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# PROGRESS IN BIOPHYSICS AND MOLECULAR BIOLOGY

## 22

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# ARTERIAL ELASTICITY AND FLUID DYNAMICS

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## CONTENTS

# 1

# ARTERIAL ELASTICITY AND FLUID DYNAMICS

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## LIST OF SYMBOLS

| | |
|---|---|
| $c$ | Measured velocity of radial (Young) waves |
| $c_0$ | Characteristic velocity of radial (Young) waves |
| $c'_0$ | Velocity of axial (Lamb) waves |
| $E$ | Modulus of elasticity, type indicated by subscript |
| $E^*$ | Complex modulus of elasticity |
| $h$ | Vessel wall thickness |
| $H$ | Effective wall thickness |
| $i$ | $\sqrt{-1}$ |
| $p$ | Pressure |
| $q$ | Volume flow rate |
| $r$ | Radial coordinate |
| $R$ | Internal vessel radius |
| $R_0$ | External vessel radius |
| $u$ | Radial fluid velocity |
| $V$ | Volume |
| $w$ | Axial fluid velocity |
| $z$ | Axial coordinate |
| $Z_0$ | Characteristic input impedance |
| $\alpha$ | $R\sqrt{\omega/\nu}$ |
| $\gamma$ | $h/R_0$ |
| $\zeta$ | Axial wall displacement |
| $\eta$ | Coefficient of wall viscosity |
| $\lambda$ | Wavelength |
| $\nu$ | Kinematic viscosity |
| $\xi$ | Radial arterial wall displacement |
| $\mu$ | Fluid viscosity |
| $\rho$ | Fluid density |
| $\rho_w$ | Vessel wall density |
| $\sigma$ | Poisson's ratio |
| $\chi$ | Peak-to-mean velocity ratio |
| $\omega$ | Angular frequency |

## I. INTRODUCTION

Since the last review on this subject appeared in this series (McDonald and Taylor, 1959) there has been a very substantial increase in research activity in both physiological and engineering laboratories. We intend to confine this chapter to a survey of the advances in the last decade in the understanding of arterial elasticity and fluid dynamics, and have not attempted a serious study of the current status of research in blood rheology nor of the proliferation of analogue computer simulations. Four major publications have appeared since 1959 (McDonald, 1960; Attinger, 1964; Fung, 1968; Wetterer and Kenner, 1968), and these, together with the review of 1959, form the best introduction to the subject.

## II. THE LINEARIZED NAVIER–STOKES EQUATIONS

The unsteady motion of a liquid through distensible vessels may be described by sets of equations whose solution under prescribed boundary conditions such as input or output flow or pressure enable the missing parameters to be determined for comparison with

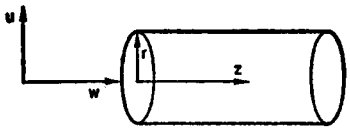


FIG. 1

experiment. If we denote the longitudinal and radial velocity components by $w$ and $u$ and the axial and radial space coordinates by $z$ and $r$ (Fig. 1) the continuity equation may be written:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

Using the assumption of incompressibility, the Navier–Stokes equations for the fluid motion may be written:

$$\frac{\partial u}{\partial t} + \underbrace{u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}}_{\text{non-linear terms}} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) \tag{2}$$

$$\frac{\partial w}{\partial t} + \overbrace{u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{3}$$

An adequate description of the mechanical behaviour of the arterial wall, providing the remaining equation which would enable the variables $p$, $u$, $w$ and $r$ to be solved, is currently the major shortcoming in our knowledge of the overall problem and the area in which most

