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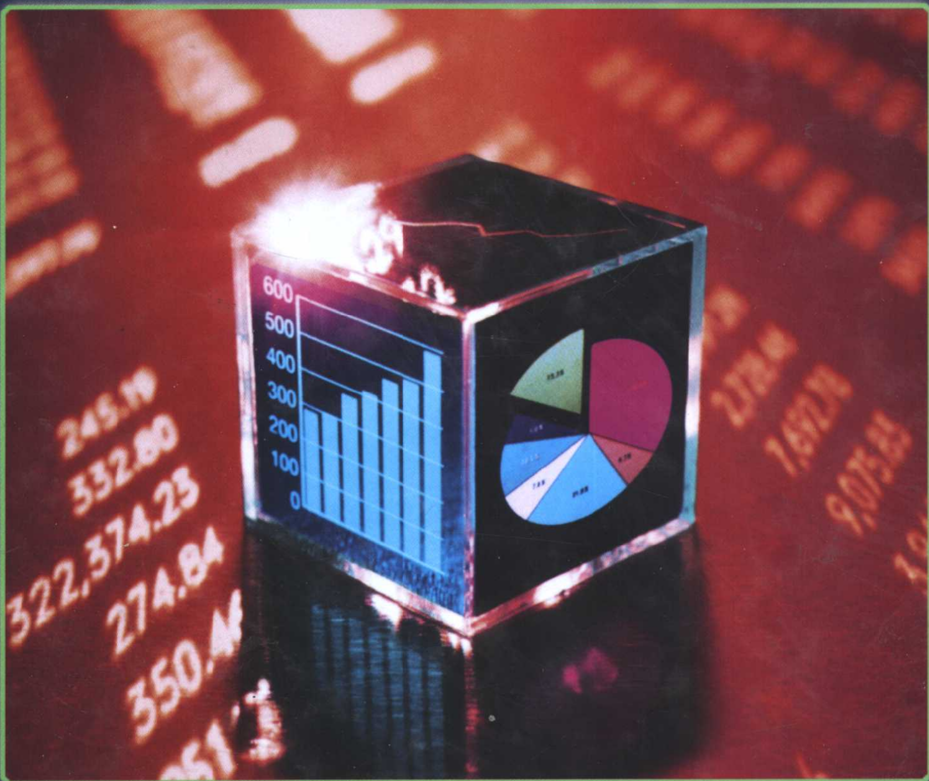
Introduction to Probability and Statistics for Engineers and Scientists



Third Edition

SHELDON M. ROSS

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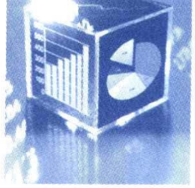

INTRODUCTION TO PROBABILITY AND STATISTICS FOR ENGINEERS AND SCIENTISTS

Third Edition

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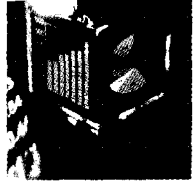

Preface

The third edition of this book continues to demonstrate how to apply probability theory to gain insight into real, everyday statistical problems and situations. As in the previous editions, carefully developed coverage of probability motivates probabilistic models of real phenomena and the statistical procedures that follow. This approach ultimately results in an intuitive understanding of statistical procedures and strategies most often used by practicing engineers and scientists.

This book has been written for an introductory course in statistics, or in probability and statistics, for students in engineering, computer science, mathematics, statistics, and the natural sciences. As such it assumes knowledge of elementary calculus.

ORGANIZATION AND COVERAGE

Chapter 1 presents a brief introduction to statistics, presenting its two branches of descriptive and inferential statistics, and a short history of the subject and some of the people whose early work provided a foundation for work done today.

The subject matter of descriptive statistics is then considered in **Chapter 2.** Graphs and tables that describe a data set are presented in this chapter, as are quantities that are used to summarize certain of the key properties of the data set.

To be able to draw conclusions from data, it is necessary to have an understanding of the data's origination. For instance, it is often assumed that the data constitute a "random sample" from some population. To understand exactly what this means and what its consequences are for relating properties of the sample data to properties of the entire population, it is necessary to have some understanding of probability, and that is the subject of **Chapter 3**. This chapter introduces the idea of a probability experiment, explains the concept of the probability of an event, and presents the axioms of probability.

