ELEMENTARY SCHOOL MATHEMATICS FOR TEACHERS

C. Alan Riedesel – Leroy G. Callahan

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C. Alan Riedesel – Leroy G. Callahan
State University of New York at Buffalo

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PREFACE

The following collection of modules on elementary school mathematics was designed to help you become a better teacher of mathematics at the elementary school level. The more knowledgeable you are about elementary school mathematics itself, the more you can concentrate on the professional aspects of teaching it.

The content and organization of the modules reflect certain beliefs about the learning of mathematical concepts. One belief is that systematic knowledge and exploratory activities are indispensable components in learning mathematics. Ideally these components should interact in an optimum way to produce learning and development. The modules are arranged and organized in order to provide the opportunity for individual progress through the program. However, learning is an active, social, and dynamic process. Individual progress does not imply isolated self-instruction. Rather, it is desirable to use the various components of the modules as a focus and motivation to stimulate interaction between yourself and others. Of course it is hoped that it may stimulate self-exploration and perhaps some discussions with yourself. But your instructor and fellow students, as well as the curriculum materials, are important to individual instruction. We hope you use all these resources, and use them wisely.

C. Alan Riedesel
Leroy G. Callahan

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INTRODUCTION

WHAT IS MATHEMATICS?

The term *mathematics*, like many terms denoting important divisions of knowledge, is difficult to define. Thus, rather than attempting an all-inclusive definition, this introduction simply illustrates some mathematical ideas and makes some partial statements.

This section gives a number of statements about mathematics and then provides a number of exploratory situations for delving into some areas of mathematics. Ideas about what mathematics is include the following:

A system of thought.

Power to solve some of the most difficult puzzles.

A search of patterns and relationships.

The study of quantities and relations through the use of numbers and symbols.

Arithmetic, algebra, geometry, trigonometry, analytic geometry, calculus, probability, statistics, and so on.

Seeing connections between things which, apart from the agency of human reason, are extremely unobvious.

Alfred North Whitehead

The science that uses easy words for hard ideas. . . .

Edward Kasner

As an enterprise mathematics is characterized by its aim, and its aim is to think rigorously whatever is rigorously thinkable in the course of the upward striving and refining evolution of ideas. . . .

Cassius J. Keyser

Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions. . . .

Felix Klien

It is remarkable that a science (probability) which began with the consideration of games of chance, should have become the most important object of human knowledge. . . .

Laplace

The language of science. . . .

Tobias Dantzig

One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them. . . .

Heinrich Hertz

They [the Alexandrians] were eventually able to measure by indirect means the radius of the Earth, the diameters of the sun and the moon, and the distances to the moon, the sun, the planets, and the stars. That we can measure such physically inaccessible lengths and do so, moreover, with an accuracy as great as we wish, seems at first blush, incredible.

Morris Kline

God created the integers, the rest is the work of man.

Leopold Kronecker

Nor more fiction for us; we calculate; but that we may calculate we had to make fiction first.

Nietzsche

THE ESSENCE OF MATHEMATICS IS FREEDOM.

Georg Cantor

For, contrary to the unreasoned opinion of the ignorant, the choice of a system of numeration is a mere matter of convention.

Pascal

Mathematicians do not deal in objects, but in relations between objects; thus, they are free to replace some objects by others so long as the relations remain unchanged. Content to them is irrelevant; they are interested in form only.

Poincaré

Because of the comparative simplicity with which they can go beyond the superficial, science and mathematics lend themselves to the development of attitudes of lifelong and general value. Among these are:

1. A healthy skepticism regarding accepted knowledge and a willingness to abandon ideas which are demonstrably erroneous.
2. The humility inherent in the realization that our understanding can never be complete, coupled with the optimism of conviction that, nevertheless, our understanding can always be increased.
3. The realization that understanding while indeed a means of power, is a job and an end in itself.

Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools.

Mathematics may be defined as the science of abstract form . . . the discernment of structure is essential no less to the appreciation of a painting or a symphony than to understanding the behavior of a physical system; no less in economics than in astronomy. Mathematics exhibit it, and in a generalized form.

"General Education in a Free Society," Harvard Commission in 1945

As the preceding statements indicate, mathematics can be thought of in a variety of ways. The layman may think of mathematics as synonymous with bookkeeping and accounting, whereas some mathematicians view mathematics as having little to do with the everyday world. In considering the mathematics an elementary school teacher should know, it is possible to think of three basic ideas: (1) mathematics as a system of thought, a search for patterns; (2) mathematics as a vehicle for the solution of problems of the "real world"; and (3) mathematics as a self-fulfilling venture of the mind. These three views should be understood by the teacher and, it is hoped, appreciated.

