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Preface

It is a great pleasure to initiate the series, Mechanics Today, with contributions from some of the most active young researchers in the field. This first volume consists of seven articles in areas of applied mechanics that are of current interest, and have enjoyed a great deal of attention in the recent past. Other areas or points of views of equal importance and interest, that are not included in this volume, will receive their due attention in future volumes.

Each article begins with a discussion of fundamentals and proceeds with a presentation of analytical and experimental (where applicable) results. The subject matter is hence developed in such a manner that the article is useful to specialists, while at the same time it should be accessible to non-experts with sufficient background.

I wish to express my gratitude to my wife Éva, and to Mrs. Erika Ivansons and Miss Cynthia S. Banta, who have assisted with the editorial tasks.

S. NEMAT-NASSER
Evanston, Illinois
July, 1972

Summary

For the convenience of the reader, an abstract of each chapter of this volume is given below.

I Dynamic Effects in Brittle Fracture *by* J. D. Achenbach, *Northwestern University*

Dynamic effects may interact in a significant manner with fracture phenomena. In this paper the importance of dynamic effects due to rapid crack propagation, and/or due to the diffraction of elastic stress waves, is examined in the context of specific examples. The examples are concerned with a crack propagating in its own plane in an unbounded, homogeneous, isotropic, linearly elastic solid. The fracture criterion is based on the balance of rates of energies. The criterion, which can be used for essentially brittle fractures, is formulated in a very simple manner in terms of the stresses and the particle velocities near the crack tip in the plane of fracture. Conditions for the onset of fracture are established and the speed of crack propagation is determined. Mathematical techniques for the computation of the two-dimensional elastodynamic fields are discussed in some detail. The extent to which dynamic effects interact in a significant manner with fracture is explored for both in-plane and anti-plane motions by investigating the four combinations of quasi-static and dynamic loading and quasi-static and dynamic fracture.

II Qualitative Theory of the Ordinary Differential Equations of Nonlinear Elasticity *by* Stuart S. Antman, *Courant Institute of Mathematical Sciences, New York University*

This article treats the qualitative behavior of solutions of boundary value problems for ordinary differential equations arising from general one-dimensional models of nonlinear elasticity. Emphasis is placed on autonomous systems. The theory, which is refined enough to describe flexure, longitudinal extension, transverse extension, and shear, is described first. Special attention is paid to the inequalities restricting the constitutive relations. Then a variety of integrals are constructed from the governing equations. For some of these integrals to be valid, the material must be hyperelastic. In the rest of the article several specific classes of problems are discussed. The flexure, extension, and shear under terminal loads are treated and specialized to the problem of an initially straight hyperelastic column under compressive end loads. The general properties of solutions are determined. It is shown that for small shears the solutions have qualitatively the same form as that of the classical (inextensional, unshearable) Euler elastica, whereas for large shear, new kinds of solutions may appear. Then flexure and extension under hydrostatic loads are discussed and specialized to an initially circular ring (not necessarily hyperelastic). Qualitative properties are determined. In particular, it is shown that every branch of noncircular solutions in a certain region of function space containing the trivial solution is characterized by its least period and that such solutions must have at least two axes of symmetry. The more complicated problem of flexure, extension, and shear of a ring under hydrostatic pressure is then examined. Finally, the deformation of an initially straight beam under tensile end loads is analyzed. Here there are no *a priori* restrictions on the nature of the deformation. It is first proved that under certain mild restrictions, the deformed axis must be straight and the only deformation possible is a contraction of the cross-section and an elongation of the axis. The properties of such solutions are determined, with emphasis on the changes that accompany the onset of necking. The mathematical characterization of the necking process is discussed. The article is concluded with a brief survey of related work and of open problems.

III Plastic Waves: Theory and Experiment *by* R. J. Clifton, *Brown University*

One-dimensional plastic waves are discussed with regard to theoretical predictions and experimental results. A summary is given of a reasonably general theory which provides a framework for including the effects of strain-hardening, rate dependent plastic flow, combined stresses, strain-rate history, and, for the case of waves of uniaxial strain, the effects of finite strains and adiabatic heating. This theory is applied to cases of uniaxial stress, uniaxial strain and combined compression and torsion of thin-walled tubes. In each case, results of experiments are summarized and interpreted for the insight that they provide in assessing the appropriateness of various constitutive assumptions which are used in the theory.

IV Modern Continuum Thermodynamics *by* Morton E. Gurtin, *Carnegie-Mellon University*

This article reviews some of the more recent work in continuum thermodynamics. It begins with a simple axiomatic treatment of the first two laws. It then goes on to discuss elastic materials, materials described by internal state variables, and the connection between thermodynamics and stability.

