

TRIGONOMETRY

THIRD EDITION

MICHAEL SULLIVAN

Trigonometry

Third Edition



Michael Sullivan

Chicago State University

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On the cover: A painting from a series, "Stations of the Cross," by California artist Tony Ligamari. The work is acrylic on canvas and measures 3 ft × 2 ft. The artist's use of vivid colors and hard-edge abstraction relates him to the abstract expressionist movement in California. He uses a masking technique to create negative space that gives a sense of depth and movement. His Italian heritage is evident in the subject matter he has chosen to abstract. Ligamari's work is in several private collections; he was a visiting artist at the American Academy in Rome in 1986 and has had shows in London, Florence, and Copenhagen as well as in the United States. He is represented by the John Natsoulas Gallery in Davis, California.

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Preface

Intent/Purpose

As a professor at an urban public university for over 25 years, I am aware of the varied needs of students—students who range from being less well-prepared and insufficiently motivated to those who are well-prepared and highly motivated. As the author of an engineering calculus text as well as texts in finite mathematics and business mathematics, I understand what students must know in order to be successful in such courses. At the same time, I am also aware of, and very concerned about, the student who has decided this trigonometry course is to be a terminal mathematics course. Based on these experiences, I have written a book to serve students with varied backgrounds and goals.

For the student who requires it, a review of basic material needed for trigonometry is contained in Chapter 1. To help further, concepts are viewed from different perspectives whenever possible, and alternate methods of solutions are given. Every effort has been made to be clear, precise, and consistent. Whenever it seemed appropriate, special encouragement has been offered; and whenever necessary, warnings have been given. To provide motivation, I have included understandable, realistic applications that are consistent with the abilities of the student. The problem sets have been carefully graded to build the student's confidence.

At the same time, a conscious attempt has been made to preview material that will be seen by students who intend to take courses such as physics or engineering calculus. These previews relate to important concepts, applications, examples, and exercises that will be encountered in later courses and can be viewed now as trigonometry problems.

To summarize, my purpose in writing this book is to serve the different needs of each type of student through suitable motivation and material.

New to This Edition

The elements of the previous editions that proved so successful remain in this edition. Nevertheless, many changes—some obvious, others subtle—have been made. Virtually every change is the result of thoughtful comments and suggestions from colleagues and students who used the previous editions. As a result of this input, for which I am sincerely grateful, this edition will be an improved teaching device for professors and a better learning tool for students.

Student Involvement In this edition, a determined effort has been made to motivate and involve students. Recent events have been used whenever possible for added interest. An increased use of visualization in the examples and exercises should help students see what is happening and thereby help them understand the underlying ideas that are being presented. To further help the students, each chapter begins with a list of topics to be reviewed in preparation for the chapter. Within each section, at selected places marked by \square , the student is encouraged to pause, work a problem related to the material just presented, and determine whether the material has been mastered before going on to the next topic. At the end of each chapter, a new component has been added, called “Sharpening Your Reasoning: Discussion/Writing/Research.” This component consists of a variety of questions—many open-ended—to encourage discussion, critical thinking, writing projects, and library work.

Calculators/Graphing Calculators This edition continues to encourage the use of a scientific calculator when one is required or helpful. For those who have access to a graphing calculator, this edition also contains comments, checks, and exploratory problems following selected examples to enhance and/or reinforce the material being presented. Exercises that are especially suited to the use of a graphing calculator also have been added.

Revised/Reorganized Material

The discussion involving the graph of a function now appears earlier in Chapter 1. Sections 1.3 and 1.4 have been reorganized so that the graph of a function appears sooner and so that some of the more difficult ideas are postponed.

The use of a calculator to evaluate trigonometric functions appears earlier (Section 2.2). Section 2.5 now contains a discussion on solving right triangles, including using a calculator to find the angles. Several new applied problems are presented.

The material on the straight line now appears in the Appendix instead of in Chapter 6, “Analytic Geometry.”

Illustrations Many new illustrations have been inserted, especially in the exercises, to assist the student in visualizing concepts and to rein-

force the graphical orientation of the text. This edition now contains over 550 illustrations.

