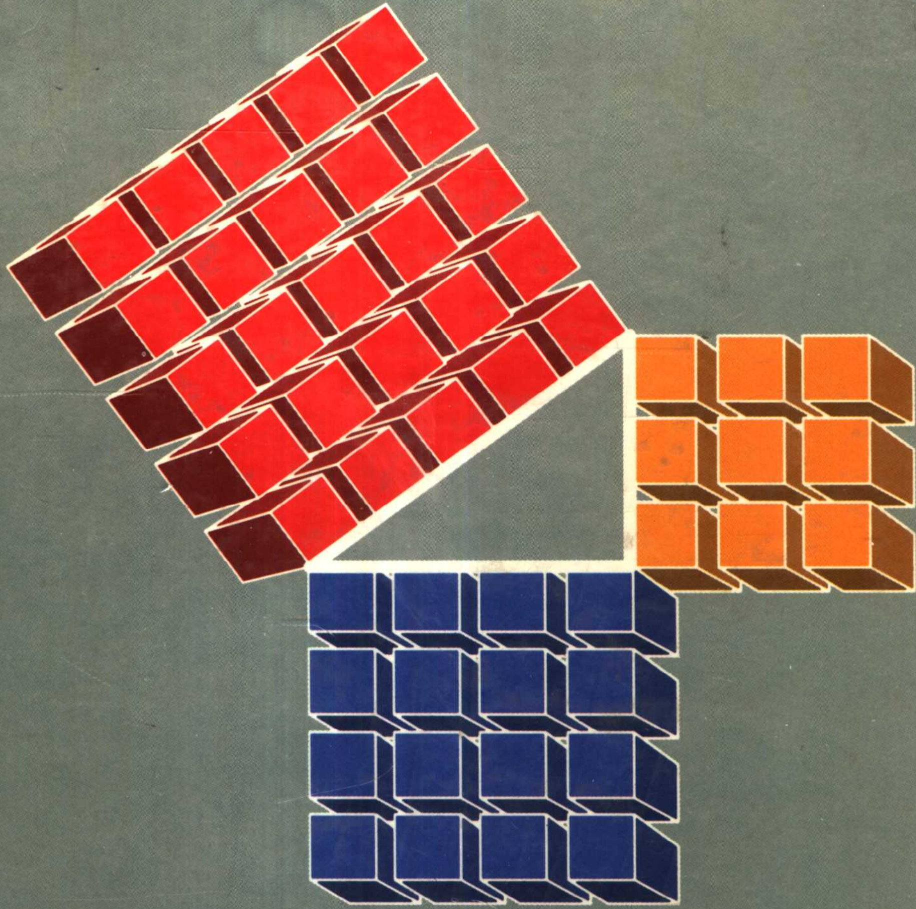

Third Edition

COLLEGE ALGEBRA

A Problem-Solving Approach

WALTER FLEMING • DALE VARBERG

THIRD EDITION

College Algebra

A PROBLEM-SOLVING APPROACH

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George Polya

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosities and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on the mind and character for a life time.

—*How to Solve It (p. v)*

Solving a problem is similar to building a house. We must collect the right material, but collecting the material is not enough; a heap of stones is not yet a house. To construct the house or the solution, we must put together the parts and organize them into a purposeful whole.

—*Mathematical Discovery (vol. 1, p. 66)*

You turn the problem over and over in your mind; try to turn it so it appears simpler. The aspect of the problem you are facing at this moment may not be the most favorable. Is the problem as simply, as clearly, as suggestively expressed as possible? Could you restate the problem?

—*Mathematical Discovery (vol. 2, p. 80)*

We can scarcely imagine a problem absolutely new, unlike and unrelated to any formerly solved problem; but, if such a problem could exist, it would be insoluable. In fact, when solving a problem, we should always profit from previously solved problems, using their result, or their method, or the experience we acquired solving them. . . . Have you seen it before? Or have you seen the same problem in slightly different form?

—*How to Solve It (p. 98)*

An insect tries to escape through the windowpane, tries the same hopeless thing again and again, and does not try the next window which is open and through which it came into the room. A mouse may act more intelligently; caught in a trap, he tries to squeeze between two bars, then between the next two bars, then between other bars; he varies his trials, he explores various possibilities. A man is able, or should be able, to vary his trials more intelligently, to explore the various possibilities with more understanding, to learn by his errors and shortcomings. "Try, try again" is popular advice. It is good advice. The insect, the mouse, and the man follow it; but if one follows it with more success than the others it is because he varies his problem more intelligently.

—*How to Solve It (p. 209)*

These quotations are taken from George Polya, *How to Solve It*, Second Edition (Garden City, NY: Doubleday & Company, Inc., 1957) and George Polya, *Mathematical Discovery*, vols. 1 and 2 (New York: John Wiley & Sons, Inc., 1962).