Our study of probability is continued in **Chapter 4**, which deals with the important concepts of random variables and expectation, and in **Chapter 5,** which considers some special types of random variables that often occur in applications. Such random variables as the binomial, Poisson, hypergeometric, normal, uniform, gamma, chi-square, t, and F are presented.

In **Chapter 6**, we study the probability distribution of such sampling statistics as the sample mean and the sample variance. We show how to use a remarkable theoretical result of probability, known as the central limit theorem, to approximate the probability distribution of the sample mean. In addition, we present the joint probability distribution of the sample mean and the sample variance in the important special case in which the underlying data come from a normally distributed population.

Chapter 7 shows how to use data to estimate parameters of interest. For instance, a scientist might be interested in determining the proportion of Midwestern lakes that are afflicted by acid rain. Two types of estimators are studied. The first of these estimates the quantity of interest with a single number (for instance, it might estimate that 47 percent of Midwestern lakes suffer from acid rain), whereas the second provides an estimate in the form of an interval of values (for instance, it might estimate that between 45 and 49 percent of lakes suffer from acid rain). These latter estimators also tell us the "level of confidence" we can have in their validity. Thus, for instance, whereas we can be pretty certain that the exact percentage of afflicted lakes is not 47, it might very well be that we can be, say, 95 percent confident that the actual percentage is between 45 and 49.

Chapter 8 introduces the important topic of statistical hypothesis testing, which is concerned with using data to test the plausibility of a specified hypothesis. For instance, such a test might reject the hypothesis that fewer than 44 percent of Midwestern lakes are afflicted by acid rain. The concept of the p-value, which measures the degree of plausibility of the hypothesis after the data have been observed, is introduced. A variety of hypothesis tests concerning the parameters of both one and two normal populations are considered. Hypothesis tests concerning Bernoulli and Poisson parameters are also presented.

Chapter 9 deals with the important topic of regression. Both simple linear regression — including such subtopics as regression to the mean, residual analysis, and weighted least squares — and multiple linear regression are considered.

Chapter 10 introduces the analysis of variance. Both one-way and two-way (with and without the possibility of interaction) problems are considered.

Chapter 11 is concerned with goodness of fit tests, which can be used to test whether a proposed model is consistent with data. In it we present the classical chi-square goodness of fit test and apply it to test for independence in contingency tables. The final section of this chapter introduces the Kolmogorov–Smirnov procedure for testing whether data come from a specified continuous probability distribution.

Chapter 12 deals with nonparametric hypothesis tests, which can be used when one is unable to suppose that the underlying distribution has some specified parametric form (such as normal).

Chapter 13 considers the subject matter of quality control, a key statistical technique in manufacturing and production processes. A variety of control charts, including not only the Shewhart control charts but also more sophisticated ones based on moving averages and cumulative sums, are considered.

Chapter 14 deals with problems related to life testing. In this chapter, the exponential, rather than the normal, distribution, plays the key role.

NEW TO THIS EDITION

New exercises and real data examples have been added throughout, including:

- The One-sided Chebyshev Inequality for Data (Section 2.4)
- The Logistics Distribution and Logistic Regression (Sections 5.4 and 9.11)
- Estimation and Testing in proofreader problems (Examples 7.2B and 8.7g)
- Product Form Estimates of Life Distributions (Section 7.2.1)
- Observational Studies (Example 8.6e)

ABOUT THE CD

Packaged along with the text is a PC disk that can be used to solve most of the statistical problems in the text. For instance, the disk computes the p-values for most of the hypothesis tests, including those related to the analysis of variance and to regression. It can also be used to obtain probabilities for most of the common distributions. (For those students without access to a personal computer, tables that can be used to solve all of the problems in the text are provided.)

One program on the disk illustrates the central limit theorem. It considers random variables that take on one of the values $0, 1, 2, 3, 4$, and allows the user to enter the probabilities for these values along with an integer n. The program then plots the probability mass function of the sum of n independent random variables having this distribution. By increasing n, one can "see" the mass function converge to the shape of a normal density function.

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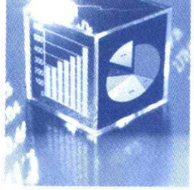

CONTENTS

* Denotes optional material.

