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PREFACE

This book was written for a one-year course in linear circuit analysis in the sophomore year. Such a course is basic in electrical engineering and is usually the first encounter of the student with his or her chosen field of specialization. It is imperative, therefore, for the textbook used to cover thoroughly the fundamentals of the subject and at the same time be as easy to understand as it is possible to make it. These have been our objectives throughout the writing of the book.

Most students, when they take this subject, will have studied electricity and magnetism in a physics course. This background is helpful, of course, but is not a prerequisite for reading the book. The material presented here may be easily understood by a student who has had a basic course in differential and integral calculus. The differential equations theory required in circuit analysis is fully developed in the book and integrated with the appropriate circuit theory topics. Even determinants, Gaussian elimination, and complex number theory are presented in appendices.

The operational amplifier is introduced immediately after the discussion of the resistor, and appears, as a matter of course, along with resistors, capacitors, and inductors, as a basic element throughout the book. Likewise, dependent sources and their construction using operational amplifiers are discussed early and are encountered routinely in almost every chapter.

To aid the reader in understanding the textual material, examples are liberally supplied and numerous exercises, with answers, are given at the end of virtually every section. Problems, some more difficult and some less difficult than the exercises, are also given at the end of every chapter. A special effort has been made to include a number of problems and exercises with realistic element values. Of course, network scaling, which is also presented, can be used to make almost all of the remaining problems practical. In particular, in the chapter on amplitude and phase responses, problems are given that relate to electric filters, which, of course, are very useful circuits. Active filters, using operational amplifiers, as well as passive filters, are used as examples. Finally, a select few of the exercises and problems are used to extend the theory discussed in the chapters. In this way, optional material is included without adding to the text of the chapter.

The first nine chapters of the book are devoted to terminology and time-domain analysis and the last nine chapters deal with frequency-domain analysis. Some sections and chapters, identified by asterisks, may be omitted without any loss in continuity. Among these is the chapter on network topology, an interesting subject which could be covered entirely, in part, or not at all. Also those on Fourier methods and Laplace transforms. These latter two topics are often reserved for a succeeding course in linear systems, but their essential ingredients are here if there is time to cover them.

There are many people who have provided invaluable assistance and advice concerning this book. We are indebted to our colleagues and our students for the form the book has taken, and to Mrs. Dana Brown and Mrs. Norma Duffy, who provided the expert typing and draftsmanship, respectively. A special note of thanks is due Professors M. E. Van Valkenburg, A. P. Sage, S. R. Laxpati, and S. K. Mitra, who reviewed the manuscript and made many helpful comments and suggestions.

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INTRODUCTION

Electric circuit analysis, in nearly every electrical engineering curriculum, is the first course taken in the major area by an electrical engineering student. Virtually all branches of electrical engineering, such as electronics, power systems, communication systems, rotating machinery, and control theory, are based on circuit theory. The only topic in electrical engineering more basic than circuits is electromagnetic field theory, and even there many problems are solved by means of equivalent electric circuits. Thus it is no exaggeration to say that the basic circuit theory course a student first encounters in electrical engineering is the most important course in his or her curriculum.

To begin our study of electric circuits we need to know what an electric circuit is, what we mean by its analysis, what quantities are associated with it, in what units these quantities are measured, and the basic definitions and conventions used in circuit theory. These are the topics we shall consider in this chapter.

1.1 DEFINITIONS AND UNITS

An electric *circuit*, or electric *network*, is a collection of electrical elements interconnected in some specified way. Later we shall define the electrical elements in a formal manner, but for the present we shall be content to represent a general *two-terminal* element as shown in Fig. 1.1. The terminals *a* and *b* are accessible for connections with other elements. Examples with which we are all familiar, and which we shall formally consider in later sections, are resistors, inductors, capacitors, batteries, generators, etc.

More complicated circuit elements may have more than two terminals. Transistors and operational amplifiers are common examples. Also a number of simple elements may be combined by interconnecting their terminals to form a single package having any number of accessible terminals. We shall consider some multiterminal elements later, but our main concern will be simple two-terminal devices.