Preface

Since publication of the second edition of this book, the mathematical community has lost one of its most gifted members. George Polya (1887–1985) was a researcher of first rank with some 250 published papers and several books on important mathematics. In addition, he was a great teacher, one who thought deeply about the essence of mathematics and how we learn it. In his books on mathematical pedogogy, he taught us that mathematics is preeminently the art of solving problems. And, better than anyone else, he described the strategies, illustrated the techniques, and explored the byways of this subject. The authors of this book are happy to acknowledge the influence of George Polya on their teaching and writing. In recognition of him, we cite on the previous page a number of quotations from his books. They capture something of the spirit of the man; they also say something about our philosophy in writing this book.

PROBLEM-SOLVING EMPHASIS

If emphasis on problem solving characterized earlier editions of this book, this emphasis is even more evident in the present edition. While we have rewritten parts of many sections to achieve greater clarity of exposition, most of our effort has gone into improving the problem sets. Every section of the book ends with an extensive problem set; and each of these is in two parts. First there is a collection of basic problems which have been carried over from the earlier edition with minor changes. These problems are quite straightforward and are meant to develop the skills and reinforce the main ideas of the section. Many of these problems are clustered around examples that appear within the problem set.

Following the basic problems, there is a set of miscellaneous problems. *These have been completely reworked for this edition*. Our aim for the miscellaneous problems is to give the student an exciting and challenging tour through the applications of the ideas of the section. We begin with a few easy review type problems, move in a carefully graded manner to harder and more interesting applications, and conclude with a *teaser* problem (about which we will say more shortly). We think both students and teachers will find our miscellaneous problems to be an outstanding feature of this edition.

THE TEASERS

Over our combined seventy years of teaching, we have collected a large number of intriguing problems. Many of them are our own creation; some are part of current mathematical lore; others come from mathematical history. As a special attraction for this edition, we have inserted one of these problems (identified as a *teaser*) at the end of each section. These teasers may appear difficult at first glance but in most cases become surprisingly easy when looked at the right way. In each case, the teaser relates to the ideas of the section in which it appears. As a group, the teasers form a collection of problems that we think would please even George Polya. We hope that you will page through the book with particular attention to the range and quality of the teasers.

How should the teasers be used in class? We suggest that teachers might select some of their favorite teasers and offer a prize to the student offering the best set of solutions at the end of the term. Or teachers might use these problems as the basis for a weekly problem-solving session (perhaps as an addition to the regular class sessions). Or a teacher might select from them a problem of the week offering a small prize for the best solution. Or they can simply be treated as extra stimulation for the very best students.

WRITING STYLE

We write as in our earlier books with the aim of being interesting and informative. This means that we address ourselves more to explaining than to proving, though we are careful to avoid logical sloppiness. Definitions are stated correctly but with a minimum of jargon. We do not prove the obvious and occasionally we omit the proof of something that is quite difficult. But we do help the student to see that mathematics is a logically coherent subject.

To capture student interest, we begin each section with a boxed display. This box may contain a historical anecdote, a famous quotation, an appropriate cartoon, a problem to solve, or a diagram illustrating a key idea of the section.

Calculators are now so common that we use them freely throughout the text though it is still true that most problems can be done without using these devices. Problems for which calculators are an important aid are labeled with the symbol Ⓒ.

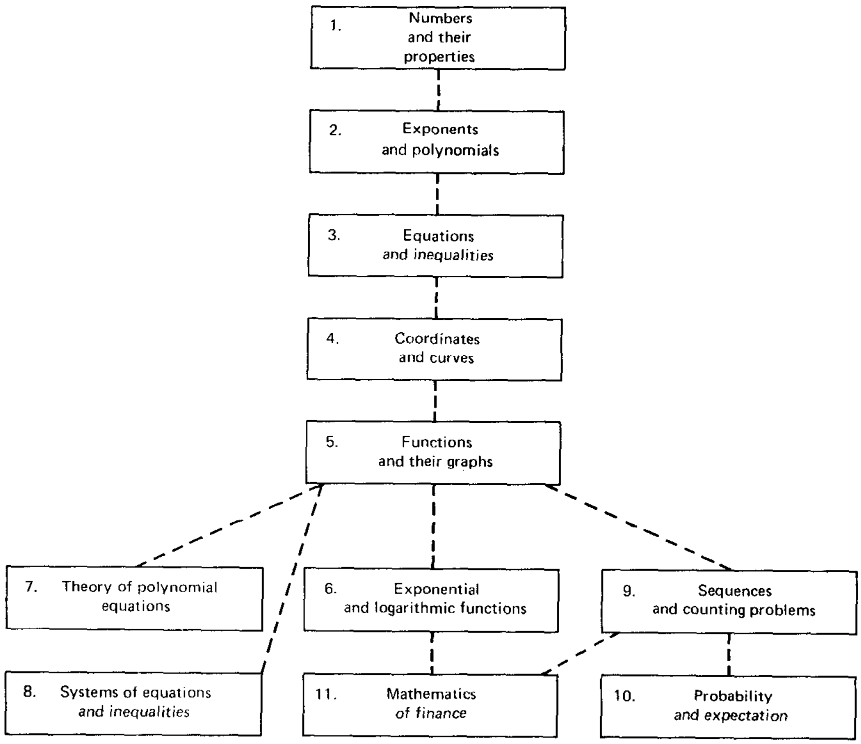
Complex numbers are introduced early (Section 1-6) but do not play a significant role until Chapter 7. For teachers who wish to deemphasize their use, we have marked those problems that use complex numbers with the symbol ⓘ.