In the following material the three ideas given in the preceding paragraph are the basis of a series of laboratory searches that also indicate a number of the areas of mathematics, such as arithmetic, algebra, geometry, probability and statistics, and so on.

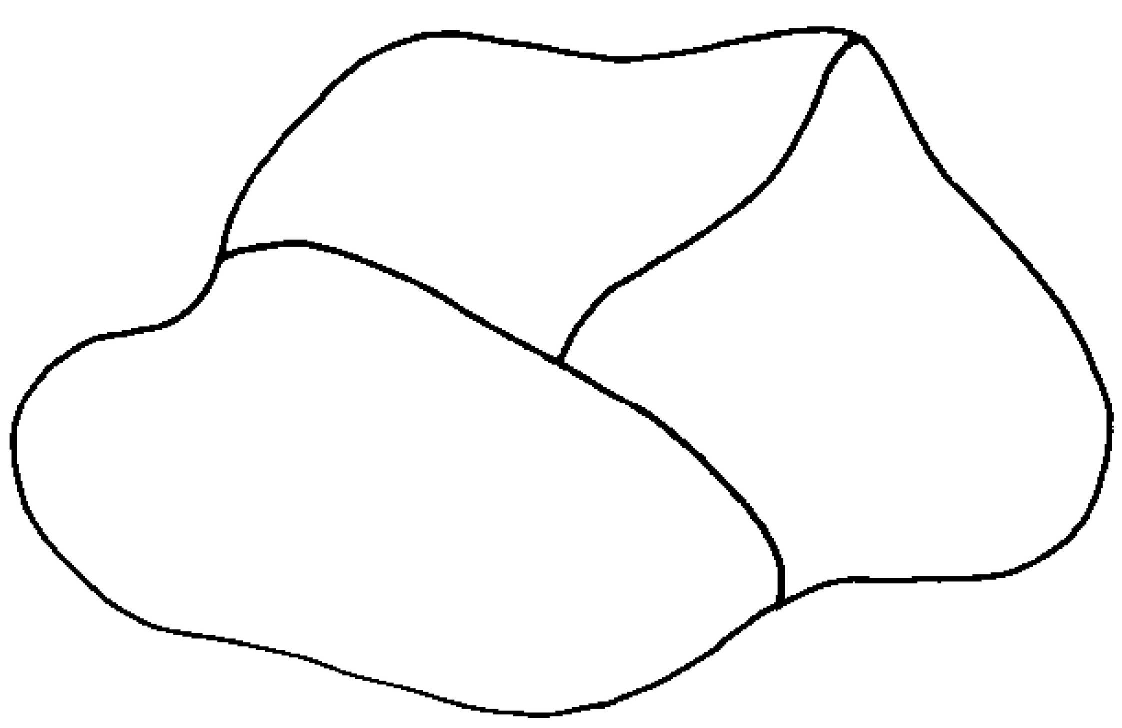
LAB I: PUZZLE-TYPE PROBLEMS

1. There were 37 starlings sitting in two bushes. After 6 birds flew from the first bush to the second and 9 starlings flew away from the second bush, there were three times as many birds in the first bush as in the second. How many starlings were in each bush originally? (Answer on p. 6)

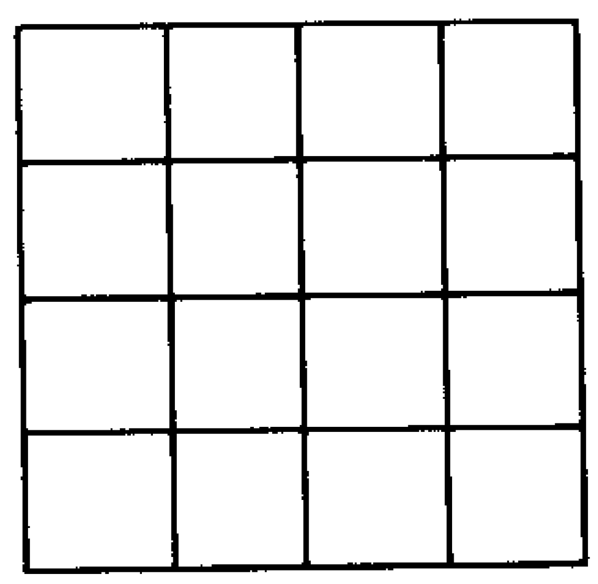
2. The members of the Mudville baseball team play cards while they travel on the train. The nine regulars form three tables of three each. But none of the three outfielders likes to play at the same table as another outfielder; the first, second, and third basemen will not sit together; while the pitcher, catcher, and shortstop say that they see enough of each other on the diamond so they will not sit together. Despite these limitations, the members of the team have been able to organize the three tables in many different arrangements. How many different arrangements are possible? (Answer on p. 6)

