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# Laser Speckle

and Related Phenomena

Edited by J. C. Dainty



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With Contributions by

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With 133 Figures

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## Preface

Since the invention of the laser there has been a growing interest in the random speckle pattern that is generated when light from this highly coherent source is scattered by a rough surface or inhomogeneous medium. In fact the speckle phenomenon was known since the time of Newton but the development of the laser has brought both a deeper understanding and many new applications.

In preparing this volume we have tried to cover the three main aspects of the subject that are of current interest. Firstly we describe the basic statistics of speckle patterns formed in coherent and partially coherent light. By considering relatively simple models for the scatterer it is possible to give quite a complete statistical description which includes most parameters of practical interest. The second aspect is the reduction of speckle in optical systems in which it forms an unwanted background 'noise', for example, in holography. The final three chapters in this volume are concerned with the wide variety of applications of speckle patterns ranging from the determination of stellar diameters in astronomy to the measurement of displacements in mechanical engineering.

This book is intended mainly for research workers and students in physics and engineering who use lasers in optical systems. Because of the wide variety of present (and, presumably, future) applications of speckle patterns we hope that it will also appeal to scientists and engineers in other disciplines.

I would like to take this opportunity of thanking my fellow contributors—A. E. ENNOS, Prof. M. FRANÇON, Prof. J. W. GOODMAN, Dr. T. S. MCKECHNIE and G. PARRY—for finding the time, energy and enthusiasm to prepare their contributions.

London, August 1975

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# 1. Introduction

J.C. DAINTY

With 3 Figures

The random intensity distribution that we now call a speckle pattern (see Fig. 1.1) is formed when fairly coherent light is either reflected from a rough surface or propagates through a medium with random refractive index fluctuations. Such patterns are clearly visible even to the casual observer when highly coherent laser light is used. In general the statistical properties of speckle patterns depend both on the coherence of the incident light and the detailed properties of the random surface or medium, although for perfectly coherent light this dependence on the random scatterer is almost negligible if the scatterer introduces path differences greater than one wavelength. Although we are con-

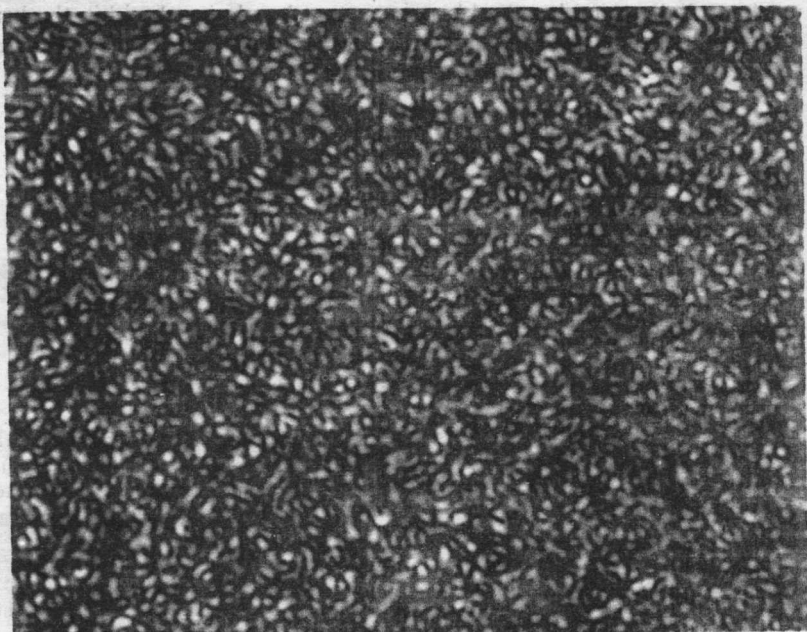


Fig. 1.1. A laser speckle pattern

cerned here with the random speckle patterns produced when visible coherent light is scattered, it should be emphasized that closely related phenomena arise in other regions of the electromagnetic spectrum and also for particles: typical examples are the scattering of X-ray liquids, radar 'clutter' and electron scattering by amorphous carbon films.

In this book we shall concentrate our attention on the development of the subject since the invention of the laser in the early 1960s. The speckle phenomenon had, however, been investigated by many scientists since the time of Newton who interpreted the fact that scintillation or twinkling may be observed for stars but not for the planets [1.1]; we might now explain this on the basis of the different spatial coherence from the two sources. Speckle patterns that are formed by starlight that has propagated through the atmosphere are in fact somewhat different in character to those which we usually meet in the laboratory and are discussed further in Chapter 7.