INTRODUCTION TO STATISTICS

1.1 INTRODUCTION

It has become accepted in today's world that in order to learn about something, you must first collect data. *Statistics* is the art of learning from data. It is concerned with the collection of data, its subsequent description, and its analysis, which often leads to the drawing of conclusions.

1.2 DATA COLLECTION AND DESCRIPTIVE STATISTICS

Sometimes a statistical analysis begins with a given set of data: For instance, the government regularly collects and publicizes data concerning yearly precipitation totals, earthquake occurrences, the unemployment rate, the gross domestic product, and the rate of inflation. Statistics can be used to describe, summarize, and analyze these data.

In other situations, data are not yet available; in such cases statistical theory can be used to design an appropriate experiment to generate data. The experiment chosen should depend on the use that one wants to make of the data. For instance, suppose that an instructor is interested in determining which of two different methods for teaching computer programming to beginners is most effective. To study this question, the instructor might divide the students into two groups, and use a different teaching method for each group. At the end of the class the students can be tested and the scores of the members of the different groups compared. If the data, consisting of the test scores of members of each group, are significantly higher in one of the groups, then it might seem reasonable to suppose that the teaching method used for that group is superior.

It is important to note, however, that in order to be able to draw a valid conclusion from the data, it is essential that the students were divided into groups in such a manner that neither group was more likely to have the students with greater natural aptitude for programming. For instance, the instructor should not have let the male class members be one group and the females the other. For if so, then even if the women scored significantly higher than the men, it would not be clear whether this was due to the method used to teach them, or to the fact that women may be inherently better than men at learning programming skills. The accepted way of avoiding this pitfall is to divide the class members into the two groups "at random." This term means that the division is done in such a manner that all possible choices of the members of a group are equally likely.

At the end of the experiment, the data should be described. For instance, the scores of the two groups should be presented. In addition, summary measures such as the average score of members of each of the groups should be presented. This part of statistics, concerned with the description and summarization of data, is called *descriptive statistics.*

1.3 INFERENTIAL STATISTICS AND PROBABILITY MODELS

After the preceding experiment is completed and the data are described and summarized, we hope to be able to draw a conclusion about which teaching method is superior. This part of statistics, concerned with the drawing of conclusions, is called *inferential statistics.*

To be able to draw a conclusion from the data, we must take into account the possibility of chance. For instance, suppose that the average score of members of the first group is quite a bit higher than that of the second. Can we conclude that this increase is due to the teaching method used? Or is it possible that the teaching method was not responsible for the increased scores but rather that the higher scores of the first group were just a chance occurrence? For instance, the fact that a coin comes up heads 7 times in 10 flips does not necessarily mean that the coin is more likely to come up heads than tails in future flips. Indeed, it could be a perfectly ordinary coin that, by chance, just happened to land heads 7 times out of the total of 10 flips. (On the other hand, if the coin had landed heads 47 times out of 50 flips, then we would be quite certain that it was not an ordinary coin.)

To be able to draw logical conclusions from data, we usually make some assumptions about the chances (or *probabilities*) of obtaining the different data values. The totality of these assumptions is referred to as a *probability model* for the data.

Sometimes the nature of the data suggests the form of the probability model that is assumed. For instance, suppose that an engineer wants to find out what proportion of computer chips, produced by a new method, will be defective. The engineer might select a group of these chips, with the resulting data being the number of defective chips in this group. Provided that the chips selected were "randomly" chosen, it is reasonable to suppose that each one of them is defective with probability p, where p is the unknown proportion of all the chips produced by the new method that will be defective. The resulting data can then be used to make inferences about p.

In other situations, the appropriate probability model for a given data set will not be readily apparent. However, careful description and presentation of the data sometimes enable us to infer a reasonable model, which we can then try to verify with the use of additional data.

Because the basis of statistical inference is the formulation of a probability model to describe the data, an understanding of statistical inference requires some knowledge of the theory of probability. In other words, statistical inference starts with the assumption that important aspects of the phenomenon under study can be described in terms of probabilities; it then draws conclusions by using data to make inferences about these probabilities.