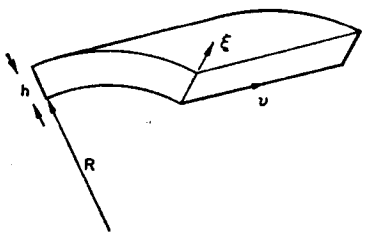


FIG. 2

work remains to be done. Fry and Greenfield (1964) have given an extensive discussion of the factors which must be considered in describing the motion of the arterial wall. Considering an element of the wall as shown in Fig. 2 it is possible to write the radial and longitudinal force balance equations for the segment as follows:

$$\underbrace{\rho_w H \frac{\partial^2 \xi}{\partial t^2}}_{\text{inertial force}} - p + \underbrace{\frac{E^* h}{1-(\sigma^*)^2}\left(\frac{\sigma^*}{R}\frac{\partial \zeta}{\partial z} + \frac{\xi}{R^2}\right)}_{\text{viscoelastic retardation}} = 0 \tag{4}$$

$$\rho_w H \frac{\partial^2 \zeta}{\partial t^2} + \rho_w H \omega^2 \zeta + \mu\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)_{r=R} - \frac{E^* h}{1-(\sigma^*)^2}\left(\frac{\partial^2 \zeta}{\partial z^2} + \frac{\sigma^*}{R}\frac{\partial \xi}{\partial z}\right) = 0 \tag{5}$$

with boundary conditions:

$$u = \frac{\partial \xi}{\partial t}, w = \left(\frac{\partial \zeta}{\partial t}\right)_{r=R+\xi}$$

These equations, (4) and (5), describe the motion of a thin-walled isotropic elastic tube with external longitudinal restraint. The problem of accounting for the tissue surrounding the vessel was initially studied by Womersley (1957), following Morgan and Ferrante (1955), by employing an effective wall thickness $H$ in terms associated with inertia. Womersley assumed no longitudinal added mass since the longitudinal motion was known to be small; he characterized the restraint by means of a natural frequency $\omega/2\pi$. The third term in eq. (4) accounts for the viscoelastic effect in the wall material. Similarly, in eq. (5) the drag force on the wall due to the fluid motion is included as the third term and the corresponding viscoelastic effect within the wall acting in the longitudinal direction as

$$\frac{E^* h}{1-(\sigma^*)^2}\left\{\frac{\partial^2 \zeta}{\partial z^2} + \frac{\sigma^*}{R}\frac{\partial \xi}{\partial z}\right\}.$$

Here $E^*$ is the complex elastic modulus whose real part is the Young's modulus and $\sigma^*$ is the complex Poisson's ratio whose real part is the conventional Poisson's ratio. The imaginary parts of $E^*$ and $\sigma^*$ account in the simplest manner for the viscous damping of the arterial wall. The assumptions involved in this simplification have again been reviewed by Fry and Greenfield (1964).

There are two types of answer sought to these problems and two major approaches which have been employed to obtain them. Information on the propagation of pressure pulses in the arterial system are of value in the interpretation of pressures recorded at distal locations and the same approach should reveal details of the velocity of propagation of the various forms of wall distortion which can occur in the aorta, i.e. radial and longitudinal. The second answer we should expect from a solution of the above equations is a detailed description of the manner in which the blood velocity alters as the ejected blood volume progresses through the arterial system. Out of this second solution should come information on the velocity distribution across the aorta at each location down the vessel. No small-scale information on the behaviour at arterial junctions could be expected from such a simplified method, and indeed in the present state of knowledge such problems are best approached from the rigid tube theory or the junction described in terms of a reflection coefficient in a manner similar to that employed by Taylor (1966a) in a study of the input impedance of the entire arterial tree.

In the first analytical approach to the solution of the equations of fluid and wall motion the non-linear terms such as $u(\partial w/\partial z)$ and $w(\partial w/\partial z)$ are omitted. Whirlow and Rouleau (1965) have given a rational argument for such linearization and have attempted a solution of the thick-walled cylinder problem. They have shown that the convective acceleration terms are of the order of $w/c$ compared with the local acceleration terms and hence may be neglected. It should be pointed out that $w/c$ may in fact approach 0·3 in the ascending aorta of man and could be as high as 0·2 in dogs. The results of Whirlow and Rouleau show pulse pressure decreasing with $z$ contrary to observation, but they have not included the effect of vessel taper and variable wall properties or reflections so that the change in wave shape which they have predicted is due only to dispersion within the wall due to frequency dependent propagation velocity. The neglect of $u(\partial w/\partial r)$ is justified on the basis of a relatively blunt velocity profile in the ascending aorta. The convective acceleration $u(\partial w/\partial z)$ is small compared with either $\partial u/\partial t$ or $\partial w/\partial t$ in vessels with modest geometric taper.