Examples New examples have been inserted where the level of content or the diversity of approach warrants. After selected examples, a reference to the graphing calculator is given that comments on the graphing calculator or asks the student to check or explore results. This edition now contains over 350 examples.

Exercises Many new exercises have been added. Many of these are visual in nature, asking the student to draw conclusions from a graph. Others are problems that require a graphing calculator. These have been selected to provide insights that are attainable using a graphing calculator. This edition now contains over 3200 exercises. As part of the chapter review section at the end of each chapter, "True/False Items" have been included. A more significant addition to the chapter review is the inclusion of a new section titled "Sharpening Your Reasoning: Discussion/Writing/Research." This section consists of a variety of relevant mathematical questions designed to stimulate classroom discussion, critical thinking, writing projects, and library research projects.

Contents and Organization

Chapter 1 consists mainly of review material. For example, Section 1.1 contains a discussion of the Pythagorean Theorem and the formulas for perimeter and area of a rectangle, area and circumference of a circle, and volume of a sphere. The first discussion of calculators is also found here.

Chapter 1 also contains all the function concepts required for the study of the trigonometric functions. Special emphasis is placed on functions and the graphs of functions. Graphing is usually done in steps, all of which are illustrated. The graphing techniques introduced in Chapter 1 are utilized and reinforced in Chapters 3 and 8 using the functions introduced there.

With Chapter 2, we begin the study of trigonometry. The trigonometric functions of a real number are introduced using the unit circle approach. This is followed by a discussion of the properties of the trigonometric functions. Right triangle trigonometry and applications close out the chapter.

Chapter 3 continues the study of trigonometric functions with a detailed presentation of their graphs. Here, we use the graphing concepts introduced in Chapter 1, as well as some additional techniques. A discussion of inverse trigonometric functions, followed by an application to simple harmonic motion rounds out the chapter.

Chapter 4 treats the analytic side of trigonometry: identities; sum and difference, double-angle, half-angle, sum-to-product, and product-to-sum formulas; and trigonometric equations.

In Chapter 5, additional applications of trigonometry to solving a general triangle are given, including a rather complete discussion of the area of a triangle. Two full sections have been devoted to polar coordinates and graphing polar equations to provide a thorough development of this subject.

Chapter 6 contains topics from analytic geometry. In a detailed presentation of the conics (Sections 6.2–6.4), the parabola, ellipse, and hyperbola are defined using geometric (distance-based) means. After a discussion of rotation of axes and the general form of a conic, a unified definition of the conics using eccentricity is given. Lastly, plane curves and parametric equations are studied.

Chapter 7 is comprised of two parts that are independent of each other and may be covered in any order. Sections 7.1 and 7.2 begin with a discussion of complex numbers, including quadratic equations and the quadratic formula, and conclude with DeMoivre's Theorem. Sections 7.3 and 7.4 provide an introduction to vectors, including the dot product and applications.

Chapter 8 treats the exponential and logarithmic functions in the detail necessary and with language consistent for subsequent use in calculus. Applications to both compound interest and growth and decay are among those given.

Finally, a detailed discussion of completing the square, including the derivation of the quadratic formula, and a discussion of the straight line are provided in the Appendix.

Applications

As we mentioned earlier, every opportunity has been taken to present understandable, realistic applications consistent with the abilities of the student, drawing from such sources as tax rate tables, the *Guinness Book of World Records*, and newspaper articles. For added interest, some of the applied exercises have been adapted from textbooks the students may be using in other courses (for example, economics, chemistry, physics, etc.). See, for example, Section 7.4.

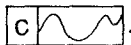
Historical Notes

William Schulz of Northern Arizona University has provided historical context and information in anecdotes that appear as introductory material and at the ends of many sections. In some cases, these comments also include exercises and discussion of comparative techniques.

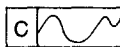
Examples, Exercises, and Illustrations

The text includes over 350 examples and 3200 exercises, of which over 400 are applied problems. The examples are worked out in appropriate

detail, starting with simple, reasonable problems and working gradually up to more challenging ones. Selected examples that allow for checking or exploration using a graphing calculator are indicated by the symbol



Exercises are numerous, well-balanced, and graduated. They usually begin with problems designed to build confidence, continue with drill-type problems that mimic worked out examples, and conclude with problems that are more challenging. Many of the beginning problems in an exercise set are visual in character—such as showing a graph and asking for conclusions. They also include a number of exercises where the student will need a scientific calculator for their solution; these are clearly marked with the symbol \boxed{C} . Others are marked with the symbol



to indicate that a graphing calculator is required. Answers are given in the back of the book for all the odd-numbered exercises.

