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A NOVEL FORMULATION FOR ACOUSTIC RADIATION FROM DIPOLE (FORCE-LIKE) SOURCES

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Introduction

This paper presents a novel application of velocity potential functions in analyses involving dipole or force sources. The motivation for the development of this approach arose in a treatment of the familiar problem of radiation of noise by helicopter rotor blades, a summary of which is included here. The initial step, representing acoustic pressure as the divergence of a vector potential function, $p = \nabla \cdot \underline{A}$, is also well-known although the full power of this approach has not been exploited. Standard treatments of rotor-noise radiation (3, 4) are burdened by coordinate transformations which are not needed in the present application. The present development has the advantage of providing a simple relationship between the acoustic pressure and fluid particle velocity which is useful in dealing with the presence of reflecting surfaces.

Furthermore, the derivation of the acoustic field equations, including both mass-flow and force sources, yields two wave equations: one obtained directly from the momentum equation and involving only the vector potential; the second wave equation involves only the classical scalar velocity potential, the wave equation arising from the mass-conservation equation. This approach can be extended to viscous or thermally conducted fluids. In the latter case, an energy-accounting equation must be incorporated. The clear advantage shown by the vector-potential approach in the moving-source problem, its adaptability to scattering and related phenomena, and its elegance in the derivation commend it.

In the remaining sections, we will present an outline of the derivation of the acoustic equations and a sketch of their application to the moving-force source problem.

The Uncoupled

In order to present the derivation in its clearest form, we will begin with the linearized fluid-dynamic equations. The conservation-of-mass equation

$$\dot{\rho} + \rho_o \nabla \cdot \underline{v} = \rho_o \dot{\mu} \qquad (1)$$

with ρ_o the ambient density, ρ its acoustic perturbation and $\underline{v}$ the acoustic particle velocity, contains a mass flow source with strength μ. The momentum equation,

$$\rho_o \dot{\underline{v}} = -\nabla p + \rho_o \underline{F} \quad (2)$$

in which p is the acoustic pressure, similarly contains a source term, a force $\underline{F}$ with strength normalized by the ambient density. Assuming a lossless medium, we may use the isentropic equation of state to relate changes in pressure and density: $dp = c^2 d\rho$.

The extended representation of the fluid particle velocity by

$$\underline{v} = \nabla\phi - c^{-2}\dot{\underline{A}} + \nabla \times \underline{B} \quad (3)$$

incorporates the scalar potential function ϕ, a vector potential $\underline{B}$ whose curl $\nabla \times \underline{B}$ represents the rotational flow, and the vector potential $\underline{B}$, which must have a vanishing curl ($\nabla \times \underline{A} = 0$) which will yield the response to force-like sources.

The derivation proceeds as follows: employ the above representation in the momentum equation and identify the relationship

$$p = \rho_o \dot{\phi} + \rho_o \nabla \cdot \underline{A} \quad (4)$$

which, with the requirement in this case that $\underline{B}$ be zero, leaves a wave equation

$$\nabla^2 \underline{A} - c^{-2}\ddot{\underline{A}} = \underline{F} \quad (5)$$

involving only the potential function $\underline{A}$ and the force. The representation in Eq. 4 combines the classical relationship between pressure and the scalar potential with that employed in analyses of radiation from dipole sources (3, 4). Upon employing the relationship between pressure and density variations and Eq. 4 in Eq. 1, the wave equation

$$\nabla^2 \phi - c^{-2}\ddot{\phi} = \dot{\mu} \quad (6)$$

is obtained. This direct derivation of the wave equations, with the momentum equation producing a response for force sources and the continuity equation, with the aid of the equation of state, producing the response to simple sources, can be extended readily to viscous media. The method of image sources can be employed for dipole sources near a rigid or pressure-release surface. The anti-symmetry vis a vis the scalar potential is noteworthy: For a simple source above a rigid surface, the image source is located equidistant from the boundary and in phase with the real source; for a dipole or force source, the potential $\underline{A}'$ associated with the image must have its component normal to the surface opposite in sign to the normal component of $\underline{A}$ for the true source. The extension to scattering by finite-impedance our forces is an intriguing possibility.

Radiation by a Moving Force

As an example of the use of the preceding formulation, we outline here the solution for radiation by a moving point force, described by

$$\underline{F}(\underline{r}, t) = \underline{f}(t)\delta(\underline{r} - \underline{r}(t)) \qquad (7)$$

By virtue of Eq. 5, Green's theorem may be applied to each component of $\underline{A}$ in turn with the Green's function $\delta(t-R/c)/4\pi R$, yielding the familiar retarded-time solution,

$$\underline{A}(\underline{r}, t) = \underline{f}(\tau)/(4\pi R(\tau)|1-M_r(\tau)|) \qquad (8)$$

in which $\underline{R}$ is the vector from source location (at time $\tau = t-R/c$) to observation point, R its magnitude. $\underline{M}_s$ is the Mach vector for the source and $M_r = \underline{M}_s \cdot \underline{R}/R$ the component in the direction of the observer (approach Mach number).

It remains to obtain the acoustic pressure field using Eq. 4. It is at this point that the ease in derivation is compensated by tedious calculation: Straightforward evaluation of the divergence of $\underline{A}$ in Eq. 8, and a modicum of regrouping terms yields (with $\hat{e}_R = \underline{R}/R$, the unit vector directed from source to receiver).

$$p(\underline{r}, t) = \frac{\mathrm{sgn}(1-M_r)}{4\pi R(1-M_r)^2}\left\{\frac{\dot{\underline{F}} \cdot \hat{e}_R}{c} + \frac{(\underline{F} \cdot \hat{e}_R)(\dot{\underline{M}}_s \cdot \hat{e}_R)}{c(1-M_r)} + \frac{(1 - M_s^2)\ \underline{F} \cdot \hat{e}_R}{R(1-M_r)} - \frac{\underline{F} \cdot \underline{M}_s}{R}\right\}\Bigg|_{\tau = t - R/e} \qquad (9)$$

This is substantially in agreement with Eqs. 17 and 18 of (3), given Lowson's interpretation of $\partial M_r/\partial t$.

Conclusion

The inclusion of a separate vector potential is the representation for the fluid particle velocity allows a derivation of linear wave equations which account directly for the presence of force-like sources as well as mass-flow sources. This is obviously not a necessary step: the restriction $\nabla \times \underline{A} = 0$ indicates that the terms $\nabla\phi - c^{-2}\underline{A}$ could be considered the gradient of another scalar potential. Taking this approach simplifies the derivation of the wave equation and the formal treatment of fields for dipole sources. Similarly, in the case of a moving force, the analysis is rendered much more direct by this representation.

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