In the linearized solutions so far presented (Womersley, 1957; Whirlow and Rouleau, 1965; Oka, 1965) it is assumed that there are no reflected waves. This apparently gross assumption is justified by the use of the pressure gradient rather than the pressure as explained by Fry and Greenfield (1964). The pressure gradient normal to the wall vanishes under the assumptions generally made in linearized solutions. The usual manner of proceeding from the above equations (1) to (5) is to prescribe a pressure of the form

$$p = f(\exp i\omega t)$$

and solutions for $p$, $w$ and $u$ are found in the form

$$w = \left[\frac{BJ_0(\alpha i^{3/2}\frac{r}{R})}{J_0(\alpha i^{3/2})} + \frac{A}{\rho c}\right]\exp\left[i\omega(t-\tfrac{z}{c})\right] \tag{6}$$

$$u = \frac{i\omega R}{2c}\left[\frac{B2J_1(\alpha i^{3/2}\frac{r}{R})}{\alpha i^{3/2}J_0(\alpha i^{3/2})} + \frac{(r/R)A}{\rho c}\right]\exp\left[i\omega(t-\tfrac{z}{c})\right] \tag{7}$$

$$p = A\exp[i\omega(t-\tfrac{z}{c})] \tag{8}$$

where $\alpha^2 = R^2\omega/\nu$ and $A$ and $B$ are constants of integration.

With the condition of no slip at the wall the above equations (7) and (8) may be reduced to

$$w = \left(B + \frac{A}{\rho c}\right)\exp\left[i\omega(t-\tfrac{z}{c})\right] \tag{9}$$

$$u = \frac{i\omega R}{2c}\left(F_{10}B + \frac{A}{\rho c}\right)\exp\left[i\omega(t-\tfrac{z}{c})\right] \tag{10}$$

where $F_{10} = \dfrac{2J_1(\alpha i^{3/2})}{\alpha i^{3/2}J_0(\alpha i^{3/2})}$.

The wall motion equations are linear and together with the fluid motion constraints at the wall results in a solution for the complex wave velocity $c^*/c = X - iY$. The phase velocity is $c_0/X$ and the damping per wavelength is $\exp(-2\pi Y/X)$. In fact, energy is transmitted at the group velocity, not at the phase velocity but for $\alpha > 3$, which is the range of application,

Womersley has shown that difference between these two velocities is small. The problem of reflected waves can be accommodated by replacing $c$ by $-c$ and superimposing the two solutions, a permissible procedure in view of the assumption of linear wall elasticity.

Wiener *et al.* (1966), in a study of wave propagation in the pulmonary circulation, compared the non-linear terms $u(\partial u/\partial x)$, $w(\partial w/\partial z)$ with $\partial w/\partial t$ and found that the non-linearities represented 13% of the total acceleration in the main pulmonary artery, although this reduced to 4% in the 6th generation of branches. Rudinger (1966) has given a useful exposition of the development of linearized solutions up to 1966 and Fry and Greenfield (1964) have discussed the linearizing assumptions and their likely effects in some detail. The ideas originally expressed by McDonald (1960) that pressure waveform changes in the descending aorta in particular are due to reflections from branches have been extended by Taylor (1966a) who has shown the effect of distributed reflection and of geometric and elastic taper. Rideout and Sims (1969) have also investigated the effect of small non-linearities in the arterial system and have included the effect of vessel taper, both of which contribute to pressure wave peaking.