V General Variational Principles in Nonlinear and Linear Elasticity with Applications *by* S. Nemat-Nasser, *Northwestern University*

This article presents a systematic account of general variational principles for nonlinear, incremental, and linear elasticity. These principles include certain general discontinuities in various field quantities, which may either be required as part of a solution (for example, when material properties change discontinuously, or when fracture occurs over an interior surface), or are introduced in order to enlarge the class of functions from which approximate solutions are sought. The effectiveness of various variational theorems and, in particular, the significance of the discontinuity which may be admitted in all or some of the field quantities and their derivatives, are brought into focus by means of several examples. The examples include the general question of eigenvalue problems for certain elliptic operators which have continuously or discontinuously varying coefficients. These operators may relate to harmonic waves in inhomogeneous elastic bodies, or they may pertain to heat conduction in composite continua. Other problems, such as Schrödinger's wave equation, are also mentioned. Finally, the application of the results to finite-element methods is discussed with an emphasis on the use of discontinuous trial functions, e.g. a piecewise constant approximation.

VI A Survey of Theory and Experiment in Viscometric Flows of Viscoelastic Liquids *by* A. C. Pipkin *and* R. I. Tanner, *Brown University*

The mechanical properties of macromolecular liquids such as molten plastics and polymer solutions are much too complicated to be described adequately by the Navier–Stokes constitutive equation. The properties that have been studied most thoroughly are the dependence of the viscosity and the normal stress differences on the shear rate in steady simple shearing and other equivalent flows, which are called viscometric flows.

In the present review the theory of viscometric flow is outlined and the flows of this kind most frequently used in experimental work are critically analysed. Since experimental errors cannot be discussed if attention is strictly limited to viscometric flows, some approximate methods for the analysis of nearly viscometric flows are also presented.

All of the main methods of measurement are considered in some detail, with particular attention to the experimental determination of the normal stress differences. Viscosity measurements are relatively straightforward, but normal stress measurements are not. Indeed, until recently, there has been confusion about even the sign of the so-called second normal stress difference for polymer solutions. It is now known that there is a large systematic error involved in the use of pressure holes in normal stress measurements, which invalidates conclusions from much of the older data. In view of this and other sources of error, an assessment of the reliability of existing data is presented. This leads to the conclusion that the second normal stress difference is negative, with a magnitude of about 15 percent of the first normal stress difference for polyisobutylene solutions.

VII Concepts in Elastic Structural Stability *by* John Roorda, *University of Waterloo*

Current efforts in the field of elastic structural stability are primarily concerned with the phenomena which occur after the onset of buckling, and with the impact that new discoveries in this area may have upon the design philosophies. An overview of some of the current thinking in this field is presented in this chapter. Certain basic concepts are derived by way of a macro-analysis of a discrete structural system. These concepts, and how they impinge on current practice in design, are illustrated at each stage by experimental results drawn from the wealth of literature now available in this field. Following a discussion of potential energy, the conditions for equilibrium and stability are applied to the two major types of structural systems, namely those which show a degree of symmetry in their post-buckling behavior and those which do not. Initial imperfections, and their effect upon the behavior, are included in the discussion, both in a deterministic sense and in a more realistic statistical context.

The possibility of different types of loading in practical applications leads to an analysis of imperfection sensitivity under dynamic loading with emphasis being placed on the case of step loading.

In a section on buckling behavior under multiple loading the stability boundary, which characterizes the interaction between different loads, is discussed in some detail. Finally, the possibility of interaction between buckling modes, and how this is related to the optimal design, is treated.

The models used and the analytical procedures employed are chosen primarily to preserve simplicity and clarity of concept. The primary purpose of this article is to present new insights into the field of structural stability and its applications.

Contents

I

Dynamic Effects in Brittle Fracture

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1 INTRODUCTION

A flaw in a stressed body gives rise to a significant increase of the stress level, particularly in the vicinity of a sharp edge of the flaw. The flaw becomes unstable and may grow into a sizable fracture surface if the external loads increase beyond certain critical magnitudes. The analysis of the stability of flaws belongs to the realm of fracture mechanics. Although fracture involves processes on an atomic level, it is a matter of wide experience that useful quantitative information can be obtained by basing computations on a continuum model for the medium.

Within the framework of a continuum model, such as the homogeneous, isotropic linearly elastic continuum, the classic analytical problem of fracture mechanics consists of the computation of the fields of stress and deformation in the vicinity of the tip of a crack, together with the application of a fracture criterion. In a conventional analysis inertia (or dynamic) effects are neglected and the analytical work is quasi-static in nature.

