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CONTENTS

VOLUME II

Chapter 5
New Concepts

Chapter 6
Lasers

Chapter 7
Ion Acceleration and Microwave

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Chapter 8
Plasma Heating

CHAPTER 5

NEW CONCEPTS

EQUILIBRIA AND STABILITY OF AN ELECTRON BEAM CONFINED IN A TORUS*

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Abstract

Relativistic, cylindrical, rigid-rotor equilibria for electrons are developed in the beam frame, and the corresponding laboratory frame equilibria are obtained by Lorentz transformation. Limitations of the two-mass approximation are thus illuminated. A toroidal equilibrium is developed that is based on the two-mass approximation and expansion in terms of the ratio of minor to major radius. The toroidal equilibrium is shown to have no unstable kink-modes or no Kruskal-Shafranov current limit as in a Tokomak. Small variations in the toroidal magnetic field around the torus result in trapped electrons if they are injected with sufficient transverse energy. The presence of both trapped and untrapped electrons leads to two-stream instabilities. Some comparisons are made with HIPAC experiments and the recent torus experiments at MLI.

* Supported by the Defense Nuclear Agency

Introduction

In recent years experiments at the University of Maryland[1,2] and at AVCO Corporation[3,4] have demonstrated that non-neutral electron clouds can be confined for periods of 5 to 10 milliseconds in either open-end or toroidal magnetic traps. This type of device was studied at AVCO[5] as a possible source of highly-stripped heavy ions. Rostoker[6] has suggested that by accelerating the electron cloud to relativistic energies, enormous energy densities (up to 100 Joules per cm^3) can be stored in the electrostatic and self-magnetic fields of the electron beam. Such an energy storage device could be utilized in a variety of applications. A discussion of its application to controlled thermonuclear fusion has been presented elsewhere[7].

A central question in the discussion of this concept concerns the existence of stable, self-consistent equilibria for non-neutral, relativistic electron beams. Equilibrium and stability calculations for non-neutral electron plasma have been reported by Krall and Davidson[8,9] and by Daugherty and Levy[10] in the non-relativistic regime. Rostoker and Hieronymus (R-H)[11,12] have considered relativistic equilibria and their stability in the framework of the "two-mass" approximation in both cylindrical and large aspect ratio toroidal geometries.

In this paper the equilibria and stability of non-neutral relativistic electron beams will be considered. A fully-relativistic equilibrium calculation for cylindrical geometry is presented and compared with the R-H model, which assumed non-relativistic perpendicular motion. The stored energy in the beam fields

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is found to be higher than the predictions of the two-mass approximation when the perpendicular kinetic energy of the electrons is made relativistic. A generalization of the cylindrical two-mass equilibria into a large aspect ratio toroidal geometry is accomplished by expansion in the inverse aspect ratio. The linear stability of the electron beam is improved over that for neutral plasma because of the absence of ions, which precludes the possibility of instabilities driven by charge separation. In particular, the linear stability of the kink mode and the absence of a Kruskal-Shafranov limit are obtained. Finally, the consequences of electron trapping due to small variations in the toroidal magnetic field will be discussed.

RELATIVISITIC RIGID-ROTOR EQUILIBRIA IN CYLINDRICAL GEOMETRY

The basic features of relativistic non-neutral equilibria have been outlined by Rostoker et al.[6,7] Consider the configuration shown in Fig. 1, where a non-neutral electron beam of uniform density, n_o, and radius, a, propagates along an external guide field, B_z, with velocity, V_o, and is enclosed by a concentric conducting shell of radius b. The radial force, $-e(E_r - \beta_o B_\theta)$, when combined with the guide field, leads to a rigid rotation of the beam, $v_\theta = \omega r$, where in this simple model $\omega = \omega_p^2/2\gamma_o^2\Omega_o$. Here $\beta_o = V_o/c$, $\gamma_o = (1-\beta_o^2)^{-\frac{1}{2}}$, $\omega_p^2 = 4\pi n_o e^2/\gamma_o m$, and $\Omega_o = eB_o/\gamma_o mc$. The beam rotation produces a diamagnetic macroscopic current density, J_θ, thereby reducing B_z within the beam and allowing equilibrium pressure balance to be achieved.