## III. PULSE WAVE VELOCITY

The theoretical predictions regarding pulse wave velocity are perhaps of more value, in that they are more open to experimental verification. The development of our ideas on the velocity of wave propagation has been well summarized by Hardung (1964c), Noordegraaf (1970) and by Skalak and Stathis (1966). In 1959 McDonald and Taylor discussed the difficulties in determining the true pulse wave velocity in the presence of reflections, and showed how the apparent phase velocity varies under the influence of peripheral reflection. They concluded that the most appropriate measurement would be that of the "foot-to-foot" velocity which appeared to correlate well with what was then known of arterial elasticity. Since then there have been a number of studies on wall properties and their relation to wave velocity, but detailed *in vivo* comparisons of theory with experiment are still needed.

The basic expression for the propagation of waves in thin-walled tubes is the Moens–Korteweg equation

$$c_0^2 = Eh/2R\rho \tag{11}$$

This refers to waves of radial dilatation, often termed Young waves after Thomas Young who originally derived a form of this expression, see Noordegraaf (1970). Axial waves travelling within the wall at a velocity $(c_0^1)^2 = E/\rho_w$ are known as Lamb waves (Klip, 1962; Van Citters and Rushmer, 1961).

A commonly quoted variant of eq. (11) was originally derived by Otto Frank and is

$$c_0^2 = \frac{V}{\rho}\left(\frac{\mathrm{d}p}{\mathrm{d}V}\right)$$

which avoids determination of the wall thickness. These two are equivalent for thin-walled tubes where $\sigma = 0$, the result of neglecting all longitudinal forces. It is more common to make measurements of external radius ($R_0$) changes and to derive a value for an incremental elastic modulus, these measurements should, of course, be dynamic. It can then be shown (Bergel, 1961b; Gow and Taylor, 1968)

$$c_0^2 = (Eh/2R_0\rho)[(2-\gamma)/2 - 2\gamma(1-\sigma-2\sigma^2) - \gamma^2(1-\sigma-2\sigma^2) - 2\sigma^2] \tag{12}$$

When $\sigma = 0{\cdot}5$ this is equivalent to

$$c_0^2 = \frac{dP}{dR_0} \cdot \frac{R_0}{2\rho}(1-\gamma) = \frac{E_p}{2\rho}(1-\gamma) \tag{13}$$

($E_p$ is the "pressure elastic modulus" used by Peterson, Jenson and Parnell, 1960, and by Patel, Greenfield and Fry, 1964.)

These simple expressions have to be modified to take account of the effects of fluid viscosity and the coupling between fluid and wall motion, and again to allow for wall viscosity. Although Witzig and Morgan and Keily discussed this earlier (see Noordegraaf, 1970), we shall start with the work of Womersley (1955, 1957) who has provided extensive tabulations of his results.

As mentioned earlier, the solution for the complex wave propagation was of the form $c^*/c = X\text{-}iY$ with velocity $c_0/X$ and attenuation $\exp[-2\pi(Y/X)\,(z/\lambda)]$. $X$ and $Y$ vary with $\alpha$ and are tabulated in Womersley (1957). It was apparent early on that the longitudinal wall motion was much less than that predicted by Womersley (1955) and a longitudinal tethering force, consisting of an undamped compliance and inertia, was introduced (Womersley, 1957). Quite recently (Patel and Fry, 1966) it has been shown that a damping factor must be introduced to describe fully the tethering forces on the thoracic aorta: the consequences of this have not been explored.

Extreme degrees of tethering ($K = -\infty$) have the effect of increasing wave velocity and slightly reducing transmission. The asymptotic wave velocity becomes $1{\cdot}15\ c^*$, with very little change for $\alpha > 4$. Transmission per wavelength increases with $\alpha$ along a sigmoid path and is about $0{\cdot}5$ at $\alpha = 7$. (At 2 Hz the values of $\alpha$ in the dog are about 9 for the aorta and 3 for the femoral artery; thus wave velocities near the asymptote are expected in all major vessels.)