FLEXIBILITY

This book is designed for the standard college algebra course that is offered at most American colleges. At some schools, it may serve as the background for a course in calculus, especially the brief calculus that is typically taken by social and behaviorial science majors. We think business students will appreciate the many references to business problems and especially the strong last chapter on mathematics of finance.

The following dependence chart will help an instructor in designing a syllabus. Keep in mind that the first three chapters are a review of basic algebra. In some classes, they can be omitted or covered rapidly. Each problem set is designed so that all students can profit from the basic problems and the first few miscellaneous problems. The later miscellaneous problems should be assigned somewhat judiciously.

DEPENDENCE CHART



SUPPLEMENTARY MATERIALS

An extensive variety of instructional aids is available from Prentice Hall.

Instructor's Manual The instructor's manual was prepared by the authors of the textbook. It contains the following items.

(a) Answers to all the even numbered problems (answers to the odd problems appear at the end of the textbook).
(b) Complete solutions to the last four problems in each problem set. This includes the teaser problem.
(c) Six versions of a chapter test for each chapter together with an answer key for these tests.
(d) A test bank of more than 1300 problems with answers designed to aid an instructor in constructing examinations.
(e) A set of transparencies that illustrate key ideas.

Prentice-Hall Test Generator The test bank of more than 1300 problems is available on floppy disk for the IBM PC. This allows the instructor to generate examinations by choosing individual problems, editing them, and if desired by creating completely new problems.

Videotapes Approximately five hours of videotaped lectures covering selected topics in college algebra are available with a qualified adoption. Contact your local Prentice Hall representative for details.

Student Solutions Manual This manual has worked-out solutions to every third problem (not including teaser problems).

Function Plotter Software A one-variable function plotter for the IBM PC is available with a qualified adoption. Contact your local Prentice Hall representative for details.

"How to Study Math" Designed to help your students overcome math anxiety and to offer helpful hints regarding study habits, this useful booklet is available free with each copy sold. To request copies for your students in quantity, contact your local Prentice Hall representative.

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The staff at Prentice Hall is to be congratulated on another fine production job. The authors wish to express appreciation especially to Priscilla McGeehon (mathematics editor), Zita de Schauensee (project editor), and Christine Gehring-Wolf (designer) for their exceptional contributions.

Walter Fleming
Dale Varberg

Contents

Numbers are an indispensable tool of civilization, serving to whip its activities into some sort of order . . . The complexity of a civilization is mirrored in the complexity of its numbers.

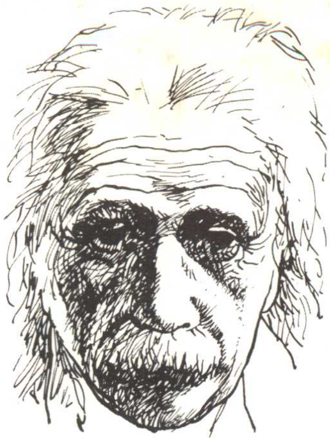
—*Philip J. Davis*

CHAPTER 1

Numbers and Their Properties

“Algebra is a merry science,” Uncle Jakob would say. “We go hunting for a little animal whose name we don't know, so we call it x. When we bag our game we pounce on it and give it its right name.”

Albert Einstein



1-1 What Is Algebra?

Sometimes the simplest questions seem the hardest to answer. One frustrated ninth grader responded, “Algebra is all about x and y, but nobody knows what they are.” Albert Einstein was fond of his Uncle Jakob's definition, which is quoted above. A contemporary mathematician, Morris Kline, refers to algebra as generalized arithmetic. There is some truth in all of these statements, but perhaps Kline's statement is closest to the heart of the matter. What does he mean?

