AN INTRODUCTION TO LOGIC

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PRENTICE-HALL, Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging-in-Publication Data

DAVIS, WAYNE A. (date)
An introduction to logic.

Includes index.
1. Logic. I. Title.
BC71.D27 1986 160 85-19127
ISBN 0-13-486259-7

Editorial/production supervision and
interior design: *Hilda Tauber*
Cover design: *20/20 Services, Inc.*
Manufacturing buyer: *Harry P. Baisley*

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

ISBN 0-13-486259-7 01

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PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
EDITORA PRENTICE-HALL DO BRASIL, LTDA., *Rio de Janeiro*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

To My Students

Preface

This introduction to logic is designed to serve two purposes. The first and primary objective is to develop the student's ability to reason logically and think critically. The second is to introduce the student to the terminology, methods, and results of modern logic. There are no prerequisites other than motivation and a good high school education. The text is aimed at the general reader, who is presumed to have little mathematical inclination and no intention of taking further courses in logic. The material is therefore presented as intuitively as possible, and the level of formality and technical difficulty begins low and increases gradually throughout the text. The student will, however, be well prepared to take more advanced courses in logic, and will hopefully be stimulated to do so. For those who have already taken courses in symbolic or mathematical logic, the text will introduce a broader range of topics, from informal inductive logic to formal and deductive logic, while also focusing attention on the application of logic to arguments presented in natural languages. While there are numerous introductions to logic with the same goals, this one is unique in several significant respects, which will be detailed below.

PLAN AND USE OF THE BOOK

The book is divided into chapters, sections, and subsections, with the section being the unit of assignment. Each section is followed by a glossary and a set of exercises. The glossary defines the main technical terms, and should be committed to memory. As in other technical disciplines, definitions are taken very seriously in logic. One little word can make a big difference. The exercises develop the student's skill at analyzing the logical structure of statements and arguments, at distinguishing sound arguments from fallacies, and at applying various formal techniques for proving arguments valid or invalid. Since the primary objective is to develop a skill, practice is essential. The exercises are therefore plentiful, and are graduated in difficulty. There are so many exercises, in fact, that students should not feel they have to do all of them. Since immediate feedback is highly beneficial, the solutions to nearly all the odd-numbered exercises are presented at the end of the book. This will enable many students to master the material without the assistance of an instructor. At the same time, enough unsolved exercises have been provided for instructors to use for homework or exams. As with any skill, "distributed" practice is more effective than "massed" practice. It is better, for example, to do logic problems an hour every day than to do them once a week in a seven-hour stint. The student is advised not to write the answers near the exercises, since this spoils their practice value. The text of each section should be read at least twice, once before and again after doing the exercises. A general review of the entire text after working through all the chapters will greatly deepen the student's understanding of many points.

There is more material than can comfortably be covered in one semester. The extra material is available for students who wish to go further, and for teachers who wish to tailor their courses to their own interests. Instructors desiring to concentrate on formal deductive logic can use Chapters 1, 2, and 5–9, for example, while those preferring a more informal focus can use Chapters 1–8. Chapters 1–4 provide a good background for courses in epistemology, philosophy of mind, and scientific methodology. I have organized the text so that the level of formality progressively increases. But other arrangements are possible. The two chapters on inductive logic can be presented after those on deductive logic, and the chapters on propositional logic can be presented before those on syllogistic logic.

A very casual approach to use-mention has been adopted. Symbolic expressions usually double as their own names, to avoid thousands of quotation marks. Quotes are occasionally used, though, for smoothness or extra clarity. Quotes or emphasis are always used to form the names of English words and sentences.

Because I have assigned a higher priority to the goal of developing logical reasoning than to that of preparing for further work in logic, I have departed from standard texts in several major ways. These include the treatment of conditionals, general categorical propositions, singular propositions, and validity. Other minor differences concern the treatment of variants of the inference rules, fallacies, inductive logic, and coverage.