Much greater attenuations result from the introduction of wall viscosity into Womersley's model. This also increases velocity somewhat. Womersley (1957) introduced a complex elastic modulus ($E^* = E_{dyn} + i\eta\omega$) and a complex Poisson's ratio and discussed the situation where wall viscosity ($\eta$) was not frequency-dependent. However, his tables allow one to insert appropriate values which is not the case with other analyses (e.g. Cox, 1968; Klip, 1962). If, for want of better information, the imaginary part of $\sigma$ is taken to be zero, then Womersley's term $\omega W$ becomes $0{\cdot}5 \tan \Delta\Phi$, where $\Delta\Phi$ is the phase angle between wall stress and strain. Very slight degrees of wall viscosity reduce transmission sharply, but the effect is $\alpha$ dependent.

Using a rubber tube subject to minimal external restraint Taylor (1959) measured wave velocity and attenuation over a very wide range of $\alpha$ values and also calculated fluid resistance and inductance. The fluid parameters showed only fair agreement with prediction. However, for $\alpha = 1\text{–}90$ the behaviour of $c$ was in excellent agreement with expectations for an infinitely tethered tube. Calculated values for wall viscosity and dynamic elasticity also conformed to those determined directly. Taylor speculated that the apparent tethering might have resulted from wall viscosity, but it is possible that this might have been due to the relatively thick-walled tube used ($\gamma = 0{\cdot}39$). These experiments remain the fullest test of Womersley's predictions, but the reasons for the poor results for fluid impedance are still obscure.

Klip and his associates (Klip, 1962; Klip, Van Loon and Klip, 1967) have published expressions for the propagation constants for radial, axial and torsional waves in thick- and thin-walled viscoelastic tubes, together with some model results. The asymptotic radial

wave velocity was that given by the Moens–Korteweg equation, but velocity variations with frequency were not well predicted for the thin tube and were only marginally better when the thick-walled model was employed. Part of the difficulty here, and with the attenuation predictions, may have been due to the use of arbitrarily chosen wall viscosity parameters.

Wave propagation in thick-walled viscoelastic tubes has also been studied by Cox (1967, 1968), who has also made a detailed comparison of the predictions of a number of authors (Cox, 1969). Cox's results do not differ greatly from Womersley's (1957) at high values of $\alpha$, but in the lower range there are significant differences both for wave propagation and fluid impedance and the agreement with measurements on dog femoral arteries is claimed to be good. It is hard to evaluate this claim, for there are considerable discrepancies between the two publications; e.g. Cox (1967), table 4 and fig. 8, and Cox (1968), table II and fig. 5.

In the last few years a series of publications from Stanford (Maxwell and Anliker, 1968; Anliker, Histand and Ogden, 1968; Anliker, Moritz and Ogden, 1968; Anliker, Wells and Ogden, 1968; Histand, 1969; Moritz, 1969) have dealt both theoretically and experimentally with wave propagation in much detail. A wide variety of wave modes are predicted, most of these being non-axisymmetrical and subject to strong attenuations. The most relevant results concern radial (Young), axial (Lamb) and torsional waves. It was shown that the radial waves should only be mildly dispersive (velocity weakly frequency dependent) and that dispersion should be stronger for the other modes. The velocities should increase in the order radial, torsional, axial (ratios *ca.* 1 : 3 : 5). Wall viscosity was again shown to be the dominant factor in attenuation. The radial wave predictions are, for $\alpha > 4$, similar to those of Womersley (1957).

The significant contribution made by this group has been the experimental study of all three wave modes in arteries (and for radial waves in veins) using small sinusoidal oscillations superimposed on the normal pulsations. Short trains of high-frequency (40–200 Hz) waves were used to avoid reflections, and thus both velocity and attenuation could be determined. The disadvantage is that there are no measurements available of wall properties in this frequency range and, further, it is in the low-frequency low $\alpha$ range that experiment might be most helpful in choosing an appropriate theoretical model for all predictions converge at high frequency.