An example of an electric circuit with six elements is shown in Fig. 1.2. Some authors distinguish a circuit from a network by requiring a circuit to contain at least one closed path such as path *abca*. We shall use the terms interchangeably, but we may note that without at least one closed path the circuit is of little or no practical interest.

FIGURE 1.1 *General two-terminal electrical element*

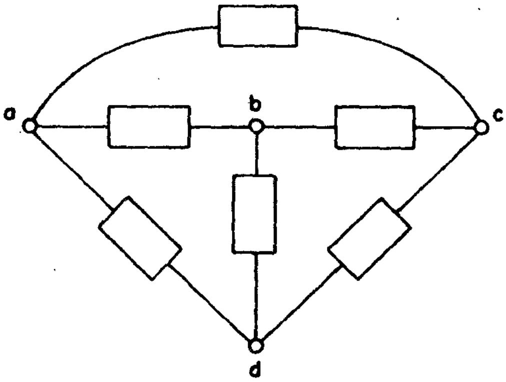

FIGURE 1.2 *Electric circuit*

To be more specific in defining a circuit element we shall need to consider certain quantities associated with it, such as *voltage* and *current*. These quantities and others, when they arise, must be carefully defined. This can be done only if we have a standard system of units so that when a quantity is described by measuring it we can all agree on what the measurement means. Fortunately, there is such a standard system of units that is used today by virtually all the professional engineering societies and the authors of most modern engineering textbooks. This system, which we shall use throughout the book, is the *International System of Units* (abbreviated SI), adopted in 1960 by the General Conference on Weights and Measures.

There are six basic units in the SI, and all other units are derived from them. Four of the basic units, the meter, kilogram, second, and coulomb, are important to circuit theorists, and we shall consider them in some detail. The remaining two basic units are the degree Kelvin and the candela, which are important to such people as the electron device physicist and the illumination engineer.

The SI units are very precisely defined in terms of permanent and reproducible quantities. However, the definitions are highly esoteric and in some cases are comprehensible only to atomic scientists.[1] Therefore we shall be content to name the basic units and relate them to the very familiar *British System of Units,* which includes inches, feet, pounds, etc.

The basic unit of length in the SI is the *meter*, abbreviated m, which is related to the British system by the fact that 1 inch is 0.0254 m. The basic unit of mass is the *kilogram*

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[1] Complete definitions of the basic units may be found in a number of sources, such as, for example, "IEEE Recommended Practice for Units in Published Scientific and Technical Work," by C. H. Page et al. (*IEEE Spectrum*, vol. 3, no. 3, pp. 169–173, March 1966).

(kg), and the basic unit of time is the *second* (s). In terms of the British units, 1 pound-mass is exactly 0.45359237 kg, and the second is the same in both systems.

The fourth unit in the SI is the *coulomb* (C), which is the basic unit used to measure electric charge. We shall defer the definition of this unit until the next section when we consider charge and current. The name coulomb was chosen to honor the French scientist, inventor, and army engineer Charles Augustin de Coulomb (1736–1806), who was an early pioneer in the fields of friction, electricity, and magnetism.

We might note at this point that all SI units named for famous people have abbreviations that are capitalized. Otherwise, lowercase abbreviations are most often used. It is also worth mentioning that we could choose units other than the ones we have selected to form the basic units. For example, instead of the coulomb we could take the *ampere* (A), the unit of electric current to be considered later. In this case the coulomb could then be obtained as a derived unit.

There are three derived units in addition to the ampere that we shall find useful in circuit theory. They are the units used to measure force, work or energy, and power. The fundamental unit of force is the *newton* (N), which is the force required to accelerate a 1-kg mass by 1 meter per second per second (1 m/s^2). Thus 1 N = 1 $kg\text{-}m/s^2$. The newton is named, of course, for the great English scientist, astronomer, and mathematician Sir Isaac Newton (1642–1727). Newton's accomplishments are too numerous to be listed in a mere chapter.