APPROACH OF THIS TEXT

Conditionals. Introductory logic texts customarily teach students to treat conditionals in English as material conditionals. The student is therefore taught that an absurd statement like "If President Nixon was a Democrat, then he was not a Democrat" is *true*, and that "If the sun has less than nine planets, then it has more than thirty" *follows validly* from the fact that the sun does not have less than nine planets. In a course designed to develop a student's ability to reason logically, this practice is counterproductive, to put it mildly. The "paradoxes of the material conditional" have been known for a long time. Recent work has shown that natural language conditionals differ from material conditionals in many further ways. Stalnaker and Lewis, for example, have produced counterexamples to contraposition and hypothetical syllogism. Some authors justify the translation of conditionals as material conditionals with the claim that the validity of valid arguments is preserved. Unfortunately, the invalidity of invalid arguments is not.

This text focuses on "strong" indicative conditionals, those expressing implication. They are treated as compounds that are partially but not completely truth functional. The strong conditional is determinately false in case its antecedent is true and its consequent false, but may be either true or false in the other three cases. This yields a truth table with seven rows, which is sufficient, among other things, to prove the validity of modus ponens, modus tollens, and the dilemmas, and to prove the invalidity of conversion, affirming the consequent, and denying the antecedent. Modus ponens and modus tollens are taken as rules of inference in the system of deduction developed for propositional logic. Contraposition and hypothetical syllogism are presented as valid only under certain commonly satisfied conditions, which are briefly and intuitively explained. While the text focuses on strong indicative conditionals, the logical properties studied are common to all natural language conditionals. For the sake of students who will be going on in logic, and for teachers who may still wish to adopt the standard approach, the material conditional is defined, and its logical properties explored, in the last section of Chapter 7.

General Categorical Propositions. Logic texts usually give general categorical propositions the Boolean interpretation. "All *S* are *P*," for example, is treated as equivalent to "Nothing is *S* and not *P*." The student is thus taught that "All flying elephants are hippos" is *true*, that "All students passed" and "No students passed" are *compatible*, and that "All musicians are humans and no humans are reptiles, so some musicians are not reptiles" is *invalid*. Students are taught in chapters on syllogistic logic that "Every thing is imperfect, therefore some thing is imperfect" is invalid, while in chapters on quantification theory they are taught that "Everything is imperfect, therefore something is imperfect" is valid. Again, when the goal is to teach logical reasoning, these practices are counterproductive.

This text distinguishes Aristotelian categoricals like "All *S* are *P*" from Venn categoricals like "Nothing is *S* and not *P*." Aristotelian categoricals are studied in Chapter 5. The method of refutation by counterinstance is used to prove invalidity, and the method of deduction is introduced to prove validity. The standard rules of "immediate inference" and a few basic syllogisms provide a powerful but easy-to-use system of natural deduction. Venn categoricals are studied in Chapter 6, where the Venn diagram test is used to prove both validity and invalidity. Corresponding Aristotelian and Venn categoricals are then distinguished in terms of their presuppositions. The former presuppose that there is something to which their subject terms apply, while the latter do not. Consequently, corresponding Aristotelian and Venn categoricals are equivalent *except* when their subject terms apply to nothing, in which case the Aristotelian categoricals are neither true nor false, while universal Venn categoricals are true and particular Venn categoricals are false. An argument involving Aristotelian categoricals is then tested for validity using Venn diagrams; both the presuppositions and the Venn transforms of the premises are diagramed, and then both the presupposition and Venn transform of the conclusion are checked to see if they have also been diagramed. Basically the same procedure is used in Chapter 9 on quantification theory. The symbols of quantification theory provide an exact translation of Venn categoricals, but not of Aristotelian categoricals. Confronted with an argument containing Aristotelian categoricals, the student symbolizes the Venn transforms and presuppositions of the premises, and tries to deduce the Venn transform and presupposition of the conclusion. In other words, to prove the validity of an argument involving Aristotelian categoricals, the student is taught to prove the validity of an argument involving Venn categoricals that is equivalent with respect to validity.