Nevertheless, the results are valuable, especially those on attenuation. The findings of the group can be summarized as follows:

Radial wave velocities are close to those found by other methods and show the expected dependence in internal pressure. There was little dispersion of radial waves and radial wave velocity added algebraically with blood flow velocity. Axial waves were more dispersive and the velocity was not affected by pressure but was increased by longitudinal stretch of the vessel (carotid). Axial velocity was only three times radial in the carotid, as opposed to the expected value of five. This was attributed to wall anisotropy which seems reasonable, but it must be pointed out that at *in vivo* lengths the aorta is *stiffer* longitudinally than circumferentially (Patel, Janicki and Carew, 1969). The velocity of torsional waves was variable, but was about 1·5—2 times that of radial waves.

All waves attenuated exponentially with distance and attenuation was too great to be attributed to fluid viscosity alone: values of $k$ (transmission $= \exp(-kz/\lambda)$) were 0·7 (aorta), 1·4 (carotid) and 1·5 (vena cava) for radial waves. Axial and torsional waves attenuated even faster, $k = 4—6$. These values for radial waves are not inconsistent with extrapolations from Bergel's (1961b) wall viscosity data, but the extrapolation is across a wide frequency range.

A few results obtained when the length of the carotid was altered showed increasing radial wave velocity with greater axial strain (Moritz, 1969). This is unexpected for the radius, and hence the radial modulus, should be decreased by stretch. This result is interesting and suggests, together with the unexpectedly low axial velocity, that our understanding of wave travel in anisotropic tubes is poor.

It is apparent from these studies that pulse wave velocity varies with both pressure and flow, and the question is then raised whether these non-linearities are so great as to invalidate Fourier methods. The difficulties met with in analysing pressure–radius relationships have been stressed by Gow and Taylor (1968) and by Gow (1969), who found marked disturbances in this relation for the higher harmonics. Similarly Dick *et al.* (1968) demonstrated non-linear interactions when pressure waves at two different frequencies were applied together to a femoral artery. There is no doubt that more sophisticated methods will be needed as measurements become more precise. This seems less of a problem in the analysis of pressure–flow relations (e.g. Bergel and Milnor, 1965; O'Rourke and Taylor, 1966, 1967). Rideout (personal communication) has suggested that since wave velocity is related to capacitance–inductance products ($CL$) and impedance to their ratio, the latter will be more stable, for a pressure increase will tend to reduce both $L$ (larger radius) and $C$ (stiffer wall).

There is still not enough evidence on the details of wave propagation to allow a choice of the most useful theoretical model. There is a need for studies of wave propagation and wall properties on the same preparation, but at least in the one investigation of this sort thus far reported (Patel *et al.*, 1969) excellent agreement was found between foot-to-foot velocity and circumferential elasticity. Neither wave velocity nor transmission can be directly determined for long wavelength disturbances *in vivo*, due to the presence of reflections, and elastic non-uniformity and geometrical taper add to the difficulties.

Methods of making these measurements have been described which involve three measurements of pressure or flow at known points along a uniform line. The method was proposed by M. G. Taylor and has been applied by Gessner and Bergel (1966) and by McDonald and Gessner (1968). The computations are lengthy and call for very high accuracy. Gessner and Bergel were able to extract reasonable values for wave velocity which were not frequency-dependent in two femoral arteries. McDonald and Gessner determined velocity and attenuation in the aorta (*in vitro*) and the carotid (*in vivo* and *in vitro*). In the aorta, in spite of taper, transmission across 10 cm. was reasonably close to that predicted by Bergel (1961b). However, transmission per wavelength was low, for the velocity was low (3·2 m/s). For the carotids transmission was lower than expected and the computed imaginary part of the elastic modulus ($\eta\omega$) was about twice as large as that found by Bergel (1961b). There was, in addition, a large variation of $c$ with $\alpha$, velocity halved with reduction in $\alpha$ from 7 to 4.

