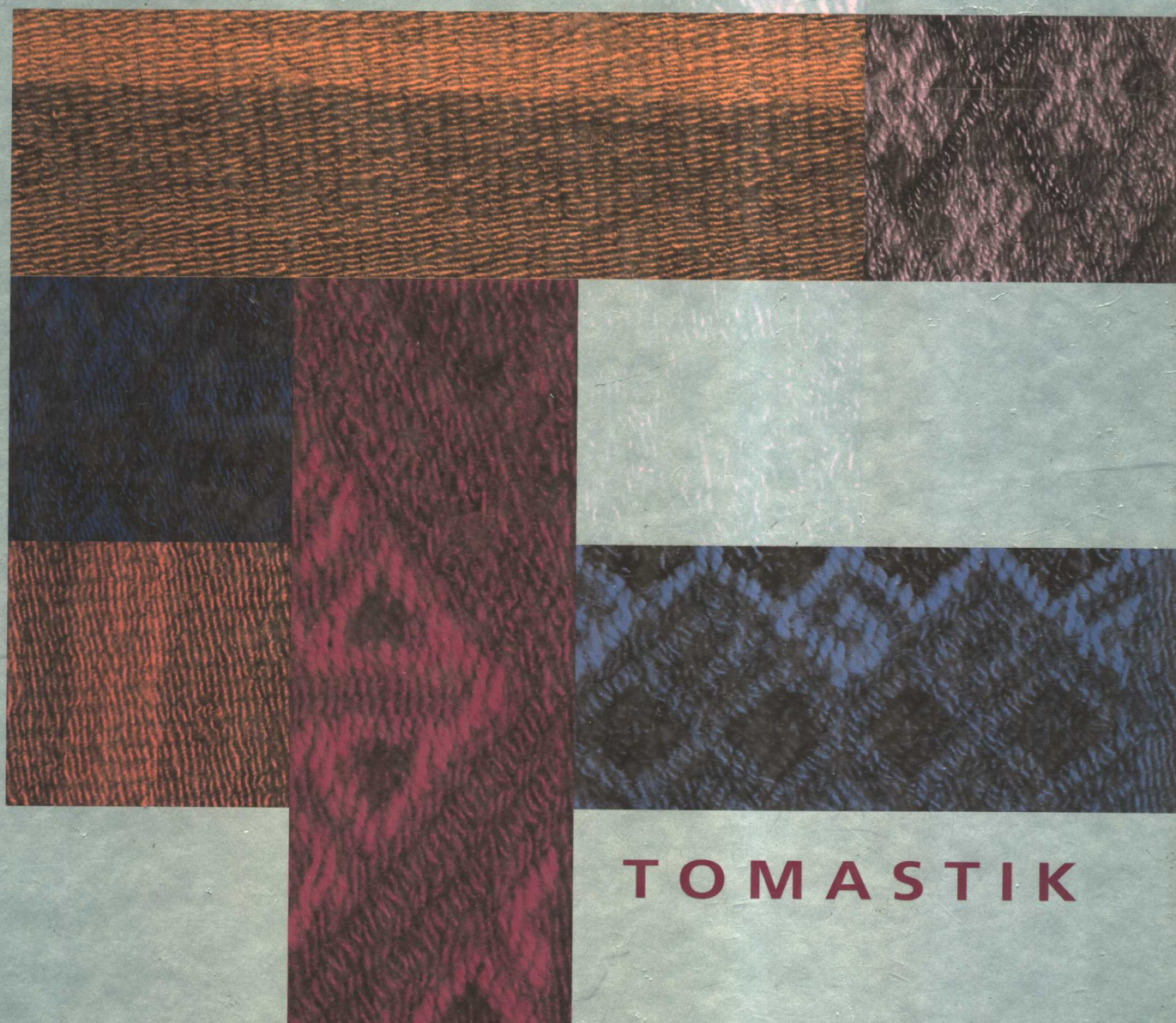
Applied Mathematics

for Business, Economics, Life and Social Sciences



TOMASTIK

APPLIED MATHEMATICS FOR BUSINESS, ECONOMICS, LIFE AND SOCIAL SCIENCES

Edmond C. Tomastik

UNIVERSITY OF CONNECTICUT

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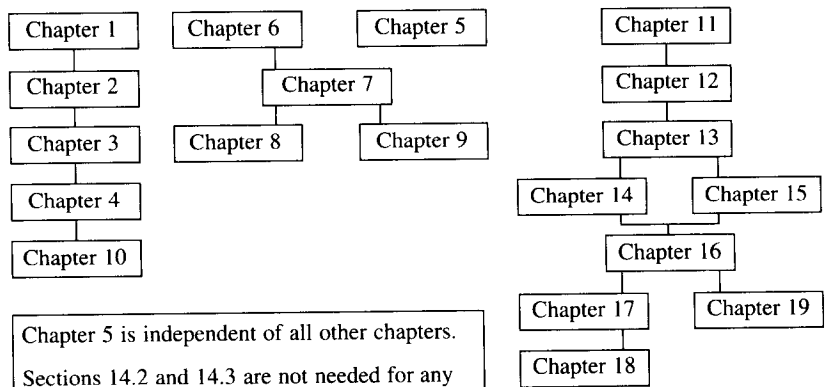

Chapter 5 is independent of all other chapters.

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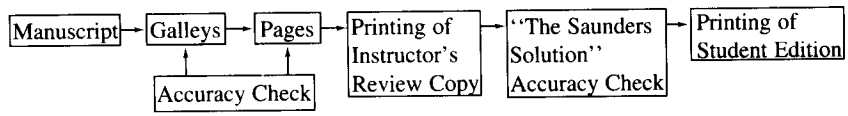

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Preface

Applied Mathematics for Business, Economics, Life and Social Sciences is designed to be used in a course that combines finite mathematics with calculus and is aimed for students majoring in business, management, economics, or the life or social sciences. The text can be understood by the average student with two years of high school algebra. A wide range of topics is included, giving the instructor considerable flexibility in designing a course. Optional graphing calculator material is included.

Distinguishing Features

I have taught both the finite mathematics course and the introductory calculus course for nearly thirty years, examined countless texts for adoption, used a variety of different texts in my classes, and, for a number of reasons, have found all of the current texts unsatisfactory. Thus I decided to write this text, which distinguishes itself from the others in the following ways.

First, applications truly play a central and prominent role in the text. Second, and in keeping within emphasis on applications, I have stressed translating applied problems into mathematical equations or into an appropriate mathematical format for solution by calculators and computers. Third, and again in keeping with an emphasis on applications, I have presented probability, not as an abstract mathematical subject, but as a natural phenomenon that arises in business and science. By beginning with empirical probability I show how the abstract definition of probability arises, resulting in deep and practical insights into probability and statistics. Fourth, I have given an easily understood geometric presentation of why and how the simple method words, giving deep insights into this fundamentally important method. The calculus is presented by relying heavily on graphs and the geometry of curves. The idea is to have the student see geometrically all