Inertia effects become of importance if the propagation of a crack is so fast, as for example in essentially brittle fracture, that rapid motions are generated in the medium. The term "dynamic fracture" will be used to denote the effects of inertia resulting from the *rapid propagation* of a crack. Inertia effects in conjunction with fracture phenomena are also important if the external loads give rise to propagating mechanical disturbances (as for impact loads and explosive charges) which strike the crack and cause fracture. Spalling is an example of a fracture phenomenon caused by the rapid application of loads. The label "dynamic loading" is attached to the effects of inertia on fracture due to *rapidly applied loads*.

In the analysis of elastodynamic problems it is often found that at certain specific locations in a body the dynamic stresses are higher than the stresses computed from the corresponding problem of static equilibrium. The reflection of elastic waves provides an example of this effect. Reflection may give rise to an increase of the stress level, as is well known for reflection of a plane wave from a rigidly clamped boundary, when the stresses actually may be doubled in the vicinity of the boundary. A comparable effect occurs when a wave is diffracted by a crack. The dynamic stress "overshoot" in the vicinity of a crack tip may be as high as 30 percent. In view of the dynamic amplification of the stress level it is entirely conceivable that there are cases for which fracture does not occur under a gradually applied system of loads, but where the material does indeed fracture when the same system of loads is rapidly applied and gives rise to waves.

To explore the extent to which inertia effects may interact in a significant and interesting manner with the fracture process we will consider the following four cases:

Case 1: quasi-static loading and quasi-static fracture
Case 2: quasi-static loading and dynamic fracture
Case 3: dynamic loading and dynamic fracture
Case 4: dynamic loading and quasi-static fracture

A number of observations on the influence of inertia effects are presented on the basis of solutions of example problems. The analytical solutions are for a two-dimensional geometry and a plane crack under various loading conditions. It is assumed that crack propagation takes place in the plane of the crack. Both anti-plane shear deformations and deformations in plane strain are considered. The case of anti-plane shear deformations, which is much easier to analyze than the case of in-plane deformations, provides us with considerable insight.

The fracture criterion which is employed is based on the postulate of an overall balance of rates of energies. This fracture criterion can be used for essentially brittle fracture. The balance of rates of energies not only serves to determine the condition for the onset of fracture, it generally also provides an equation for the computation of the speed of crack propagation. In its original form the energy criterion was advanced as a criterion for the onset of quasi-static fracture by Griffith[1] in 1921.

In the work that will be discussed in this paper the attention is limited to the case that the material fractures without appreciable plastic flow in the vicinity of the crack tip, i.e. we consider essentially brittle fracture. Some materials such as glass and metals at low temperatures fail in a brittle fashion even under quasi-static conditions. Under gradually applied loads many materials do, however, develop a considerable region of plastic flow in the vicinity of the edge of a flaw before the material ruptures, because there is sufficient time for plastic flow to develop. Under dynamic loading conditions the stress level near the flaw rises, however, very quickly and it is possible that fracture takes place so rapidly that sufficient time is not available for a substantial zone of plastic flow to be established. The zone of yielding may then remain a very small region near the moving crack tip, and as a good approximation it may be assumed that the fracture is essentially brittle fracture. Thus, steel which certainly shows ductility effects for quasi-static loading may behave as a brittle material under conditions of dynamic loading and dynamic fracture. It should be remarked, however, that the significance of effects of ductility for rapid crack propagation is still a matter of further investigation. It might be noted that it is not difficult to account for some yielding by extending the Dugdale model to dynamic problems.

What appears to be the first work on fracture under dynamic loading conditions was both experimental and analytical in nature, and it was carried out by J. Hopkinson[2]. Hopkinson measured the strength of steel wires when they were suddenly stretched by a falling weight. He explained the results in terms of elastic waves propagating up and down the wire. The next significant investigation was carried out by B. Hopkinson[3], who detonated an explosive charge in contact with a metal plate. In his work, B. Hopkinson demonstrated the effect of "spalling" or "scabbing," which occurs when the compressive pulse generated by the explosive is reflected at the opposite side of the plate as a tensile pulse. The reflected pulse produces tensile fractures, and a disk of metal roughly in the shape of a spherical cap breaks away from the surface directly opposite the explosive charge. After the work of B. Hopkinson very little research was apparently carried out in this area until the Second World War. In his book *Stress Waves in Solids*, Kolsky[4] devoted a chapter to fractures produced by stress waves, in which the work of J. and B. Hopkinson is described and further work dating from the late forties and the early fifties is discussed.

In recent years much additional experimental work has been carried out with a large number of materials and with specimens of various geo-