Modern Network Analysis

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PREFACE

Undergraduate students in electrical engineering ordinarily have completed a number of networks courses. These include the usual first courses in a-c and d-c circuits; a course in network analysis which might include some work on general networks, filters, and transmission lines; a course or more in electron-tube circuits; and perhaps even some required or elective work in networks or advanced electronics. All of these serve to provide the student with a detailed study of selected network topics. Generally, however, these studies of networks are done from a limited viewpoint. Little emphasis is placed upon the system function, the pole-zero configuration of the system function, the independent loop currents or node-pair potentials and the associated topological considerations, and the ties between linear systems theory and the mathematics of matrices and linear algebra.

In planning the contents of this book, the authors set as their goal a text that would provide the student with a broad understanding of modern network analysis at a modest level of sophistication. Such an understanding is important in itself, and also, it provides the background necessary for subsequent work in the field of network analysis and synthesis. The book may be used to provide the needs of (1) the first graduate course on linear networks for those who have not been fully exposed to the ideas of modern network theory, or (2) a course at the advanced undergraduate level where an introductory course on the subject having a similar philosophy is available. The book does not seek to provide a high level of mathematical development for those who might have professional interest in network theory. Our object is, in fact, to provide a link between modern introductory circuit theory texts (such as those by E. A. Guillemin, R. Scott, and others) and the professional work of M. Bayard, H. W. Bode, W. Cauer, R. M. Foster, and E. A. Guillemin.

As usually presented, the study of linear systems proceeds from considerations of circuits involving passive R, L, C elements. The integro-differential equations that arise through an application of the Kirchhoff potential and current laws are the basis for a generalized treatment.

Often at some subsequent point in the development the exponential excitation of the form Ee^{st} or the Laplace transform is introduced. This ultimately leads to a discussion of the complex-frequency or s plane, which is of fundamental significance in network analysis and synthesis.

In the present text, the viewpoint is adopted that the ordinary linear elements are representative of constants, differentiating devices, and integrating devices. Such linear elements constitute the building blocks for network analysis or network synthesis. Intimately involved, therefore, is the study of a system of interconnected elements. To demonstrate the interconnection aspects of the physical array, it has been found convenient to use the theory of linear graphs, a branch of topology. Moreover, since Kirchhoff's equations associated with a linear graph are studied most conveniently by means of the methods of linear algebra, the techniques of matrix methods are introduced early. Furthermore, the analysis of networks proceeds most conveniently when referred to the complex-frequency or s plane, and therefore the s plane, and its place in network theory, is introduced early in the text. Thus what is done, following an introductory chapter which distinguishes between linear and nonlinear elements and networks containing such elements, is to introduce the linear system from the broad viewpoint and then to introduce in successive chapters the s plane, and with it certain of the elements of complex-function theory, the properties of matrices and their use in the formulation of network problems, and finally, sufficient network topology to permit an intelligent study of the geometrical properties of electrical networks. The network topological considerations of linear graphs yield directly information concerning the constraints on the system, the number of independent variables, and an insight into the natural modes of a network and the functional structure of system functions.

With the s plane and its relation to network theory understood, the relation of the exponential excitation to any general excitation is stressed. The use of the complex variable s proves to be considerably more general than the use of $j\omega$, which is dominant in the texts of the previous decade. Similarly, with a knowledge of matrix methods, any problems which involve a study of matrix algebra (such as network formulations by Kirchhoff laws, two-port networks, and transmission lines) are direct applications.

Considerable emphasis is directed to the role of the system function **T**(s) both in the steady-state and in the transient-response performance of a system. It is shown that the knowledge of **T**(s) is necessary and sufficient to the complete description of the performance of the system, and this, together with the general exponential excitation, which had previously been shown to be sufficiently general to include most important specified excitations, permits a completely general formulation of the system response. The final compact expression that is deduced for the initially relaxed network is a form of the Heaviside expansion theorem, although this is accomplished without the mathematical processes employed using the Heaviside or the Laplace-transform methods. Provision is also made for the solution of systems which are not initially relaxed.