the concepts of the calculus. Furthermore, as in the finite mathematics part, a wide variety of timely applications are used to motivate the material.

Let me amplify on these points, one at a time. First, this text is written for *users* of mathematics. Thus, for example, a concrete applied problem is presented first as a motivation before developing a needed mathematical topic. After the mathematical topic has been developed, further applications are given so that the student understands the practical need for knowing the mathematics. This is done so consistently and thoroughly that after going through a number of chapters, the student should come to believe that mathematics is everywhere.

Second, I believe that no other skill is more important than the ability to translate a real-life problem into an appropriate mathematical format for finding the solution. Students often refer to this process as "word problems." Whereas linear systems of equations, linear programming problems, and financial problems, for example, can easily be solved by computers and some calculators, no computer or calculator, now or in the foreseeable future, can translate these applied problems into the necessary mathematical language. Thus students, in their jobs, will most likely use their mathematical knowledge to translate applied problems into necessary mathematical formats for solution by computers.

To develop these needed skills many word problems, requiring the writing of one linear equation, are given in all of the sections in the first introductory chapter. This prepares the student for the many word problems that require creating systems of linear equations in the second chapter on linear systems and matrices. The word problems continue in the third chapter on linear programming, with the last section of this chapter devoted entirely to setting up linear programming problems for solution on computers. If desired, students can then use the software packages that are free to adopters of this text to solve the linear programming problems that have been written. Word problems then continue in the next chapter on the simplex method and beyond. In the calculus many timely applications are considered, such as the optimal management of renewable resources, the Laffer curve in federal tax policy, the logistic equation in population dynamics, to name just a few.