Illustrations are abundant, numbering over 550. Full use is made of a second color to help clarify and highlight.

A Word about Format and Design

Each chapter-opening page contains a list of topics the student should review before beginning the chapter, a table of contents for the chapter, and an overview—often historical—of the contents.

New terms appear in boldface type where they are defined. The most important definitions are shown in color.

Theorems are set with the word “Theorem” in the margin for easy identification; if it is a named theorem, the name also appears in the margin. When a theorem has a proof given, the word “Proof” appears in the margin to mark clearly the beginning of the proof, and the symbol ■ is used to indicate the end of the proof.

All important formulas and procedures are enclosed by a box and shown in color.

Examples are numbered within each section and identified clearly with the word “Example” in the margin. The solution to each example appears immediately following the example with the word “Solution” in the margin to identify it. The symbol ■ indicates the end of each solution.

Within each section, several references, marked by a □, are given that direct the student to “Now work Problem . . .” These are provided so that the student can pause, work a problem based on the material just presented, and then ascertain whether the subject matter has been mastered.

Each section ends with an exercise set. Each chapter ends with a chapter review containing a list of “Things to Know” (a summary of the key elements of the chapter), a selection of fill-in-the-blank items and

true/false items (to test vocabulary and formulas), a set of review exercises, and a collection of questions to encourage classroom discussion, critical thinking, written projects, and library research.

Supplementary Material

For the Instructor:

Instructor's Solutions Manual to Accompany Trigonometry, by Katy Murphy and Michael Sullivan, contains worked-out solutions to all the even-numbered problems. If used in conjunction with the *Student Solutions Manual*, the instructor will have worked-out solutions for every problem in the book.

Instructor's Resource Material contains approximately 150 transparency masters that duplicate important illustrations in the text.

Instructor's Computer Test Generator, DellenTest 4.0 (MS-DOS version) and DellenTest MAC 2.0, contain 25–30 model test questions for the objectives of each chapter in the text. These test generators are available in both free response and multiple choice formats. Carefully constructed algorithms use random number generators to produce different yet equivalent versions of these model questions each time they are printed. By using these test generators it is possible to create an unlimited number of test items comparable to the question models, each different from the other yet each testing the same concept at the same level. This system thus provides a virtually unlimited number of quizzes, chapter tests, or final examinations. The system also generates an answer key and student worksheet with an answer column that exactly matches the column on the answer key. Graphing grids are included for all problems requiring graphs. The multiple choice versions of DellenTest 4.0 and DellenTest MAC 2.0 utilize distractors that are representative of commonly made student errors. These versions are machine-gradable.

DellenTest 4.0 utilizes MS-DOS and will produce high-quality output on an IBM-compatible dot matrix printer, a Hewlett-Packard Laserjet II® compatible laser printer, or a DeskJet® printer. DellenTest MAC 2.0 is available for the Macintosh with a minimum of 1 MEG RAM running under system 6.0 or higher. Both MS-DOS and Macintosh platforms incorporate a unique editing function that allows the instructor to create additional problems or to alter any of the existing problems in the test generator using a full set of mathematics notation.

Both DellenTest 4.0 and DellenTest MAC 2.0 are accompanied by user notes and annotated printouts of the test models on a chapter-by-chapter basis.

For the Student:

Student Solutions Manual to Accompany Trigonometry, by Katy Murphy and Michael Sullivan, contains complete step-by-step solutions to all the odd-numbered problems.

Acknowledgments

Textbooks are written by an author, but evolve from an idea into final form through the efforts of many people. Before initial writing began, a survey was conducted which drew nearly 250 instructor responses. The manuscript then underwent a thorough and lengthy review process, including class-testing at Chicago State University. I would like to thank my colleagues and students at Chicago State, who cooperated and contributed to this text while it was being class-tested.