Singular Propositions. After discussing the logic of general categorical propositions, logic texts generally teach students to treat singular propositions as general. Thus "Socrates is a man" is equated with something like "All people who are Socrates are men," and "Socrates is not a man" is equated with something like "No people who are Socrates are men." This erroneously makes the two singular propositions contrary rather than contradictory (or neither contrary nor contradictory if the Boolean interpretation of general propositions is adopted). This practice furthermore conflicts with the chapters on quantification theory in the very same texts, where singular and general propositions are treated very differently. In this text singular propositions are introduced at the end of Chapter 5, where relationships among singular propositions and between singular and general propositions are discussed. The methods of refutation and deduction are then applied to arguments involving singular propositions. In Chapter 6, the Venn diagram test is applied to singular propositions. "Socrates is a man" is diagramed by putting an *s* (instead of the *x* used to diagram particular propositions) in the man circle, and "Socrates is not a man" is diagramed by putting an *s* outside the man circle. In Chapter 9, finally, the transition to quantification theory is made simply by telling the student that "*s* is *P*" will now be reduced to *Ps*, and "*s* is not *P*" to $-Ps$.

The student is similarly *not* taught that "Most *S* are *P*" can be "rendered" as "Some *S* are *P*," nor that "Almost all *S* are *P*" should be "translated" as "Some *S* are *P* and some *S* are not *P*," or anything of the kind. This procedure is misleading at best. While students should be encouraged to generalize what they have learned, they should not be trained to misapply it. In general, the customary section ("the Procrustean Bed") in which the student is shown how to "reduce" other forms of argument to standard-form categorical syllogisms has been deleted. Even when legitimate, this procedure is pointless, given the methods of deduction and refutation, and given the chapters on propositional logic and quantification theory. A section on suppressed premises has been included, however, since they are common phenomena, and since failure to acknowledge them constitutes misunderstanding an argument.

Validity and Soundness. Introductions to logic often begin by defining a valid argument as one in which the conclusion follows with necessity from the premises, and end up defining a valid argument as one in which it is impossible for the premises to be true and the conclusion false. These definitions are not exactly equivalent. For example, "The sky is blue, therefore two is the square foot of four" is valid on the second definition but not the first. Since such an argument constitutes poor reasoning, and could not possibly count as a proof of its conclusion, the first definition is preferable given the priorities of this text. Even the first definition is not perfectly suitable, however, since many inductive arguments constitute excellent reasoning even though their conclusions do not follow with necessity. Consequently, this text defines a valid argument as one in which the conclusion follows with necessity *or* probability, and distinguishes between "deductively valid" and "inductively valid" arguments. I have adopted this terminology to emphasize that whether or not the premises support the conclusion is more important than whether the premises support the conclusion with necessity or probability.

Many texts claim that deductive validity depends exclusively on form. But this is incorrect on either definition given above. For example, while "This is red, therefore it is colored" is deductively valid, most arguments with the same form are invalid. This text explicitly distinguishes deductive validity from formal validity. Finally, a sound argument is customarily defined as a valid argument with true premises. This leads to the result that question-begging arguments with true premises are sound, as are valid arguments with wholly unwarranted premises that happen to be true, even though such arguments constitute poor reasoning. Soundness is therefore defined more stringently in this text: it refers to a good argument, one that counts as proving or verifying its conclusion. Since students are generally puzzled about the logician's distinction between soundness and validity, I spend more time than usual on it (see Chapter 2).