LAB II: USING GEOMETRIC THINKING

1. Mr. Herbert left his land to his three sons. They divided it so that each of them had one piece of land, and each piece of land had a common boundary with each of the other pieces. The map shows this arrangement. How could four children divide the land so that each piece would have common boundary with the other pieces? (Answer on p. 6)



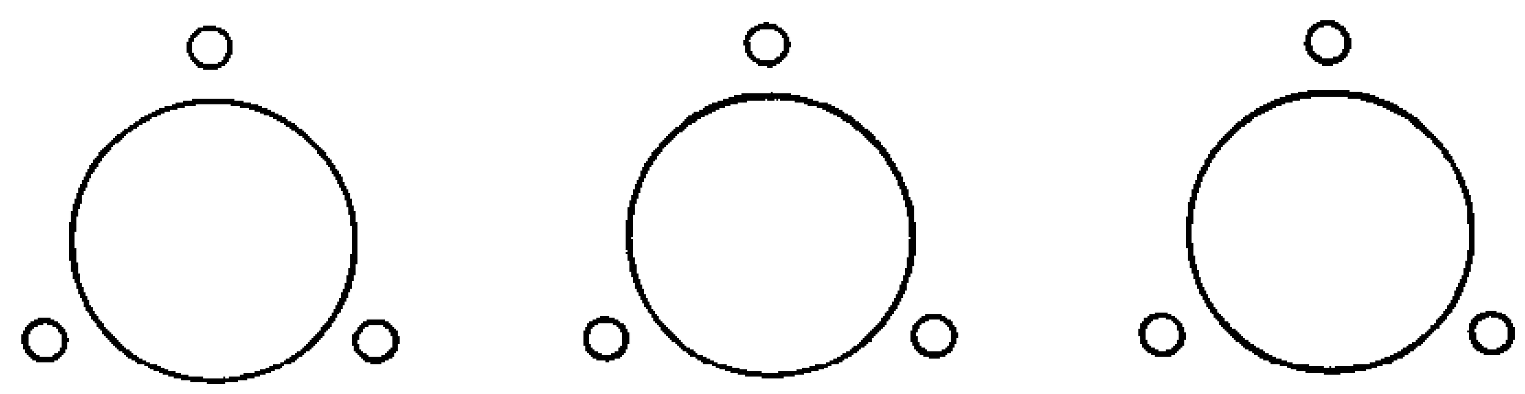
2. How many different squares can you find in this figure? (Answer on p. 6)



ANSWERS FOR LAB I

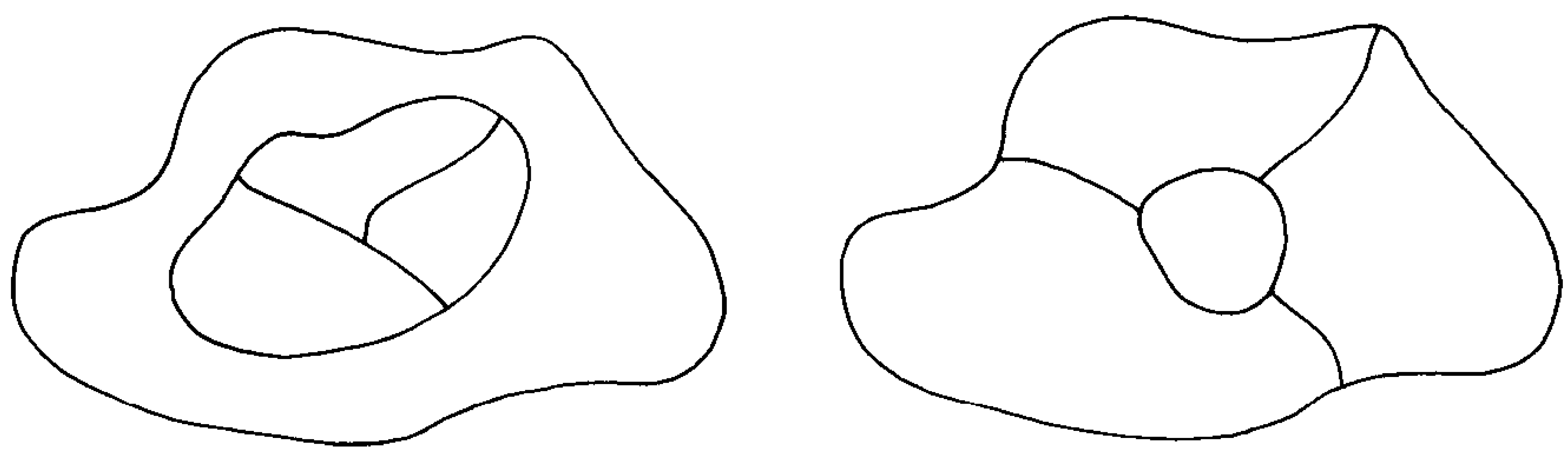
1. You can start with the situation at the end. That is, 9 starlings flew away—thus, there are 37 − 9 or 28 birds on the two trees. If there are three times as many birds on the first tree as the second, 3 out of 4 or 3/4 of the birds are on the first tree. So, 3/4 × 28 or 21 birds are on the first tree and 7 birds are on the second. Now, backtrack to find the original number of birds on each tree. The problem states that 6 flew away from the first tree, so 21 plus 6 or 27 were on the first tree at the beginning. If 27 are on the first tree, 10 must be on the second. To check this remember: 7 were on the second tree; at the end 9 had flown away from it, so this would give us a total of 16; then send the 6 back to the first tree, giving a total of 10. *First Tree—27 Second Tree—10*