A summary of the Laplace-transform method as a tool in network analysis serves to formalize the understanding of the s-plane analysis, the significance of the system function **T**(**s**), and the system response to any excitation, including singularity functions. However, we have not based our presentation on Laplace-transform techniques. A thorough coverage of this theory requires an adequate knowledge of the convergence and contour integration in the complex plane. Such an acquaintance with function theory is not generally provided in the undergraduate curriculum. Moreover, as has been elegantly demonstrated by E. A. Guillemin in his "Introductory Circuit Theory," it does not seem necessary to proceed with a long introductory discussion of Laplace-transform theory, since the same results can be obtained by the simple classical exponential function.

An important feature of the book is the consideration of the properties of system functions. Among these are the natural modes of a network, the general aspects of stability and physical realizability, and the special properties of networks containing only two types of elements.

Because of the extent to which the present study must go to cover the limited class of linear bilateral networks, nothing has been said about nonlinear networks, and only little has been done with networks which include dependent sources. A significant omission related to this latter topic is the lack of a discussion of signal flow graphs and their methods of analysis. Such discussions would have extended the present work beyond the original plans of the authors. Likewise, it was felt that a coverage of the recent interesting and useful methods of scattering matrices would also carry the book beyond the original plans. In short, we have tried to present the material for an intermediate-level course on modern linear systems.

The book is organized and divided into three parts. Part 1, which comprises Chaps. 1 through 5, presents a systematic introduction to the study of network analysis, primarily under steady-state exponential excitation. Part 2, which includes Chaps. 6 through 8, discusses some of the classical applications of Part 1 to filters and transmission lines. Part 3, comprising Chaps. 9 through 13, presents a study of linear networks in the transient state and also a number of consequent general properties of such systems. If time is not available to study the entire book, Parts 1 and 3 should be studied in sequence, as these two parts contain the essential aspects of network analysis. Part 2 may be omitted or deferred without disturbing the continuity of the study.

We wish to acknowledge helpful suggestions by many of our colleagues. Particular thanks are due to Mr. Hwei-Piao Hsu of Case Institute of Technology for his careful checking of the book while in note form. We would be remiss if we did not acknowledge the influence of Professor E. A. Guillemin of the Massachusetts Institute of Technology in the development of our text, both of us being intimately acquainted with his writings, one of us (F. M. R.) having been his colleague for a number of years.

The book has been classroom-tested at Syracuse University and at Case Institute of Technology. We are indebted to many of our colleagues for their generous help in this connection.

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CONTENTS

PART 1

FUNDAMENTALS OF NETWORK ANALYSIS

CHAPTER 1

PRELIMINARY CONSIDERATIONS

This chapter serves to introduce some of the concepts and some of the terminology which will be encountered throughout the book. Since our study will be limited to linear networks, techniques are given for testing whether the networks are linear. Such tests are shown to apply whether the network behavior is formulated in differential-equation or in integral-equation form.

To establish a systematic method for the solution of network problems necessitates a means for selecting a set of independent loop-current or node-pair potential variables. A powerful method involving a study of the linear graph of the network is introduced, although the details of the topological considerations of such network graphs are deferred until a later chapter.

1-1. Classes of Networks. The term *network theory* has come to mean a variety of things, and in the interests of clarity it is necessary to specify what the term is intended to encompass in the present work. As will be discussed at some length in what follows, a network will be composed of a number of network elements. Network elements may be linear, bilateral, and passive. Other network elements may be nonlinear; some are passive and others are active. The exact definition of these terms will be given below, but for the present it is necessary only to understand that a network is an interconnected aggregate of such elements. The study here contemplated is that of the behavior of the system as a whole and not a study of the elements. Thus, while the elements which make up the network will be carefully classified, the details of these elements will not be of direct concern to us. Of major interest will be the behavior of the over-all system with respect to specified terminals.

General network systems can usually be classified into three essential categories, these being dependent upon the character of the elements contained therein. These may be designated for convenience as class A, class B, and class C systems. In class A systems, all elements of the system are linear, whence the system is said to be linear. Ordinarily an element or a system is said to be linear when its behavior can be characterized by a linear differential equation. That is, if $\theta_i(t)$ and $\theta_o(t)$

denote respectively the stimulus or input and the response or output functions of a linear system as functions of time, then $K\theta_i(t)$ and $K\theta_o(t)$, with K a constant, will be an admissible pair of excitation and response functions for the same system. Moreover, if $(\theta_{i1},\theta_{o1})$ denotes one admissible input-output pair function, and if $(\theta_{i2},\theta_{o2})$ denotes a second admissible input-output pair function, then for a linear system, $(K_1\theta_{i1} + K_2\theta_{i2}, K_1\theta_{o1} + K_2\theta_{o2})$ will also denote an admissible input-output pair function, where K_1 and K_2 are constants.