Similar measurements by Gabe, Fry, Patel and Plexico (personal communication) on thoracic aortas resulted in negative values for attenuation at frequencies up to 20 Hz. Values for $c$ were as expected. Such behaviour might be due to taper. Yet another method purporting to extract "true" phase velocity (Malindzak and Stacy, 1966) shows considerable variation with frequency. It appears that this last method is not reliable, but the discrepancies between the results of McDonald and Gessner and of Gabe *et al.* is hard to explain, and the position with regard to relatively low frequencies remains uncertain.

## IV. THE PROPERTIES OF THE ARTERIAL WALL

### A. *Arterial Elasticity*

It has always been realized that the elastic properties of the arterial wall are one dominant factor in arterial fluid dynamics, and there has been considerable advance in our understanding of wall mechanics in the last ten years. Attention is now being focused on the relation between the details of wall structure and their relation to function, the mechanical properties of vascular smooth muscle and the analysis of anisotropic arterial properties.

The largest "elastic" arteries are constructed of medial lamellar units (Wolinsky and Glagov, 1967). The thickness of these units is relatively constant, about 15$\mu$M under physiological conditions, and thus their number increases with wall thickness and with vessel radius. Assuming uniform intramural stress distribution, the stress per lamellar unit is remarkably constant in mammalian aortae ranging in size from the mouse to the pig. Each lamellar unit consists of a pair of elastic tissue rings obliquely cross-linked by muscle cells (Bierring and Kobayashi, 1962). It appears that collagen fibres in the media of all arteries do not insert into any other tissue but that they lie within the wall forming a parallel elastic mesh. However, the relation of all wall materials to the mucopolysaccharide ground material is not known.

With increasing distance from the heart arterial structure transforms to that of the muscular arteries. The essential difference is that the muscle cells come to lie in rings around the lumen (Pease and Molinari, 1960). Tension does not seem to be transmitted by direct cell-to-cell coupling (Wiederhielm, 1965), but coupling could equally well be assured by the surrounding collagen fibre meshwork (Mullins and Gunteroth, 1965).

Chemical assays of arterial walls show that the elastin/collagen ratio is high (*ca.* 2) for intrathoracic vessels except the coronaries (Fischer and Llaurado, 1966; McDonald, 1960), and that the ratio is reversed elsewhere. Many studies of elastin/collagen ratios with age (see Bader, 1964) and hypertension (Fischer and Llaurado, 1967; Fiegl, Peterson and Jones, 1963) serve to emphasize the rather poor correlation between mechanical properties and total wall constituents.

Arterial elasticity measurements are generally analysed on the assumption that the wall is incompressible, homogeneous and isotropic. The first assumption is justified (Carew, Vaishnav and Patel, 1968), i.e. Poisson's ratio ($\sigma$)=0·5, but not the others. The question of wall anisotropy will be discussed later. It will be a considerable time before the effects of inhomogeneity can be realistically assessed; one analysis of a single small artery (Wiederhielm *et al.*, 1968) illustrates the complexities of the situation.

In the first analysis a measurement of incremental stiffness will be sufficient to predict performance of the vessel as part of a circulatory system. Thus elastic strain is best defined as strain relative to some mean strain (Wetterer and Kenner, 1968). Such measurements have been published as either an incremental Young's modulus $E_{\text{inc}}$ (Bergel, 1961a; Learoyd and Taylor, 1966; Gow and Taylor, 1968) or as a pressure–strain modulus $E_p$ (Peterson, Jensen and Parnell, 1960; Patel, Greenfield and Fry, 1964). This latter index avoids the necessity of determining wall thickness, but it is important that the reference radius be that at an appropriate mean pressure rather than at zero pressure. Using the expression for $E_{\text{inc}}$ given by Bergel (1961a) and assuming a constant vessel length, the relation between the two moduli can be shown (Gow and Taylor, 1968) to be

$$E_{\text{inc}} = E_p\, 2(1-\sigma^2)(1-\gamma)^2/(1-(1-\gamma)^2) \qquad (14)$$

Thus if $\sigma$ is 0·5 and $\gamma$ is 0·1, $E_{\text{inc}} = ca.\ 6E_p$, but the relation is very sensitive to changes in

relative wall thickness. The coefficient of volume distensibility, $\kappa = dP/dV$ (Bader, 1964) is, for small volume changes, equal to $0{\cdot}5E_p$.