2. Draw the three outfielders at separate tables. Then the first baseman may sit at any of 3 tables, and after he is placed, the second baseman may sit at either of the other two tables. Then there is only one place for the third baseman. Thus, for the six men there are 3 × 2 × 6 different arrangements. The pitcher, catcher, and shortstop can be seated in 6 different ways, so the total for the nine is 6 × 6 or 36. (The same 3 players will play together 4 times, but on each of these times there will be a variation in the make-up of the other two tables.)

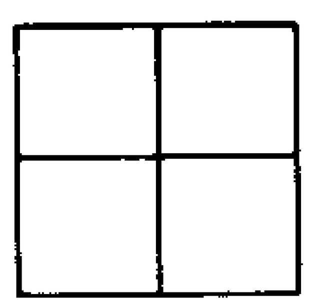
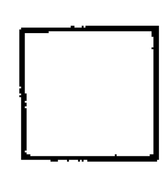


ANSWERS FOR LAB II

1. Four sons could have divided the land in either of the ways shown:

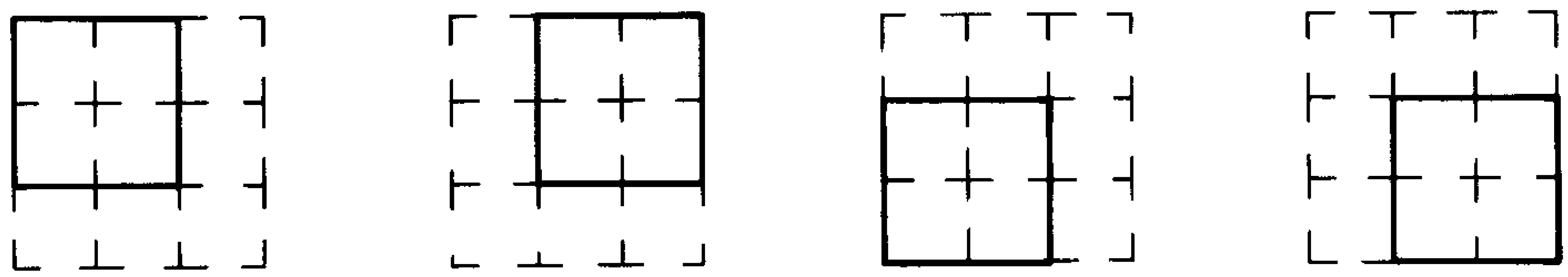


2. A 2-by-2 square is made up of four 1-by-1 squares and the outside 2-by-2 square. There are therefore five squares in the 2-by-2 square. Four are this size:

