty}^{x} \delta(x')\, dx' = \begin{cases} 0 & x < 0 \\ 1/2 & x = 0 \\ 1 & x > 0 \end{cases}$	Unit step function
(65) $\operatorname{Sgn} x = 2U(x) - 1$	Signum function
(66) $G_X(x) = \begin{cases} 1 & \lvert x\rvert < X/2 \\ 0 & \lvert x\rvert > X/2 \end{cases}$	Gate function
(67) $\delta_X(x) = \sum_{n=-\infty}^{\infty} \delta(x - nX)$	Dirac comb function. Equispaced impulse function
(68) $\operatorname{Str} x = \int_{-\infty}^{\infty} \delta_X(x)\, dx$	Unit staircase function
(69) $\operatorname{Sa} x = \frac{\sin x}{x}$	Sampling function
(70) $\mathcal{F}, \mathcal{F}^{-1}$	Direct and inverse Fourier transform operations
(71) $\mathcal{F}_c, \mathcal{F}_c^{-1}$	Direct and inverse Fourier cosine transform operations
(72) $\mathcal{F}_s, \mathcal{F}_s^{-1}$	Direct and inverse Fourier sine transform operations
(73) $\mathcal{L}, \mathcal{L}^{-1}$	Direct and inverse Laplace transform operations
(74) $\mathcal{M}, \mathcal{M}^{-1}$	Direct and inverse Mellin transform operations
(75) $\mathcal{H}_\nu, \mathcal{H}_\nu^{-1}$	Direct and inverse Hankel transform operations
(76) $\mathcal{H}, \mathcal{H}^{-1}$	Direct and inverse Hilbert transform operations
(77) $\mathcal{Z}, \mathcal{Z}^{-1}$	Direct and inverse z transform operations

1.4. Mathematical Formulas and Data

1.4a. Real Number Systems. A real number system can be devised using two or more counting symbols. The base of the number system is the number of different symbols used and is called the radix r of the number system: for example, radix = 10 for our familiar decimal number system. The r different symbols are associated with values from 0 to $r-1$: for example, 0 to 9 in the decimal number system. The r symbols a_i ($i = 0, 1, 2, \ldots, r-1$) are ranked in integer increasing value.

TABLE 1.13 Names of Number System

Base (radix)	Name	Base (radix)	Name
2	Binary	8	Octal
3	Ternary	9	Nonary
4	Quaternary	10	Decimal
5	Quinary	11	Undenary
6	Senary	12	Duodenary
7	Septenary	16	Hexadecimal

Any positive real number of value x can be uniquely represented in the number system radix r by writing a row of $m + k$ symbols, with each of the $m + k$ symbols being one of the r different symbols. These symbols are ordered in the row from the right (least significant symbol) to the left (most significant symbol). The column positions of the $m + k$ symbols are numbered from right and left of a dot called the radix point.

(1)
$$x = (a_m \; a_{m-1} \; \cdots \; a_2 \; a_1 \; a_0 \cdot a_{-1} \; a_{-2} \; \cdots \; a_{-k})_r$$

The value of this number radix r is

(2)
$$x = a_m r^m + a_{m-1} r^{m-1} + \cdots + a_2 r^2 + a_1 r^1 + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \cdots + a_{-k} r^{-k} = \sum_{i=-k}^{i=m} a_i r^i$$

It is generally convenient to use the r symbols of number system radix r to be identically the same as the ranked symbols of the decimal number system. Number designations in several different number systems are illustrated in Table 1.14.

Negative numbers can be accommodated by introducing two sign symbols + and − placed before the most significant symbol. The radix point can be moved about as desired (floating point) by factoring out a power of r. Thus for $r = 2$, $m + k = 6$, radix point just to the left of the most significant symbol, first symbol to indicate sign, and last two symbols to indicate the radix power, the number $(110110111)_2$ would be $-0.101101_2 \times 2^3 = -101.101_2 = -5.625_{10}$.

TABLE 1.14 Number Designations in Different Number Systems

N_2	N_3	N_4	N_6	N_8	N_{10}
0.	0.	0.	0.	0.	0.
1.	1.	1.	1.	1.	1.
10.	2.	2.	2.	2.	2.
11.	10.	3.	3.	3.	3.
100.	11.	10.	4.	4.	4.
101.	12.	11.	5.	5.	5.
110.	20.	12.	10.	6.	6.
111.	21.	13.	11.	7.	7.
1000.	22.	20.	12.	10.	8.
1001.	100.	21.	13.	11.	9.
1010.	101.	22.	14.	12.	10.
1011.	102.	23.	15.	13.	11.
1100.	110.	30.	20.	14.	12.
1101.	111.	31.	21.	15.	13.
1110.	112.	32.	22.	16.	14.
1111.	120.	33.	23.	17.	15.
10000.	121.	100.	24.	20.	16.
10001.	122.	101.	25.	21.	17.
10010.	200.	102.	30.	22.	18.
10011.	201.	103.	31.	23.	19.
10100.	202.	110.	32.	24.	20.

Occasionally for a variety of reasons it is necessary to encode the number as written in the original number system. A few different encoding schemes are illustrated in Table 1.15.

TABLE 1.15 Illustrative Encoding Schemes

N_{10}	N_x	$(N_x)_{\text{enc.}}$	Code name	Encoding rules
415	$N_8 = 637$	141	Radix complement $= r^{m+1} - N_x$	Replace each column symbol by (radix −1) complement, and add 1 to the final number.
13	$N_2 = 1101$	1011	Reflected binary (gray)	Column symbol is 1 if corresponding column symbol + symbol to left is 1. Otherwise it is zero.
13	$N_2 = 1101$	10000	Excess 3	Add binary 3 to N_x.
13	$N_2 = 1101$	11000	Reflected binary, excess 3	Excess 3 + reflected binary codes.
659	$N_{10} = 659$	0110 0101 1001	Binary Coded Decimal	Encode each symbol as a binary number.
415	$N_8 = 637$	110 011 111	Binary Coded Octal	Encode each symbol of N_x as a binary number.