The reasons for preferring incremental elastic moduli are strong. They relate directly, as was discussed earlier, to the radial wave velocity at the relevant mean pressure, and when comparison is made between incremental elasticities at any given wall circumferences it greatly clarifies the analysis of arteries in different states, e.g. smooth muscle activation (see later) or length changes (Bergel, 1961a).

There is, of course, a fundamental objection to the use of classical small strain elastic theory in the study of highly extensible materials. This is only partly avoided by the use of incremental methods. However, it has yet to be shown that other approaches (King, 1957; Opatowski, 1967; Fung, 1967) give information that is either more useful or biologically meaningful. We shall not discuss these methods here and will continue to refer to "elastic moduli" which refer only to the mean pressure or mean strain at which they were determined.

It must be remembered that measurements of arterial elasticity have been made under various conditions and that the results of each form of experiment will not be the same. We may list four classes of experiment used:

A. *In vitro,* static or dynamic, vessel held at *in vivo* length (Bergel, 1961a, b; Learoyd and Taylor, 1966; Dobrin and Rovick, 1969).

B. *In vitro,* static, vessel allowed to change length (Roach and Burton, 1959; Schönenberger and Müller, 1960; Bader, 1964, 1967; Tickner and Sacks, 1967).

C. *In vitro,* static and dynamic, measurements on strips or rings (Hardung, 1970; Apter, Rabinowitz and Cummings, 1966; Apter and Marquez, 1968).

D. *In vivo,* constant length, dynamic (at heart frequency) (Peterson *et al.,* 1960; Patel *et al.,* 1964; Gow and Taylor, 1968).

There is general agreement that arterial walls become stiffer as the strain increases. This is true both circumferentially and longitudinally, though it is not to be expected that the relation between stiffness in the two directions will remain constant and there is some evidence (Tickner and Sacks, 1967) to show that it does not. Pressure/volume plots obtained from class B experiments tend to show an exaggeratedly sigmoid shape (Bergel, 1961a). This has led to the conclusion that the pulse wave velocity should be rather constant over the physiological pressure range, but this has not been verified (Anliker, Histand and Ogden, 1968).

There is also general agreement that the elastin of the wall dominates behaviour at low pressure and that the increase in wall stiffness with distension is due to collagen recruitment (Roach and Burton, 1959; Bader, 1964; Wolinsky and Glagov, 1964; Goto and Kimoto, 1966; Apter, 1967). This simple model must not be taken too far for single elastin fibres themselves are markedly non-linear (Carton, Dainauskas and Clark, 1962).

Stress relaxation (Zatzman *et al.,* 1954; Goto and Kimoto, 1966; Apter *et al.,* 1966; Mikami and Attinger, 1968) and creep (Wiederhielm, 1965) are prominent in all arterial tissues and more marked in muscular vessels. The behaviour is not that of a simple viscoelastic material, e.g. for stress relaxation tension falls approximately linearly with log time (Zatzman *et al.,* 1954). Such properties, also seen in rubber and artificial polymers, can be modelled by assuming a number of relaxing elements with a broad distribution of time constants; the more the number of such units the better the fit (Cox, 1967; Goto and Kimoto, 1966), but this array does not mean very much to the physiologist. A more illuminating approach was suggested by Zatzman *et al.* (1954) and later used by Wiederhielm (1965), based on the kinetics of reversible short–long transformations of some tissue element associated with wall muscle.