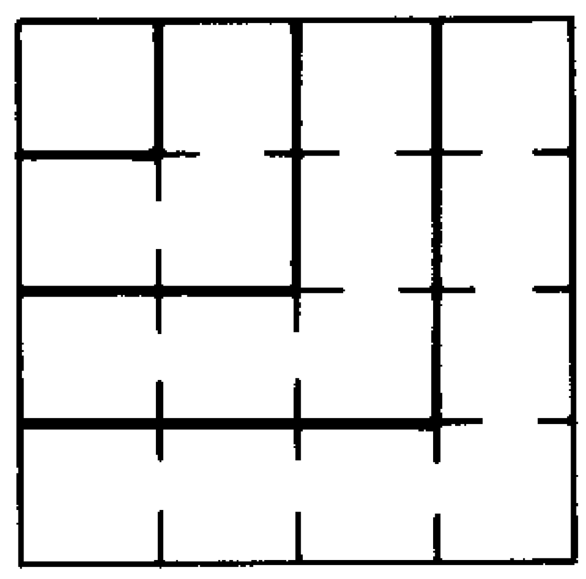
 and one is this size: 

The figure may be built up by using squares cut from cardboard plywood, or any other suitable material; by using matchsticks or simply by drawing it.

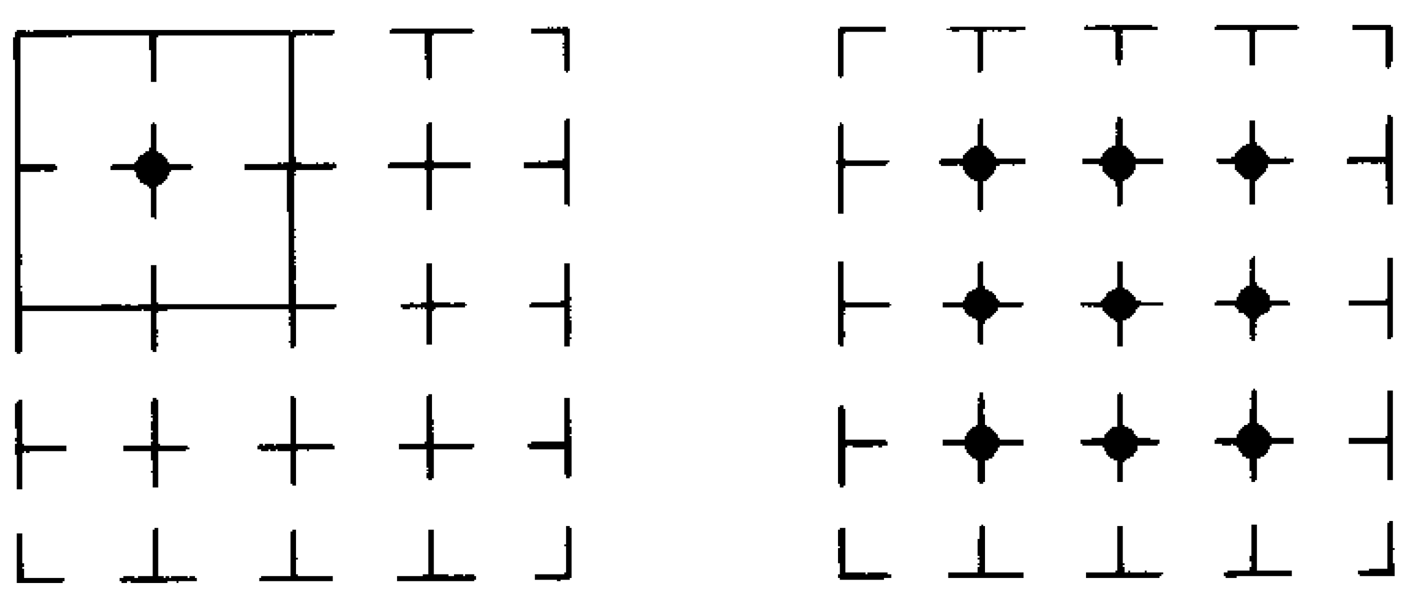
A 3-by-3 square contains nine 1-by-1 squares and one 3-by-3 square, so you can find at least ten squares in this figure. But the figure also contains some 2-by-2 squares, as shown by the heavy lines in the diagram:



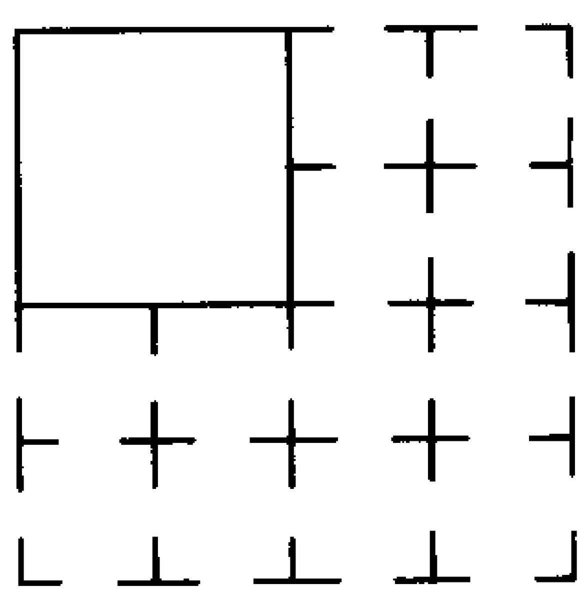
The 4-by-4 square also contains 3-by-3 squares and 2-by-2 squares as well as sixteen 1-by-1 squares.