The form of the total kinetic energy of the electron, $(\gamma-1)mc^2$, where

$$\gamma = \frac{1}{\sqrt{1-\beta_o^2-(v_\theta/c)^2}} = \frac{\gamma_o}{\sqrt{1-(\gamma_o v_\theta/c)^2}} , \qquad (1)$$

shows immediately that $\gamma_o v_\theta < c$ is required. That is, in the beam frame (denoted by prime), $v'_\theta = \gamma_o v_\theta < c$. In the limit, $\gamma_o v_\theta \ll c$, an expansion of γ in powers of $\gamma_o v_\theta/c$ yields the two-mass approximation.

From the requirement that the electron orbits be stable, it is evident that there is no equilibrium when $B_o = 0$. In fact, in the simple model depicted in Fig. 1, the guide field must be large enough to satisfy

$$q^2 \equiv \frac{\omega_p^2}{2\gamma_o^2\Omega_o^2} \leq \frac{1}{4} \quad .$$

Using the two-mass approximation equations of motion and performing an expansion to second order in the assumed small parameter, q, R-H[11,12] obtain the single-particle equations of motion

$$\gamma_o m \left[\frac{dv_r}{dt} - \frac{v_\theta^2}{r}\right] = -e\left[E_r + \frac{1}{c}\left(v_\theta B_z - v_z B_\theta\right)\right] \tag{2}$$

$$\gamma_o m \left[\frac{dv_\theta}{dt} + \frac{v_r v_\theta}{r}\right] = \frac{e}{c} v_r B_z$$

$$\gamma_o^3 m \frac{dv_z}{dt} = -\frac{e}{c} v_r B_\theta = \frac{e}{c}\frac{dA_z}{dt} \simeq 0 \quad .$$

The single-particle constants of motion which follow from the equations are

$$H_\perp = \tfrac{1}{2}\gamma_o m(v_r^2 + v_\theta^2) - e(\Phi - \beta_o A_z) \tag{3}$$

$$P_\theta = \gamma_o m r v_\theta - \frac{e}{c} r A_\theta$$

$$v_z = V_o \quad ,$$

to order q^2, where all quantities are assumed to depend only on the radial coordinate.

The equilibrium distribution function can be any function of the constants of motion. In particular, R-H examined the function

$$F(\underset{\sim}{r},\underset{\sim}{v}) = \frac{\gamma_o m n_o}{2\pi}\,\delta[H_\perp - \omega P_\theta - \mathcal{E}_o]\,\delta[v_z - V_o] \;, \tag{4}$$

corresponding to a uniform density, finite radius beam, which rigidly rotates at the frequency

$$\omega = \frac{\Omega_o}{2\left[1+\frac{1}{4}\,\frac{\omega_p a}{c}\right]^2}\left\{1 - \sqrt{1 - \frac{2}{\gamma_o^2}\left(\frac{\omega_p}{\Omega_o}\right)^2\left[1+\frac{1}{4}\left(\frac{\omega_p a}{c}\right)^2\right]\left[1+\frac{2\gamma_o^2\langle V_\perp^2\rangle}{\omega_p^2 a^2}\right]}\right\}$$

$$\simeq \frac{1}{2\gamma_o^2}\,\frac{\omega_p^2}{\Omega_o}\left[1+\frac{2\gamma_o^2\langle V_\perp^2\rangle}{\omega_p^2 a^2}\right] , \tag{5}$$

where $\langle V_\perp\rangle \equiv [\langle v_r^2\rangle + \langle v_\theta^2\rangle]_{r=0}$ is a measure of transverse temperature, and $\langle\ \rangle$ denotes the velocity-space average of the enclosed quantity weighted by $F(\underset{\sim}{r},\underset{\sim}{v})$.