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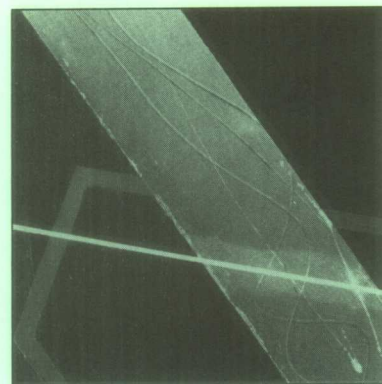
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CHAPTER 1

Functions and Graphs

- 1.1 Review Topics from Algebra and Geometry
- 1.2 Rectangular Coordinates and Graphs
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Chapter Review

Perhaps the most central idea in mathematics is the notion of a *function*. This important chapter deals with what a function is, and how to graph functions.

The word *function* apparently was introduced by René Descartes in 1637. For him, a function simply meant any positive integral power of a variable x . Gottfried Wilhelm von Leibniz (1646–1716), who always emphasized the geometric side of mathematics, used the word function to denote any quantity associated with a curve, such as the coordinates of a point on the curve. Leonhard Euler (1707–1783) employed the word to mean any equation or formula involving variables and constants. His idea of a function is similar to the one most often used today in courses that precede calculus. Later, the use of functions in investigating heat flow equations led to a very broad definition, due to Lejeune Dirichlet (1805–1859), which describes a function as a rule or correspondence between two sets. It is his definition that we use here.

1.1

Review Topics from
Algebra and
Geometry

When we want to treat a collection of similar but distinct objects as a whole, we use the idea of a **set**. For example, the set of *digits* consists of the collection of numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we use the symbol D to denote the set of digits, then we can write

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

In this notation, the braces $\{ \}$ are used to enclose the objects, or **elements**, in the set. Another way to denote a set is to use **set-builder notation**, where the set D of digits is written as

$$D = \{ x \mid x \text{ is a digit} \}$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

Read as “ D is the set of all x such that x is a digit.”

- Example 1** (a) $E = \{x \mid x \text{ is an even digit}\} = \{0, 2, 4, 6, 8\}$
 (b) $O = \{x \mid x \text{ is an odd digit}\} = \{1, 3, 5, 7, 9\}$ ■

If every element of a set A is also an element of a set B , then we say that A is a **subset** of B . If two sets A and B have the same elements, then we say that A **equals** B . For example, $\{1, 2, 3\}$ is a subset of $\{1, 2, 3, 4, 5\}$; and $\{1, 2, 3\}$ equals $\{2, 3, 1\}$.

Real Numbers

Real numbers are represented by symbols such as

$$25, 0, -3, \frac{1}{2}, -\frac{5}{4}, 0.125, \sqrt{2}, \pi, \sqrt[3]{-2}, 0.666\dots$$

The set of **counting numbers**, or **natural numbers**, is the set $\{1, 2, 3, 4, \dots\}$. (The three dots, called an **ellipsis**, indicate that the pattern continues indefinitely.) The set of **integers** is the set $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. A **rational number** is a number that can be expressed as a quotient a/b of two integers, where the integer b cannot be 0. Examples of rational numbers are $\frac{3}{4}$, $\frac{5}{2}$, $\frac{0}{4}$, and $-\frac{2}{3}$. Since $a/1 = a$ for any integer a , every integer is also a rational number. Real numbers that are not rational are called **irrational**. Examples of irrational numbers are $\sqrt{2}$ and π (the Greek letter pi), which equals the constant ratio of circumference to diameter of a circle. See Figure 1.

Real numbers can be represented as **decimals**. Rational real numbers have decimal representations that either **terminate** or are non-terminating with **repeating** blocks of digits. For example, $\frac{3}{4} = 0.75$, which terminates; and $\frac{2}{3} = 0.666\dots$, in which the digit 6 repeats indefinitely. Irrational real numbers have decimal representations that neither repeat nor terminate. For example, $\sqrt{2} = 1.414213\dots$ and $\pi =$

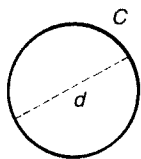


Figure 1

$$\pi = \frac{C}{d}$$