The third major area that distinguishes this text from others, is the discussion found on probability. Other finite mathematics texts present probability as a *mathematical* subject with an *abstract mathematical* definition of probability given to the reader to accept on faith. If, on occasion, a text attempts to explain why this definition is used, the explanation is always confined to the equally likely outcomes case.

Since this text is not meant for mathematics majors but for business, finance, and science majors, the approach is to first present probability as a *natural* phenomenon that arises in concrete problems in business, finance, and science. Thus empirical probability in a natural context is presented first. An experiment is performed N times or a survey of N people is taken. The frequency, $f(E)$, that some particular event occurs is noted. The significance of the fraction $\frac{f(E)}{N}$, which is called the empirical probability, is then easily explained. The student readily understands that changing N will normally result in changing the empirical probability and that making N large should result in this empirical probability being

very close to some number, which is then defined as the actual probability. This, of course, is precisely how probability arises everyday in business and science. The basic properties of empirical probability are easily grasped, extended to probability, and then provide a basic motivation for the mathematical definition of probability. The mathematical definition of probability then arises in a natural context. Relating probability to relative frequencies is then used throughout the text to give deep insights into other subjects.

For example, in statistics, the text easily demonstrates that after N trials of an experiment with outcomes $x_1, x_2, \ldots, x_n$, the *average* outcome is just

$$x_1 \frac{f(x_1)}{N} + x_2 \frac{f(x_2)}{N} + \cdots + x_n \frac{f(x_n)}{N},$$

where x_1 is the first outcome, x_2 the second, and so on, while $f(x_1)$ is the frequency of occurrence of the first outcome, $f(x_2)$ is the frequency of occurrence of the second, and so on. Now when N is large, we expect the relative frequency $\frac{f(x_1)}{N}$ to be approximately the probability p_1 of the first outcome, $\frac{f(x_2)}{N}$ to be approximately the probability p_2 of the second outcome, and so on. Thus we have a natural definition of expected value as

$$x_1 p_1 + x_2 p_2 + \cdots + x_n p_n,$$

and the student sees the somewhat mysterious expected value arising from an everyday *average*. Additionally, these insights are valuable when studying continuous probability using the calculus.

Fourth, a problem with current texts in finite mathematics is their presentation of the simplex method as a mechanical process with only a hint as to why the method might work. In this text I give a geometrical presentation that my students find easy to follow and that shows precisely how the simplex method works. The students see precisely the roles played by the various steps, ratios, and pivots. With these insights, the students are able to learn the additional topics involved in the dual, the extended simplex method, and post-optimal analysis.

Finally, the calculus is presented in a very intuitive and geometric manner, with emphasis on a clear understanding of the fundamentals. Applications are woven into the development of the material and are also used to motivate and to maintain interest. For example, average and instantaneous velocity are introduced at the beginning. This is followed immediately with obtaining the slope of the tangent line as the limit of the slope of secant lines. These ideas are presented in expanded form over several sections. Thus the student is immediately presented with both a physical and geometric interpretation of the notion of the derivative before the derivative is even defined.

Great care is taken to present the reader with the basic assumptions, in order to avoid possible confusion. For example, we state clearly that we only deal with functions that have a finite number of critical points on any finite interval. Thus, a continuous function with a positive derivative at one point will be increasing

on some interval that contains the point, a fact which need not be true for functions with an infinite number of critical points on a finite interval.

Important Features

Style. The text is interesting and can be understood by the average student with a minimum of outside assistance. Material on a variety of topics is presented in an interesting, informal, and student-friendly manner without compromising the mathematical content and accuracy. Concepts are developed gradually, always introduced intuitively, and culminate in a definition or result. Where possible, general concepts are presented only after particular cases have been presented. Scattered throughout the text, and set-off in boxes, are historical and anecdotal comments. The historical comments are not only interesting in themselves, but also indicate that mathematics is a continually developing subject. The anecdotal comments relate the material to contemporary problems.

Applications. The text includes many meaningful applications drawn from a variety of fields. For example, nearly every section opens by posing an interesting and relevant applied problem using familiar vocabulary, which is then solved later in the section after the appropriate mathematics have been developed. Applications are given for all the mathematics that are presented and are used to motivate the student.