The fundamental unit of work or energy is the *joule* (J), named for the British physicist James P. Joule (1818–1889), who shared in the discovery of the law of conservation of energy and helped establish the idea that heat is a form of energy. A joule is the work done by a constant 1-N force applied through a 1-m distance. Thus 1 J = 1 N-m.

The last derived unit we shall consider is the *watt* (W), which is the fundamental unit of power, the rate at which work is done or energy is expended. The watt is defined to be 1 J/s and is named in honor of James Watt (1736–1819), the Scottish engineer whose engine design first made steam power practicable.

Before we leave the subject of units we should point out that one of the greatest advantages the SI has over the British system is its incorporation of the decimal system to relate larger and smaller units to the basic unit. The various powers of 10 are denoted by standard prefixes, some of which are given, along with their abbreviations, in Table 1.1.

TABLE 1.1 *Prefixes in the SI*

Multiple	*Prefix*	*Symbol*
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	k
10^{-3}	Milli	m
10^{-6}	Micro	μ
10^{-9}	Nano	n
10^{-12}	Pico	p

As an example, at one time a second was thought to be a short time, and fractions such as 0.1 or 0.01 of a second were unimaginably short. Nowadays in some applications, such as digital computers, the second is an impracticably large unit. As a result, times such as 1 nanosecond (1 ns or 10^{-9} s) are in common use. Another common example is 1 gram (g) = 10^{-3} kg.

EXERCISES

1.1.1 Find the number of millimeters in 10 km. *Ans.* 10^7

1.1.2 If a mile equals 5280 ft, how many miles in 10 km? *Ans.* 6.2137

1.1.3 Find the work in millijoules done by a constant force of 25 μN applied to a mass of 4 g for a distance of 10 m. *Ans.* 0.25

1.2 CHARGE AND CURRENT

We are all familiar with gravitational forces of attraction between bodies, which are responsible for holding us on the earth and which cause an apple dislodged from a tree to fall to the ground rather than to soar upward into the sky. There are bodies, however, that attract each other by forces far out of proportion to their masses. Also, such forces are observed to be repulsive as well as attractive and are clearly not gravitational forces.

We explain these forces by saying that they are electrical in nature and caused by the presence of *electrical charges*. We explain the existence of forces of both attraction and repulsion by postulating that there are two kinds of charges, positive and negative, and that unlike charges attract and like charges repel.

As we know, according to modern theory, matter is made up of atoms, which are composed of a number of fundamental particles. The most important of these particles are protons (positive charges) and neutrons (neutral, with no charge) found in the nucleus of the atom and electrons (negative charges) moving in orbit about the nucleus. Normally the atom is electrically neutral, the negative charge of the electrons balancing the positive charge of the protons. Particles may become positively charged by losing electrons to other particles and become negatively charged by gaining electrons from other particles.

As an example, we may produce a negative charge on a balloon by rubbing it against our hair. The balloon will then stick to a wall or the ceiling, which are uncharged. Relative to the negatively charged balloon the neutral wall and ceiling are oppositely charged.

We now define the *coulomb* (C), discussed in the previous section, by stating that the charge of an electron is a negative one of 1.6021×10^{-19} coulombs. Putting it another way, a coulomb is the charge of about 6.24×10^{18} electrons. These are, of course, mind-boggling numbers, but their sizes enable us to use more manageable numbers, such as 2 C, in the circuit theory to follow.

The symbol for charge will be taken as Q or q, the capital letter usually denoting constant charges such as $Q = 4$ C, and the lowercase letter indicating a time-varying charge. In the latter case we may emphasize the time dependency by writing $q(t)$. This practice involving capital and lowercase letters will be carried over to the other electrical quantities as well.

The primary purpose of an electric circuit is to move or transfer charges along specified paths. This motion of charges constitutes an *electric current*, denoted by the letters i or I, taken from the French word "intensité." Formally, current is the time rate of change of charge, given by

$$i = \frac{dq}{dt} \tag{1.1}$$

The basic unit of current is the *ampere* (A), named for André Marie Ampère (1775–1836), a French mathematician and physicist who formulated laws of electromagnetics in the 1820s. An ampere is 1 coulomb per second.