In the later part of the nineteenth century considerable interest was shown in interference phenomena in scattered light such as Newton's diffusion rings or Quêtelet's fringes<sup>1</sup> and also in Fraunhofer's diffraction rings which are produced when fairly coherent light is diffracted by a glass plate covered with small particles. In 1877, EXNER [1.2, 3] sketched the radially granular speckle pattern that he observed within the bright central Fraunhofer ring and this is shown in Fig. 1.2. Nearly forty years

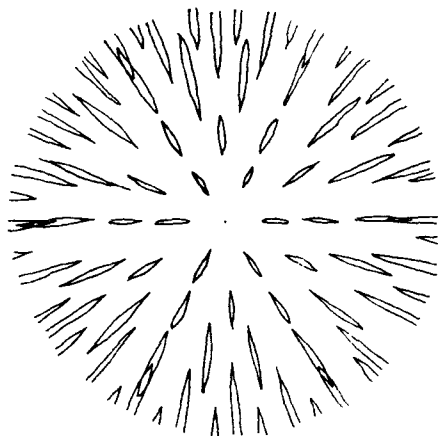


Fig. 1.2. EXNER's sketch of the radial granular structure he observed in the diffraction pattern of a glass plate on which he had breathed. He used a candle as a light source

<sup>1</sup> Quêtelet's fringes can easily be observed by looking at the image of a torch or flashlight held close to the eye in a plane back-silvered mirror the front surface of which is slightly dusted with a fine powder (or breathed upon).

later in 1914 VON LAUE [1.4] published a photograph of Fraunhofer rings obtained from a plate covered with lycopodium powder which clearly shows the radially granular structure noted by EXNER (see Fig. 1.3). EXNER attributed the radial nature of the pattern to the fact that the light source used was not very monochromatic and this was later confirmed by DE HAAS [1.5]; the effect of non-monochromaticity (partial temporal coherence) will be discussed in detail in Chapter 3.

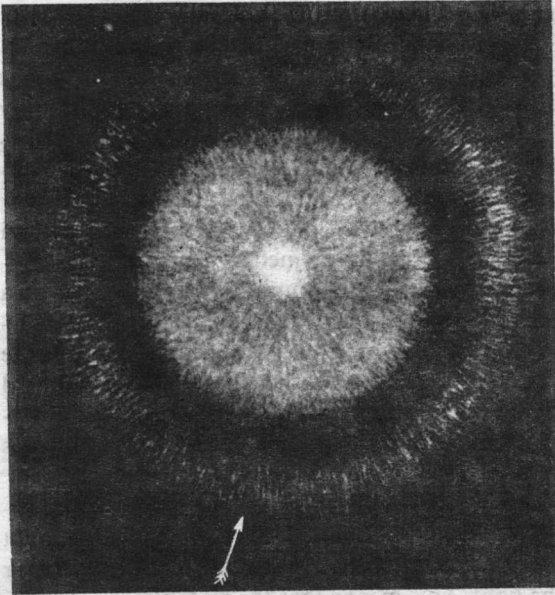


Fig. 1.3. Photograph by M. VON LAUE of the diffraction pattern produced by a glass plate covered with lycopodium powder. Light from a carbon arc lamp was passed through a prism and the region 420–430 nm was used to illuminate the sample

VON LAUE [1.6, 7] gave a fairly complete description of the statistical properties of the speckle pattern found within the central Fraunhofer ring including a derivation of the second-order probability density function and the autocorrelation function of the intensity. Some of this early work has been reviewed by SCHIFFNER [1.8a] and HARIHARAN [1.8b].

At the present time interest in speckle patterns lies in approximately six main areas:

- (i) fundamental statistical properties,
- (ii) reducing speckle in optical and holographic systems.

- (iii) measurement of surface roughness,
- (iv) applications in image processing,
- (v) applications in metrology, and
- (vi) stellar speckle interferometry.

In general, the statistical properties of speckle patterns may be quite complicated and detailed analyses for both coherent and partially coherent light are given in Chapters 2 and 3, respectively. However, if (a) the light is perfectly coherent, (b) the random medium introduces phase fluctuations greater than  $2\pi$ , (c) the medium does not depolarise the light, and (d) a large number of scattering centres contribute to the intensity at a point in the observing plane, then, using the central limit theorem, it is relatively straightforward to show that the intensity at a point in such a speckle pattern has a negative exponential probability density function. The ratio of the standard deviation to the mean is unity for this distribution, so that we might say that the speckle 'contrast' is equal to one; it is also interesting to note that the most probable intensity at any point in such a speckle pattern is zero. The lateral structure of a random pattern is strictly defined in terms of the auto-correlation function. Loosely speaking, the minimum speckle 'size' in a speckle pattern for which the above conditions are valid is approximately equal to the Airy disc that would be produced in the absence of the random medium. This means that the speckle in an optical image has dimensions of the same order of magnitude as the (aberration-free) resolution limit. A more detailed treatment of the basic statistics in monochromatic light is given in Chapter 2.

As we have already remarked, speckle patterns formed in the Fraunhofer plane of a random object illuminated by polychromatic light have a radial structure that depends both on the nature of the scatterer and the bandwidth of the light. This means that the statistical properties vary with overall position in the observation plane (i.e. are non-stationary) and a formal treatment of this is given in Chapter 3. It turns out that the most elegant way to examine this case is using an auto-correlation function in terms of the radial and azimuthal scattering angles rather than the more usual rectangular ( $x, y$ ) coordinates. Partial spatial coherence does not affect the stationarity of speckle patterns but basically lowers the contrast and may cause the speckle 'size' to increase in certain cases. This is also examined in detail in Chapter 3.