Variants of the Rules of Inference. In many texts, the rules of inference are defined by listing a set of argument forms. For example, simplification is commonly defined as the argument form "p & q ∴ p." Thus defined, the student is allowed to infer "Aristotle was a philosopher" but not "Aristotle was a mathematician" from "Aristotle was a philosopher and mathematician" by simplification. The student has to commute first, and then simplify, to get "Aristotle was a mathematician." This strikes most students as needlessly arbitrary, and they feel unjustly penalized when they lose points for "simplifying from the right." One solution is to include "p & q ∴ q" as one of the forms defining simplification. But if such additions are made for all similar cases, the result is an unmanageable list of forms. Another solution is to give verbal statements of the rules. Simplification would then be "From a conjunction, you can infer either conjunct." But verbally stated rules of inference are not as graphic, and therefore not as easy to use. This text combines the two approaches. The rules are given a general verbal definition and presented with a representative argument form. The student is allowed to use that representative form or any of its variants.

Fallacies. The customary chapter on informal fallacies has been omitted. Instead, there is a short section on fallacies in Chapter 2, where the purpose is to clarify by contrast the concept of a sound argument. Begging the question, equivocation, *ad hominen* arguments, and arguments from ignorance are briefly discussed there because they illustrate important general points about sound arguments. Other informal fallacies are described throughout the book in connection with related forms of valid inductive argument. Thus *post hoc* arguments are discussed with Mill's methods, and hasty generalization with enumerative induction. This procedure makes the student "compare and contrast," and thereby increases his or her understanding of both the valid and the invalid arguments. Appeals to pity, force, and the like are classified as "biasing influences," not fallacies, because they are seldom if ever presented as arguments. Appeals to authority are discussed not as fallacies but as arguments that are inductively valid under specified conditions, since much of any individual's knowledge is based on arguments from authority. Some commonly discussed fallacies, like composition and division, are omitted altogether because they hardly ever occur in practice. On the other hand, an entire section of Chapter 3 is devoted to what I call the "fallacy of exclusion," which results when undermining evidence is excluded. This fallacy is possibly the most common of all in practice, and is very important theoretically, yet most texts ignore it. I have omitted Latin names unless they have become part of English (like "*ad hominem*") or are firmly entrenched in modern logic (like "modus ponens").

Names like "*argumentum ad verecundiam*," rare even in educated discourse, serve only to annoy the student.

Inductive Logic. The primary focus of this text is on the formulation and evaluation of patterns of argument. To maintain this focus throughout required a departure from typical treatments of inductive logic, which concentrate on methods of inquiry like Mill's methods and the hypothetico-deductive method. This text focuses on the argument forms that underlie the methods of inquiry. More important, I have tried to formulate with greater explicitness than usual the conditions under which specific types of inductive argument are valid. And I have devoted a lengthy section to the criteria determining the "best" explanation of a given phenomenon.

Coverage. The traditional three rules of syllogistic logic have been omitted, since they do not exemplify any generally applicable technique in logic. The method of indirect proof and a simple method of conditional proof are introduced, since they are important and can be learned very easily once the method of direct proof has been mastered. (These sections can, however, be omitted.) Quantification theory is presented as a *generalization* of syllogistic logic. And I have gone more deeply into monadic quantification theory than is usual, if only to keep students from thinking that quantifiers must always come at the beginning of a line. The rules of quantifier negation are given more than the usual emphasis, since they forge an important link with the chapters on syllogistic logic, and formulate simple but extremely useful conceptual relationships. I believe that the student's understanding of quantification theory can be greatly deepened with little additional effort. The chapter on quantification theory concludes with a brief section on relational statements, to show students the limits of what they have learned, and to invite them to take more advanced logic courses. The quantification rules are not generalized to cover relational statements, since the added complexity requires weeks to master.

A subsection on modality appears in Chapter 2 because a large percentage of arguments found in ordinary discourse contains modal terms, while the main part of introductory logic avoids them. The purpose of the section is to enable the student in a rough and ready way to apply what is subsequently learned to discourse that does contain modal notions. The first section in Chapter 10 goes more deeply into modal logic without rising above the general level of conceptual difficulty represented by the rest of the text. I believe a treatment of elementary modal logic is very desirable in an introductory logic text, given that students generally have difficulty with even the simplest modal arguments, and given the close connections between modal logic and syllogistic logic, and between modal logic and probability theory. The section should be especially useful for philosophy students. Finally, Chapter 10 includes a section on digital logic circuits, which shows how propositional logic can be applied outside its usual domain. It should also be intrinsically interesting in this age of computers.