1.4 POPULATIONS AND SAMPLES

In statistics, we are interested in obtaining information about a total collection of elements, which we will refer to as the *population*. The population is often too large for us to examine each of its members. For instance, we might have all the residents of a given state, or all the television sets produced in the last year by a particular manufacturer, or all the households in a given community. In such cases, we try to learn about the population by choosing and then examining a subgroup of its elements. This subgroup of a population is called a *sample*.

If the sample is to be informative about the total population, it must be, in some sense, representative of that population. For instance, suppose that we are interested in learning about the age distribution of people residing in a given city, and we obtain the ages of the first 100 people to enter the town library. If the average age of these 100 people is 46.2 years, are we justified in concluding that this is approximately the average age of the entire population? Probably not, for we could certainly argue that the sample chosen in this case is probably not representative of the total population because usually more young students and senior citizens use the library than do working-age citizens.

In certain situations, such as the library illustration, we are presented with a sample and must then decide whether this sample is reasonably representative of the entire population. In practice, a given sample generally cannot be assumed to be representative of a population unless that sample has been chosen in a random manner. This is because any specific nonrandom rule for selecting a sample often results in one that is inherently biased toward some data values as opposed to others.

Thus, although it may seem paradoxical, we are most likely to obtain a representative sample by choosing its members in a totally random fashion without any prior considerations of the elements that will be chosen. In other words, we need not attempt to deliberately choose the sample so that it contains, for instance, the same gender percentage and the same percentage of people in each profession as found in the general population. Rather, we should just leave it up to "chance" to obtain roughly the correct percentages. Once a random sample is chosen, we can use statistical inference to draw conclusions about the entire population by studying the elements of the sample.

1.5 A BRIEF HISTORY OF STATISTICS

A systematic collection of data on the population and the economy was begun in the Italian city states of Venice and Florence during the Renaissance. The term *statistics*, derived from the word *state*, was used to refer to a collection of facts of interest to the state. The idea of collecting data spread from Italy to the other countries of Western Europe. Indeed, by the first half of the 16th century it was common for European governments to require parishes to register births, marriages, and deaths. Because of poor public health conditions this last statistic was of particular interest.

The high mortality rate in Europe before the 19th century was due mainly to epidemic diseases, wars, and famines. Among epidemics, the worst were the plagues. Starting with the Black Plague in 1348, plagues recurred frequently for nearly 400 years. In 1562, as a way to alert the King's court to consider moving to the countryside, the City of London began to publish weekly bills of mortality. Initially these mortality bills listed the places of death and whether a death had resulted from plague. Beginning in 1625 the bills were expanded to include all causes of death.

In 1662 the English tradesman John Graunt published a book entitled *Natural and Political Observations Made upon the Bills of Mortality*. Table 1.1, which notes the total number of deaths in England and the number due to the plague for five different plague years, is taken from this book.

TABLE 1.1 *Total Deaths in England*

Year	Burials	Plague Deaths
1592	25,886	11,503
1593	17,844	10,662
1603	37,294	30,561
1625	51,758	35,417
1636	23,359	10,400

Source: John Graunt, Observations Made upon the Bills of Mortality. 3rd ed. London: John Martyn and James Allestry (1st ed. 1662).

Graunt used London bills of mortality to estimate the city's population. For instance, to estimate the population of London in 1660, Graunt surveyed households in certain London parishes (or neighborhoods) and discovered that, on average, there were approximately 3 deaths for every 88 people. Dividing by 3 shows that, on average, there was roughly 1 death for every 88/3 people. Because the London bills cited 13,200 deaths in London for that year, Graunt estimated the London population to be about

$$13{,}200 \times 88/3 = 387{,}200$$

Graunt used this estimate to project a figure for all England. In his book he noted that these figures would be of interest to the rulers of the country, as indicators of both the number of men who could be drafted into an army and the number who could be taxed.

Graunt also used the London bills of mortality — and some intelligent guesswork as to what diseases killed whom and at what age — to infer ages at death. (Recall that the bills of mortality listed only causes and places at death, not the ages of those dying.) Graunt then used this information to compute tables giving the proportion of the population that

TABLE 1.2 *John Graunt's Mortality Table*

Age at Death	Number of Deaths per 100 Births
0–6	36
6–16	24
16–26	15
26–36	9
36–46	6
46–56	4
56–66	3
66–76	2
76 and greater	1

Note: The categories go up to but do not include the right-hand value. For instance, 0–6 means all ages from 0 up through 5.

dies at various ages. Table 1.2 is one of Graunt's mortality tables. It states, for instance, that of 100 births, 36 people will die before reaching age 6, 24 will die between the age of 6 and 15, and so on.