Worked Examples. About 700 worked examples, including about 200 Self-Help Exercises mentioned below, have been carefully selected to take the reader progressively from the simplest idea to the most complex. All the steps needed for the complete solutions are included.

Self-Help Exercises. Immediately preceding each exercise set is a set of Self-Help Exercises. These 200 exercises have been very carefully selected to bridge the gap between the exposition in the chapter and the regular exercise set. By doing these exercises and checking the complete solutions provided, students will be able to test or check their comprehension of the material. This, in turn, will better prepare them to do the exercises in the regular exercise set.

Exercises. The book contains over 4600 exercises. Each set begins with drill-type problems to build skills, and then gradually increases in difficulty. The exercise sets also include an extensive array of realistic applications from diverse disciplines. Graphing calculator exercises are included. (See below.)

Graphing Calculator Exercises. These optional exercises appear at the end of appropriate exercise sets under the heading of Graphing Calculator Exercises, marked with the icon  . They are provided for those instructors who wish to supplement and complement the material by using graphing calculators.

Graphing Calculator Appendix. Located at the end of the text, this appendix gives step-by-step explanations on how to use the TI-81 graphing calculator. Topics covered include: basic graphing, the Gauss–Jordan method, matrix inverses, and statistical calculations. Sample programs are also provided.

Students Aids

- **Boldface** is used when defining new terms
- **Boxes** are used to highlight definitions, theorems, results, and procedures.
- **Remarks** are used to draw attention to important points that might otherwise be overlooked.
- **Warnings** alert students to common mistakes.
- **Titles** for worked examples help to identify the subject.
- **Chapter summary outlines**, at the end of each chapter, conveniently summarize all the definitions, theorems, and procedures in one place.
- **Review exercises** are found at the end of each chapter.
- **Answers** to odd-numbered exercises and to all the review exercises are provided in an appendix.
- A student's **solution manual** that contains completely worked solutions to all odd-numbered exercises and to all chapter review exercises is available.
- The software package **MathPath** by George Bergeman, Northern Virginia Community College, is available free to users of this text. It supplies graphical and computational support for many of the important topics in each chapter. Available for the IBM or IBM-compatible.
- **Graph 2D/3D**, a software package by George Bergeman, Northern Virginia Community College, is also available free to users. This software graphs functions in one variable and graphs surfaces of functions in two variables. It also provides computational support for solving calculus problems and investigating concepts. Available for the IBM or IBM-compatible.

Instructor Aids

- An **instructor's manual** with completely worked solutions to the even-numbered exercises is available free to adopters. A **student's solution manual** is free to adopters and contains the completely worked solutions to all odd-numbered and Chapter Review exercises.
- The **printed test bank** contains short-answer and multiple-choice test questions for the instructor to use in their own format. Approximately 100 test questions per chapter are given. The printed test bank is available free to adopters.
- A **computerized test bank** allows instructors to quickly create, edit, and print tests or different versions of tests from the set of test questions accompanying the text. It is free to adopters and is available in IBM or Mac versions.
- The software package MathPath and Graph 2D/3D (see *Student Aids*) are free to adopters.

Content Overview

Chapter 1. The first three sections contain review material on sets, Cartesian coordinates, and lines. An introduction to the theory of the firm with some necessary economics background is provided to take into account the students' diverse backgrounds. The fourth (optional) section on least squares provides an example of the use of linear equations.

Chapter 2. Linear systems and the Gauss–Jordan method are covered in the first two sections of this chapter. The next three sections cover the basic material on matrices. Although many applications are included in the first five sections, the sixth section of this chapter is entirely devoted to input-output analysis, which is an application of linear systems and matrices used in economics.

Chapter 3. The first section presents linear inequalities with an emphasis on translating applied problems into an appropriate mathematical format. Basic material on geometric linear programming is included in the second section. The third section stresses translating more complicated applied problems into the linear programming format. If computers are available, all of these problems can be solved on the computer using the softwave available free to adopters.

Chapter 4. The first two sections present the simplex method for standard maximization problems and give considerable geometric insight into why the method works. The third section shows how to solve minimization problems by solving the dual problem, while the fourth section considers more general linear programming problems. The last section presents some post-optimal analysis.

Chapter 5. This chapter on finance does not depend on any of the other material and can be covered at any point in the course.

Chapter 6. This chapter contains basic material on sets and counting, which is used extensively in the next chapter on probability.