This self-consistent equilibrium reproduces the simple model described earlier with the inclusion of $\langle V_\perp^2\rangle$. The conditions for this equilibrium to exist are $\langle V_\perp^2\rangle > 0$ and

$$q^2 \equiv \frac{1}{2\gamma_o^2}\left(\frac{\omega_p}{\Omega_o}\right)^2 \le \left\{4\left[1+\frac{1}{4}\left(\frac{\omega_p a}{c}\right)^2\right]\left[1+\frac{2\gamma_o^2\langle V_\perp^2\rangle}{\omega_p^2 a^2}\right]\right\}^{-1} \tag{6}$$

which we plot in Fig. 2. From the figure we see that $q^2 \ll 1$ is required when $\omega_p a/c \gg 1$, which corresponds to the limit of high ν/γ_o.

The existence of a small parameter, q, in non-relativistic equilibria was noted by Daugherty and Levy.[10] For fixed values of density and magnetic field the relativistic value of q is

$1/\gamma_o$ - fold smaller than its non-relativistic value. The advantage of using a relativistic electron beam instead of a non-relativistic electron cloud is that equilibria can sustain γ_o - times higher electron density at a given value of B_o. The field energy is therefore γ_o^2 - fold higher in the relativisitic system. This improvement is due to the reduction in the self-field stress, $(E_r^2 - B_\theta^2)/8\pi$, when the system is relativistic, since $B_\theta = \beta_o E_r$. While this stress is very small for a relativistic system, the energy density, $(E_r^2 + B_\theta^2)/8\pi$, associated with the beam self-fields can be quite high, limited only by ion field emission from the conducting outer shell. Self-consistent equilibria in which the beam-associated field energy density exceeds both the guide field energy density and the particle kinetic energy density are therefore possible.

To verify this model and to obtain the effects associated with high perpendicular particle energy, $\gamma_o v_\theta/c \lesssim 1$, a fully-relativistic formulation of the equilibrium is required. In terms of the fully-relativistic single-particle constants of motion in the beam frame (prime),

$$H' = mc^2 \sqrt{1 + (p'/mc)^2} - e\Phi' \tag{7}$$

$$P_\theta' = r'(p_\theta' - \frac{e}{c} A_\theta')$$

$$P_z' = p_z' ,$$

where $p_z' = \gamma' m v_z'$ and $p_\theta' = \gamma' m v_\theta' = (p_y' x' - p_x' y')/r' = p_\perp' \sin(\varphi - \theta)$, consider an equilibrium distribution function of the form

$$F(\underset{\sim}{r}', \underset{\sim}{p}') = K'\delta\,[H' - \omega' P'_\theta - \mathcal{E}'_o]\,\delta(p'_z)\ . \qquad (8)$$

Introducing the variable $y = \sqrt{1 + (p'_\perp/mc)^2}$, the density and macroscopic velocity may be evaluated as follows

$$n'(r') = \int F(r', \underline{p}')\, d\underline{p}'$$

$$= K'm \int_1^\infty y\ dy \int_o^{2\pi} d\varphi\,\delta \left[y - \frac{\omega' r'}{c}\sqrt{y^2 - 1}\ \sin(\varphi - \theta) - g'(r') \right]$$

$$n'(r')\langle v'_\theta \rangle = K'm \int_1^\infty y\ dy \int_o^{2\pi} d\varphi\ \frac{c\sqrt{y^2 - 1}}{y}\ \sin(\varphi - \theta)$$

$$\delta \left[y - \frac{\omega' r'}{c}\sqrt{y^2 - 1}\ \sin(\varphi - \theta) - g'(r') \right] ,$$

where

$$g'(r') = \frac{1}{mc^2}\left[\mathcal{E}'_o - \frac{e\omega' r'}{c}\ A'_\theta + e\Phi' \right] .$$

To carry out the indicated integrations, the δ-function may be expressed as

$$\frac{1}{\frac{\omega' r'}{c}\sqrt{y^2 - 1}}\ \frac{2}{|\cos\varphi_o|}\ \delta(\varphi - \varphi_o) ,$$

where

$$\cos\varphi_o = \frac{\left\{ \left(\frac{\omega' r'}{c}\right)^2 (y^2 - 1) - \left[y - g'(r') \right]^2 \right\}^{\frac{1}{2}}}{\left(\frac{\omega' r'}{c}\right)^2 \sqrt{y^2 - 1}}$$

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