ACKNOWLEDGMENTS

It gives me great pleasure to thank the many people who have contributed to this text. My major debt is to Irving M. Copi, whose fine *Introduction to Logic* served as a model (and also provided some exercises). I am indebted principally to P. F. Strawson, Robert C. Stalnaker, and David Lewis for the intellectual developments leading to the major innovations in this text. Especial gratitude is due to Robert B. Barrett of Washington University and Donald Kalish of UCLA. The former provided encouragement at two key stages of the project and wrote detailed chapter-by-chapter comments, nearly all of which led to improvements. The latter's incisive criticism of the chapter on quantification theory suggested how to increase the degree of rigor without exceding the level of formality aimed at. David Benfield of Montclair State College also played a key role, first reviewing the project for Prentice-Hall and later trying it out on his classes and conveying a steady stream of constructive feedback. Thanks are also due to Jeremy Horne of Louisiana State University, Evelyn M. Barker of the University of Maryland, Baltimore County, and the other (anonymous) reviewers for Prentice-Hall. Readers seldom appreciate how much a textbook owes to the services of reviewers. I would also like to thank Doris S. Michaels, whose enthusiasm and wise advice as editor for Prentice-Hall turned a modest endeavor into a major production, and Emily Baker, who took over as philosophy editor with equal devotion. Hilda Tauber, production editor, carefully and sympathetically transformed a difficult manuscript into a beautiful book. Working with her was a delight.

Closer to home, my friend and colleague Tom L. Beauchamp gave me helpful advice on a variety of matters. I am grateful to Georgetown University for a summer grant in 1983 that enabled me to complete the second draft, and for providing me with the hundreds of fine logic students who inspired the project in the first place. I am deeply indebted to those students who used the book in manuscript form, and had to suffer its inevitable typos, errors, and infelicities. Among the students who were kind enough to bring errors to my attention, Chris Bard, Bob Nucci, Reggie Noble, Rita Dillon, Anthony Lepore, and Sharon Fekrat were especially helpful. Thanks are due also to Jameson Kurasha, my TA ("trusty assistant") for many years. Above all, I am indebted to Kathryn M. Olesko, who *qua* historian of science provided assistance with the chapters on inductive logic, and *qua* friend and wife provided the background of happiness necessary for such a large project.

Contents

1

Arguments

§1.1 LOGIC, REASONING, AND ARGUMENTS

Logic. Reasoning is a vital human activity. For unlike some animals able to function instinctively, we need knowledge in order to survive. At the very least, knowledge facilitates the pursuit of happiness. Some knowledge can be gained directly. In this way we know, for example, that an object in front of us looks orange and tastes sweet. But we cannot know that it is edible and nutritious, or that it contains vitamin C, which prevents scurvy, without a process of reasoning. Similarly, we do not need reasons to believe that every triangle has three angles. But we cannot know that the angles of a triangle add up to 180° without evidence or proof. The vast bulk of human knowledge is based on reasoning. Indeed, our knowledge can be described as a pyramid, in which what is directly evident provides the foundation on which all other beliefs are based.

Not just any old reasoning gives us knowledge. Knowledge—as opposed to misguided opinion—requires *correct* reasoning, reasoning that is logical and rational. Everyone can reason correctly to some extent. It is a skill we acquire while growing up. But the ability can be improved by study and reflection. People differ just as markedly in their native logical ability as in their athletic ability. Natural talent can be augmented by training in both areas.

Logic is the discipline that studies correct reasoning. Logic seeks in particular to develop methods for distinguishing correct from incorrect reasoning. Psychologists also study reasoning, but their viewpoint is quite different. Psychologists seek to describe how people actually do reason. Psychologists are