You can best count the 2-by-2 squares by observing that each intersection of the lines inside the figure is the center of a 2-by-2 square. Since there are nine such intersections, there are nine such squares in the figure.



Another way of finding the number of squares is to cut out a sqaure of paper the size of a 2-by-2 square and fit it onto the pattern of lines.



It can be placed in nine positions in the figure; the boundary of the paper will coincide with each of the nine 2-by-2 squares.

Tabulating our results we find:

Number of Squares

Size of Square	1 by 1	2 by 2	3 by 3	4 by 4	Total
1 by 1	1				1
2 by 2	4	1			5
3 by 3	9	4	1		14
4 by 4	16	9	4	1	30

The set of numbers $\{1, 4, 9, 16, \ldots\}$ is the set of square numbers. By continuing the pattern that is apparent in this table you can first guess and then confirm by experiment that the 5-by-5 square contains $25 + 16 + 9 + 4 + 1 = 55$ squares and the 6-by-6 square contains $36 + 25 + 16 + 9 + 4 + 1 = 91$ squares.

MASTERY TEST: PRETEST

The following is a general survey test on the mathematics of the elementary school program. Take the test, and when you have completed it, you can correct it using the scoring key on page 306.

The results of this preliminary survey test will give you and your instructor an indication your knowledge of the mathematics of the elementary school program. A second form of this test is given at the end of this book; it can be used to measure your growth during the course.

TEST RESPONSE SHEET

1 ______	19 ______	37 ______	55 ______
2 ______	20 ______	38 ______	56 ______
3 ______	21 ______	39 ______	57 ______
4 ______	22 ______	40 ______	58 ______
5 ______	23 ______	41 ______	59 ______
6 ______	24 ______	42 ______	60 ______
7 ______	25 ______	43 ______	61 ______
8 ______	26 ______	44 ______	62 ______
9 ______	27 ______	45 ______	63 ______
10 ______	28 ______	46 ______	64 ______
11 ______	29 ______	47 ______	65 ______
12 ______	30 ______	48 ______	66 ______
13 ______	31 ______	49 ______	67 ______
14 ______	32 ______	50 ______	68 ______
15 ______	33 ______	51 ______	69 ______
16 ______	34 ______	52 ______	70 ______
17 ______	35 ______	53 ______	71 ______
18 ______	36 ______	54 ______	72 ______

MATH SUMMARY TEST (FORM A)

Directions: Choose the best response and indicate your choice on the answer sheet.

1. Which of the following sets is equal to the set $V = \{a, e, i, o, u\}$?

a. $\{1, 2, 3, 4, 5\}$ **c.** both (a) and (b)

b. $\{u, o, i, e, a\}$ **d.** neither (a) nor (b)

2. Which of the following sets is equivalent to the set $V = \{a, e, i, o, u\}$?
   a. $\{1, 2, 3, 4, 5\}$ c. both (a) and (b)
   b. $\{u, o, i, e, a\}$ d. neither (a) nor (b)

3. Set $A = \{5, 6, 7, 8, 9\}$. The symbol $n\{A\}$ suggests which of the following?
   a. 9 c. {whole numbers}
   b. {integers} d. 5

4. The term associated with the number property of a set, or the "how many?" attribute of a set, is which of the following?
   a. ordinality c. cardinality
   b. commutativity d. associativity