Chapter 7. The basic material on probability is covered in this chapter. The second section introduces probability in a natural way by emphasizing empirical probability and the long-term behavior of the relative frequency of an event. This sets the background for the mathematical definition of probability given in the third section, motivates the definition of expected value and variance of a random variable given in the next chapter, and provides the background needed to understand the law of large numbers also given in the next chapter. Conditional probability, Bayes' theorem, and Bernoulli trials round out this chapter.

Chapter 8. This chapter contains the basic material on statistics. The expected value and variance are motivated using the notions of empirical probability and the long-term behavior of the relative frequency given in Section 7.2. Thus the discussion in Section 7.2 allows a deeper coverage of statistics than otherwise possible with other texts. The normal distribution and its approximation by the

binomial distribution are covered in the fourth and fifth sections. The law of large numbers is found in Section 8.5.

Chapter 9. The basic material on Markov processes, covering both regular and absorbing Markov processes, is presented in this chapter.

Chapter 10. Game theory and its important connection to linear programming is presented in this chapter. This material gives the basics on the extensive interrelationship between linear programming and the celebrated theory of games developed by von Neumann and important in economic theory.

Chapter 11. A review of some needed precalculus subjects makes up this chapter, with a primary emphasis on functions. The first section contains the definition of function, function notation, and the vertical line test. Quadratic and other elementary functions are considered in the second section. The third section considers combinations and composition of functions.

Chapter 12. The study of calculus begins here. In the first section, limits are introduced by using both geometric and numerical examples. The two fundamental problems of determining the instantaneous velocity and slope of the tangent line are introduced also in order to motivate the need to study limits and to set the stage for the later material on derivatives. The rules of limits are also given. Continuity, rates of change, and the derivative are found in the next three sections. The last three sections contain the rules of differentiation, including the chain rule.

Chapter 13. The first section of this chapter uses the sign of the first derivative to determine where functions are increasing and decreasing, while the second section considers relative extrema and the first derivative test. The third section discusses concavity and the second derivative test. These first sections also include a discussion of federal tax revenue and the Laffer curve and also the management of renewable natural resources. The fourth section contains the material on limits at infinity and some extended material on curve sketching. The fifth section considers absolute extrema on both infinite and finite intervals and the last section contains many applications of optimization problems.

Chapter 14. Several additional topics on the derivative are included in this chapter. The first section contains the differential and the linear approximation using differentials, the second section covers implicit differentiation, while the third section includes both related rate problems and a discussion of elasticity of demand.

Chapter 15. The first section of this chapter develops exponential functions and the second section develops logarithmic functions. The third section gives the derivative formulas for the exponential and logarithmic functions, while the last section gives extensive applications, including population growth, radioactive

decay, and the logistic equation with derived estimates of the limiting human population of the earth.

Chapter 16. This chapter begins the theory of integration. The antiderivative and substitution are given in the first two sections. The definite integral as a limit of Riemann sums is the subject of the third section. The fundamental theorem of calculus and area between curves is covered in the last two sections. Extensive applications are given throughout this chapter, including applications to the theory of the firm and economics, average value, density, consumer's and producer's surplus, and Lorentz's curves.

Chapter 17. This chapter rounds out additional topics in integration. Integration by parts and by using tables is covered in the first two sections. An extensive section on numerical integration, including the trapezoid and Simpson's rule and errors, is given in the third section. Improper integrals, with applications, is contained in the fourth section and continuous money flow is the subject of the last section.

Chapter 18. Continuous probability distributions using calculus is the subject of the first section, with the mean and variance covered in the second. The normal distribution is covered in the last section. This last section overlaps the material covered in Section 8.4, giving two options of when to cover the normal distribution.

Chapter 19. The first section presents an introduction to functions of several variables, including cost and revenue curves, Cobb-Douglas production functions, and level curves. The second section then introduces partial derivatives with applications that include competitive and complementary demand relations. The third section gives the second derivative test for functions of several variables and applied applications on optimization. The fourth section is on Lagrange multipliers and carefully avoids algebraic complications. The differential and approximations is given in the fifth section. The last section on double integrals covers double integrals over general domains, Riemann sums, and applications to average value and density.

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Edmond Tomastik

November 1993

The operation of a factory involves both fixed and variable costs.

Contents