In arithmetic we are concerned with numbers and the four operations of addition, subtraction, multiplication, and division. We learn to understand and manipulate expressions like

$$16 - 11 \qquad \frac{3}{24} \qquad (13)(29)$$

In algebra we do the same thing, but we are more likely to write

$$a - b \qquad \frac{x}{y} \qquad mn$$

without specifying precisely what numbers these letters represent. This determination to stay uncommitted (not to know what x and y are) offers some tremendous advantages. Here are two of them.

GENERALITY AND CONCISENESS

All of us know that $3 + 4$ is the same as $4 + 3$ and that $7 + 9$ equals $9 + 7$. We could fill pages and books, even libraries, with the corresponding facts about other numbers. All of them would be correct and all would be important.

But we can achieve the same effect much more economically by writing

$$a + b = b + a$$

The simple formula says all there is to be said about adding two numbers in opposite order. It states a general law and does it on one-fourth of a line.

Or take the well-known facts that if I drive 30 miles per hour for 2 hours, I will travel 60 miles, and that if I fly 120 miles per hour for 3 hours, I will cover 360 miles. These and all other similar facts are summarized in the general formula

$$D = RT$$

which is an abbreviation for

$$\text{Distance} = \text{rate} \times \text{time}$$

FORMULAS

The formula $D = RT$ is just one of many that scientists use almost without thinking. Among these formulas are those for area and volume, which have been known since the time of the Greeks. As a premier example, we mention the formula for the area of a triangle (Figure 1), namely,

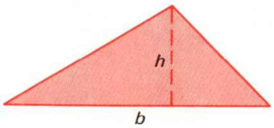


Figure 1

$$A = \frac{1}{2}bh$$

a formula that we will have occasion to use innumerable times in this book. Here b denotes the length of the base and h stands for the height (or altitude) of the triangle. Thus, a triangle with base $b = 24$ and height $h = 10$ has area

$$A = \frac{1}{2}bh = \frac{1}{2}(24)(10) = 120$$

Of course, we must be careful about units. If the base and height are given in inches, then the area is in square inches.

A more interesting formula is the familiar one

$$A = \pi r^2$$

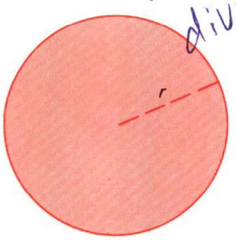


Figure 2

for the area of a circle of radius r (Figure 2). It is interesting because of the appearance of the number π. Perhaps you have learned to approximate π by the fraction 22/7, actually, a rather poor approximation. In this course, we suggest that you use the decimal approximation 3.14159 or the even better approximation that your calculator gives (it should have a π button). Thus a circle of radius 10 centimeters has area

$$A = \pi r^2 = (3.14159)(10)(10) = 314.159$$

The answer is in square centimeters.

We are confident that you once learned all the important area and volume formulas but, because your memory may need jogging, we have listed those we will need most often in Figures 3 and 4, which accompany the first problem set.

PROBLEM SOLVING

Uncle Jakob's definition of algebra hinted at something that is very important; we call it problem solving. Algebra, like most of mathematics, is full of problems. Often these problems involve finding a number that is initially unknown but that must satisfy certain conditions. If these conditions can be translated into the symbols of algebra, it may take only a simple manipulation to find the answer or, as Uncle Jakob put it, to bag our game. Here is an illustration.

> Roger Longbottom has rented a motorboat for 5 hours from a river resort. He was told that the boat will travel 6 miles per hour upstream and 12 miles per hour downstream. How far upstream can he go and still return the boat to the resort within the allotted 5-hour time period?

We recognize immediately that this is a distance-rate-time problem; the formula $D = RT$ is certain to be important. Now what is it that we want to know? We want to find a distance, namely, how far upstream Roger dares to go. Let us call that distance x miles. Next we summarize the information that is given, keeping in mind that, since $D = RT$, it is also true that $T = D/R$.

	GOING	*RETURNING*
Distance (miles)	x	x
Rate (miles per hour)	6	12
Time (hours)	$x/6$	$x/12$

There is one piece of information we have not used; it is the key to the whole problem. The total time allowed is 5 hours, which is the sum of the time going and the time returning. Thus

$$\frac{x}{6} + \frac{x}{12} = 5$$

After multiplying both sides by 12, we have

$$2x + x = 60$$

$$3x = 60$$

$$x = 20$$

Roger can travel 20 miles upstream and still return within 5 hours.

We intend to emphasize problem solving in this book. To be able to read a mathematics book with understanding is important. To learn to calculate accurately and to manipulate symbols with ease is a worthy goal. But to be able to solve problems, easy problems and hard ones, practical problems and abstract ones, is a supreme achievement.

It is time for you to try your hand at some problems. If some of them seem difficult, do not become alarmed. All of the ideas of this section will be treated in more detail later. As the title "What Is Algebra?" suggests, we wanted to give you a preview of what lies ahead.