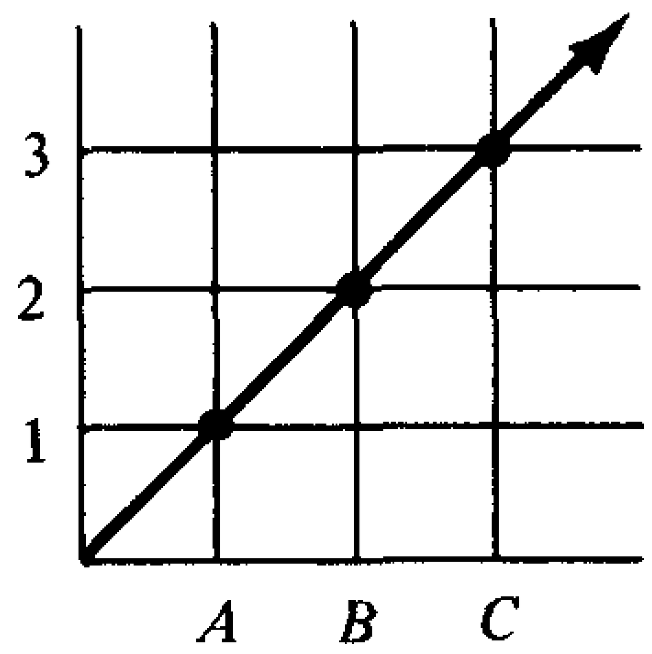
$X = \{B, I, G\}$
$Y = \{S, M, A, L, L\}$

5. Examine the correspondence between the sets X and Y at the side. Which of the statements is true of the sets?
   a. $n\{X\} = n\{Y\}$ c. $n\{X\} > n\{Y\}$
   b. $n\{X\} \sim n\{Y\}$ d. $n\{X\} < n\{Y\}$

$U = \{$□, ○, △, ◖$\}$
$S = \{$○, △$\}$

6. Given the two sets U and S shown at the side, which of the following sets is the complement of S?
   a. { } c. {□, ◖}
   b. {□} d. {◖}

7. The operation on two sets that yields a third set containing all elements common to the two sets is called which of the following?
   a. complementation c. union
   b. intersection d. addition



8. Which of the following represents the set of ordered pairs suggested by the graph at the side?
   a. $\{(A, 1), (B, 2), (C, 3)\}$ c. $\{(1, 1), (2, 2), (3, 3)\}$
   b. $\{(A, 1), (B, 1), (C, 1)\}$ d. $\{(A, 3), (B, 2), (C, 1)\}$

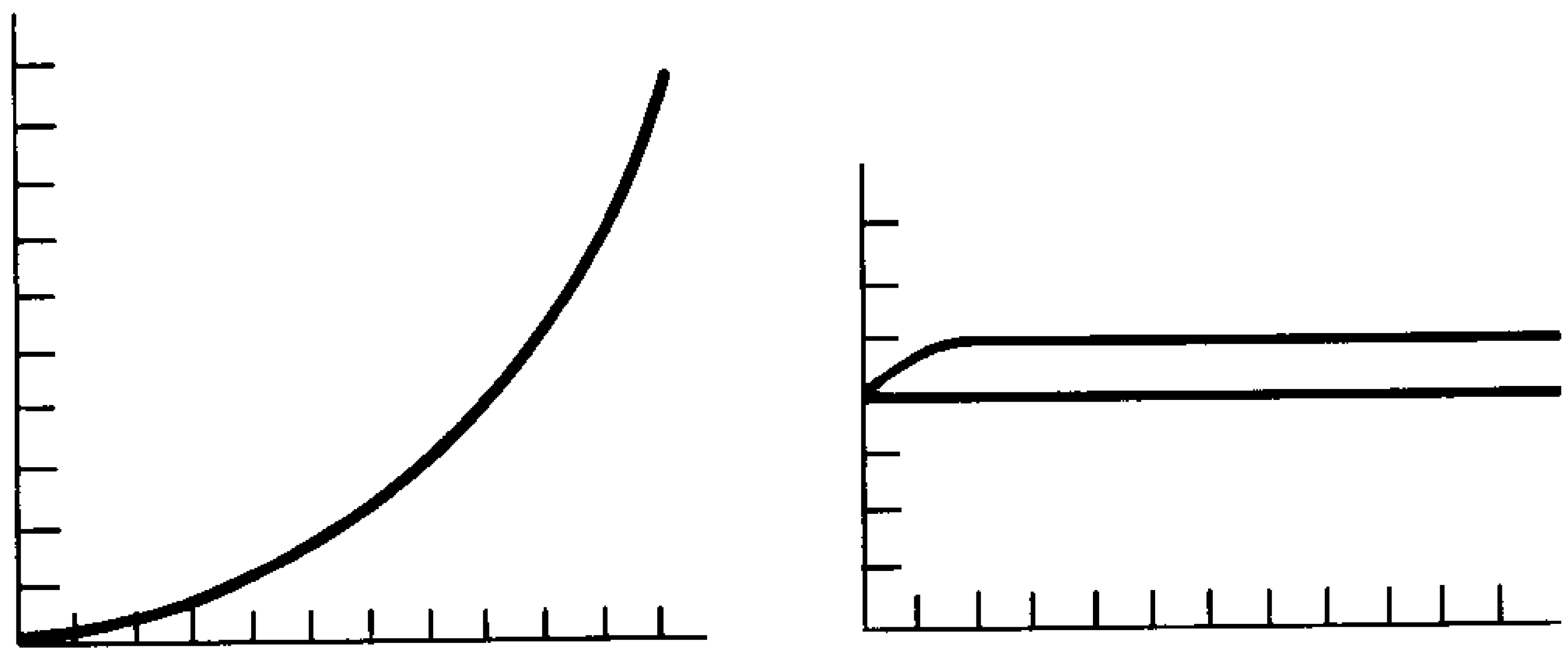
9. Look at the mappings under the following operations.
   (i) $2, 3 \xrightarrow{+} 5$ (add)
   (ii) $4 \xrightarrow{\sqrt{\ }} 2, -2$ (square root)
   (iii) $6, 8, 10 \xrightarrow{\text{GCF}} 2$ (greatest common factor)
   (iv) $2, 3, 4, 5, 6 \xrightarrow{M} 4$ (arithmetic mean)