It is found that when a Fresnel hologram is made of a non-diffusing object, such as a photographic transparency immersed in a medium of matching refractive index, the quality of the reconstruction is strongly influenced by any scratches or marks on the hologram itself. This influence can be reduced to negligible proportions if a diffuser such as ground glass is placed before the transparency so that the object is

illuminated by a speckle pattern ('diffuse coherent illumination'). Unfortunately we also obtain a speckle pattern on reconstruction. For objects that are inherently optically rough we, of course, always get speckle noise in the reconstruction. Various attempts have been made to reduce speckle in reconstructions of diffusing objects and these are described in Chapter 4. It is, in fact, not possible to reduce the speckle noise in such cases and still retain a coherent image; all methods of speckle reduction involve introducing a certain degree of partial coherence. For non-diffusing objects, however, it is possible to design phase screens that increase the redundancy of the hologram without introducing speckle noise, but we cannot classify this as speckle reduction since speckle is not present in the first place.

In recent years interest has turned from the unwanted aspects towards the uses of speckle patterns and a very wide variety of applications have been found. Perhaps the most obvious application is to the measurement of surface roughness; if a speckle pattern is produced by coherent light incident on a rough surface then surely its statistics must depend on the detailed surface properties. Whilst this is undoubtedly true, it is general very difficult to extract meaningful surface parameters from speckle patterns, especially for very rough surfaces in monochromatic light where the dependence on roughness is almost negligible. This difficulty is common to many optical methods of evaluating surface structure. There are two particular cases in which the extraction of surface roughness information is relatively straightforward: (a) surfaces whose root-mean-square roughness is less than one wavelength illuminated by fully coherent light, and (b) surfaces whose root-mean-square roughness is greater than one wavelength illuminated by polychromatic, spatially coherent light. These cases are discussed in Chapters 2 and 3, respectively.

The other main area of application of speckle patterns concerns their use in what we may broadly term 'information processing'. As we have mentioned in connection with holography, the introduction of a diffuser that scatters uniformly within the angle of acceptance of an optical system (i.e. uniformly over the spatial frequency bandwidth of the system) will code information about an object in an efficient way yielding a high degree of redundancy. Because of this, a speckle pattern may be used as a random carrier of information (e.g. in the detection of the difference between two images) or for data storage. These applications are described in Chapter 5. Another related application of growing importance in engineering is the use of speckle patterns in the study of object displacements and distortion that arise in non-destructive testing of mechanical components ('speckle interferometry' and 'speckle photography'). The key advantage of speckle methods in this case is



that the speckle size may be adjusted to suit the resolution of the most convenient detector whilst still retaining information about displacements on an interferometric scale if required. 'Electronic' speckle pattern interferometry based on the use of a television camera and video-tape recorder has been developed to a high level of sophistication. These applications in metrology are discussed in Chapter 6.

One of the most exciting applications of speckle techniques at the present time is in astronomy. If a short-exposure photograph is taken of a magnified image of an unresolved star (point source) the picture has a speckle-like structure. The speckle is in many ways similar to that produced in the laboratory using a laser, and in particular the speckle size is of the same order as the Airy disc of the telescope. This means that the short-exposure photograph of a resolvable object (e.g. a binary star) contains information about the object down to the diffraction limit of the telescope; this is approximately 0.02 arcsec. for the 5 m Mt. Palomar telescope, whereas the limit usually set by atmospheric seeing for conventional long-exposure photographs is approximately 1 arcsec. This technique has many similarities with the speckle methods used in metrology and is discussed in Chapter 7.

There are a variety of applications in which speckle patterns are used as test patterns for optical and other recording systems such as cameras, photographic emulsions and the eye. These are not discussed in detail in the following chapters and are therefore briefly described below.

If a parallel laser beam enters a camera it is ideally focussed to a small point. If the camera is not focussed at infinity then the film is illuminated by a small roughly circular area. The light reflected back by the diffusing surface of the film may be observed as a speckle pattern at the plane of the lens diaphragm with the speckle size inversely proportional to the size of the illuminated spot. If the spot moves, so do the speckles and the rate of movement is also inversely proportional to the spot size. Thus we have two criteria—spot size and rate of movement—for determining the focus error or the precise location of focus [1.9]. This technique can be extended to the measurement of all the primary aberrations (other than distortion) of the camera/film system.

Another example of the use of speckle patterns as test objects is in the measurement of the modulation transfer function of photographic emulsions [1.10]. In this case a very fine speckle pattern is used as a 'white' noise exposure distribution. Under these conditions it can be shown that the power or Wiener spectrum of the recorded noise pattern is proportional to the square of the modulation transfer function of the photographic emulsion. This is a very simple and fast way of measuring this function and does not rely on the use of sinusoidal charts which tend to be difficult to construct and use.