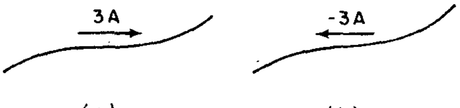
In circuit theory current is generally thought of as the movement of positive charges. This convention stems from Benjamin Franklin (1706–1790), who guessed that electricity traveled from positive to negative. We now know that in metal conductors the current is the movement of electrons that have been pulled loose from the orbits of the atoms of the metal. Thus we should distinguish *conventional* current (the movement of positive charges), which is used in electric network theory, and *electron* current. Unless otherwise stated, our concern will be with conventional current.

As an example, suppose the current in the wire of Fig. 1.3(a) is $I = 3$ A. That is, 3 C/s pass some specific point in the wire. This is symbolized by the arrow labeled 3 A, whose direction indicates that the motion is from left to right. This situation is equivalent to that depicted by Fig. 1.3(b), which indicates -3 C/s or -3 A in the direction from right to left.

Figure 1.4 represents a general circuit element with a current i flowing from the left toward the right terminal. The total charge entering the element between time t_0 and t is found by integrating (1.1). The result is

$$q_T = q(t) - q(t_0) = \int_{t_0}^{t} i \, dt \tag{1.2}$$

We should note at this point that we are considering the network elements to be *electrically neutral*. That is, no net positive or negative charge can accumulate in the element. A positive charge entering must be accompanied by an equal positive



(a) (b)

FIGURE 1.3 *Two representations of the same current*

FIGURE 1.4 *Current flowing in a general element*

charge leaving (or, equivalently, an equal negative charge entering). Thus the current shown entering the left terminal in Fig. 1.4 must leave the right terminal.

There are several types of current in common use, some of which are shown in Fig. 1.5. A constant current, as shown in Fig. 1.5(a), will be termed a *direct current*, or dc. An *alternating current*, or ac, is a sinusoidal current, such as that of Fig. 1.5(b). Figures 1.5(c) and (d) illustrate, respectively, an *exponential* current and a *sawtooth* current.

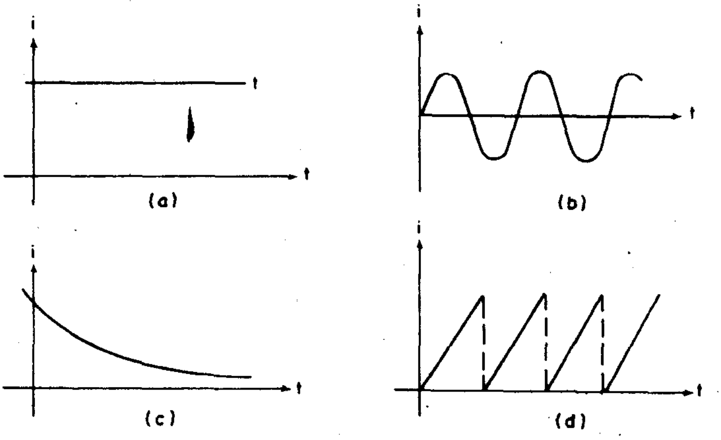


FIGURE 1.5 *(a) dc; (b) ac; (c) Exponential current; (d) Sawtooth current*

There are many commercial uses for dc, such as in flashlights and in power supplies for electronic circuits, and, of course, ac is the common household current found all over the world. Exponential currents appear quite often (whether we want them or not!) when a switch is actuated to close a path in an energized circuit. Sawtooth waves are useful in equipment, such as oscilloscopes, used for displaying electrical characteristics on a screen.

EXERCISES

1.2.1 Find the charge in picocoulombs represented by 10,000 electrons.

Ans. 0.0016021

1.2.2 The total charge entering a terminal of an element is given by

$$q(t) = 10t^2 - 2t \text{ C}$$

Find $i(t)$ at $t = 0$ and at $t = 1$ s. *Ans.* −2 A, 18 A