Which illustrates a binary operation?

a. (i) only **c.** (iv) only

b. (ii) and (iii) only **d.** (i), (ii), and (iii) only

10. Which graph below suggests a mathematical function?



A *B*

a. *A* **c.** both *A* and *B*

b. *B* **d.** neither *A* nor *B*

11. Which of the following represents the value of the 3 in the number 5382?

a. $3(10^2)$ **c.** $3(10^3)$

b. $3(10^1)$ **d.** $3(10^4)$

12. Large multidigit numbers are organized into three-digit sets called periods. Each period is separated by commas. What is the name given the underlined period in the number 859, <u>672</u>, 854?

a. units **c.** hundreds

b. millions **d.** thousands

13. Which of the following would express the value of *ABCD* in base *X* in the manner of the Hindu-Arabic numeration system?

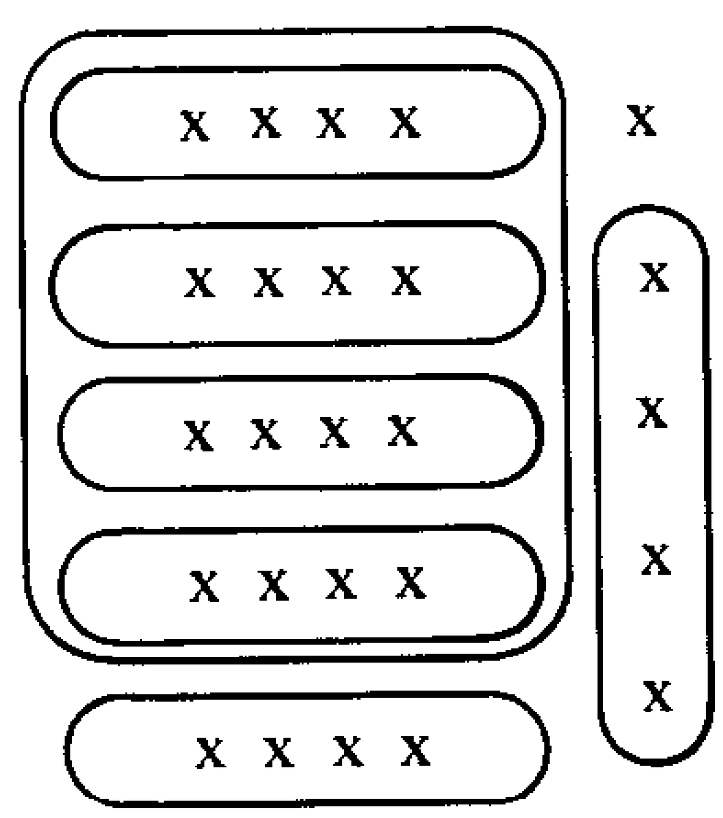
a. $A(X^4) + B(X^3) + C(X^2) + D(X^1)$

b. $X(A^4) + X(B^3) + X(C^2) + X(D^1)$

c. $A(X^3) + B(X^2) + C(X^1) + D(X^0)$

d. $X(A^3) + X(B^2) + X(C^1) + X(D^0)$

14. The set of *X*'s at the side was bundled into subsets as shown. The notation used to convey the number property of the set was 121. Which of the following identifies the base of the numeration system being used?



a. 2 **c.** 4

b. 3 **d.** 5