Graunt's estimates of the ages at which people were dying were of great interest to those in the business of selling annuities. Annuities are the opposite of life insurance in that one pays in a lump sum as an investment and then receives regular payments for as long as one lives.

Graunt's work on mortality tables inspired further work by Edmund Halley in 1693. Halley, the discoverer of the comet bearing his name (and also the man who was most responsible, by both his encouragement and his financial support, for the publication of Isaac Newton's famous *Principia Mathematica*), used tables of mortality to compute the odds that a person of any age would live to any other particular age. Halley was influential in convincing the insurers of the time that an annual life insurance premium should depend on the age of the person being insured.

Following Graunt and Halley, the collection of data steadily increased throughout the remainder of the 17th and on into the 18th century. For instance, the city of Paris began collecting bills of mortality in 1667; and by 1730 it had become common practice throughout Europe to record ages at death.

The term *statistics*, which was used until the 18th century as a shorthand for the descriptive science of states, became in the 19th century increasingly identified with numbers. By the 1830s the term was almost universally regarded in Britain and France as being synonymous with the "numerical science" of society. This change in meaning was caused by the large availability of census records and other tabulations that began to be systematically collected and published by the governments of Western Europe and the United States beginning around 1800.

Throughout the 19th century, although probability theory had been developed by such mathematicians as Jacob Bernoulli, Karl Friedrich Gauss, and Pierre-Simon Laplace, its use in studying statistical findings was almost nonexistent, because most social statisticians at the time were content to let the data speak for themselves. In particular, statisticians of that time were not interested in drawing inferences about individuals, but rather were concerned with the society as a whole. Thus, they were not concerned with sampling but rather tried to obtain censuses of the entire population. As a result, probabilistic inference from samples to a population was almost unknown in 19th century social statistics.

It was not until the late 1800s that statistics became concerned with inferring conclusions from numerical data. The movement began with Francis Galton's work on analyzing hereditary genius through the uses of what we would now call regression and correlation analysis (see Chapter 9), and obtained much of its impetus from the work of Karl Pearson. Pearson, who developed the chi-square goodness of fit tests (see Chapter 11), was the first director of the Galton Laboratory, endowed by Francis Galton in 1904. There Pearson originated a research program aimed at developing new methods of using statistics in inference. His laboratory invited advanced students from science and industry to learn statistical methods that could then be applied in their fields. One of his earliest visiting researchers was W. S. Gosset, a chemist by training, who showed his devotion to Pearson by publishing his own works under the name "Student." (A famous story has it that Gosset was afraid to publish under his own name for fear that his employers, the Guinness brewery, would be unhappy to discover that one of its chemists was doing research in statistics.) Gosset is famous for his development of the t-test (see Chapter 8).

Two of the most important areas of applied statistics in the early 20th century were population biology and agriculture. This was due to the interest of Pearson and others at his laboratory and also to the remarkable accomplishments of the English scientist Ronald A. Fisher. The theory of inference developed by these pioneers, including among others

TABLE 1.3 *The Changing Definition of Statistics*

Statistics has then for its object that of presenting a faithful representation of a state at a determined epoch. (Quetelet, 1849)

Statistics are the only tools by which an opening can be cut through the formidable thicket of difficulties that bars the path of those who pursue the Science of man. (Galton, 1889)

Statistics may be regarded (i) as the study of populations, (ii) as the study of variation, and (iii) as the study of methods of the reduction of data. (Fisher, 1925)

Statistics is a scientific discipline concerned with collection, analysis, and interpretation of data obtained from observation or experiment. The subject has a coherent structure based on the theory of Probability and includes many different procedures which contribute to research and development throughout the whole of Science and Technology. (E. Pearson, 1936)

Statistics is the name for that science and art which deals with uncertain inferences — which uses numbers to find out something about nature and experience. (Weaver, 1952)

Statistics has become known in the 20th century as the mathematical tool for analyzing experimental and observational data. (Porter, 1986)

Statistics is the art of learning from data. (this book, 2004)