If one or more of the elements of the system are nonlinear, then two subclasses of networks, class B and class C, are possible. In class B networks it is convenient to linearize the nonlinear element, at least over a range of operation, or to replace the nonlinear element by a roughly equivalent piecewise-linear element. In this case, a substantially linear network-analysis technique may be employed in the subsequent analysis of the network behavior. In class C networks, a linear equivalent model cannot be found within a permissible range of approximation, and the special techniques of nonlinear analysis must be employed. We shall limit ourselves to linear systems.

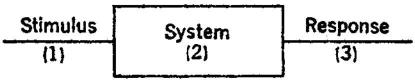


FIG. 1-1. The essential factors in network theory.

Suppose that the system is illustrated in block form, as in Fig. 1-1. Three positions are designated in this figure: (1) the input (stimulus), (2) the system, and (3) the output (response). Two fundamental classes of network problems exist:

a. Network analysis—given (1) and (2), to find (3)
b. Network synthesis—given (1) and (3), to find (2)

Clearly, therefore, the study of linear systems must actually be divided into two main studies, analysis and synthesis.

Attention is called to the limited objectives of the present volume. We shall be concerned principally with the development of methods for the analysis of systems composed of linear lumped bilateral elements. Also some consideration will be given to a study of systems composed of distributed linear bilateral elements. Network synthesis is outside the scope of this book. But since synthesis bears heavily on the general properties of networks which are deduced from an analysis of the general behavior of networks when subject to specified stimuli, many of the general aspects of network functions which will be developed in our study are of considerable importance to the synthesis problem.

1-2. Circuit Terminology. To avoid ambiguities in terminology, the more important terms are defined. When these terms are subsequently used, they will be used in the sense given by these definitions.

Ideal Potential Source. An ideal potential source is a hypothetical device which maintains the potential† at its terminals according to some prescribed constraint (law of variation) independently of the current in its leads, as long as the current is finite. When the current is infinite, as for a short-circuited source, the notion of an independent source becomes meaningless. In general for the ideal potential source, the law of variation is a characteristic of the source—it may be a sinusoidal function of time, it may produce a sequence of pulses of specified shape, it may be a constant, etc. For example, the specification that $e(t)$ in

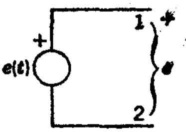


FIG. 1-2. An ideal potential source.

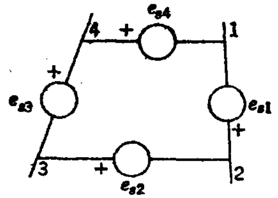


FIG. 1-3. Potential sources with internal constraints.

Fig. 1-2 is an ideal potential source means that the potential difference between the terminals indicated will maintain the specified form no matter what the nature might be of a system connected to the specified terminals, if a finite source is assumed. The reference polarity denoted by the symbol }+ establishes the relative potential of the terminals. This means that if the symbol which represents the potential is positive, the polarity corresponds to the reference polarity. In general, potential differences are specified in the customary mks unit, the *volt*.

Attention is called to the possibility that a number of apparently independent potential sources may not, in fact, all be independent. For instance, the four sources of Fig. 1-3 are not independent. In this illustration, an internal linear constraint exists among the sources and is such that the potential sources between junctions 1-2, 2-3, and 3-4 impose a constraint on the potential source between the junctions 1-4. This constraint of the potential may or may not be equal to e_{s4}.

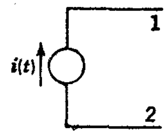


FIG. 1-4. An ideal current source.

Ideal Current Source. An ideal current source is a hypothetical device which maintains the current at its terminals according to some prescribed constraint (law of variation) independently of the terminal potential. The law of variation is characteristic of the source.

Refer to Fig. 1-4, which illustrates a current source that maintains a specified current through its terminals. The specified current will be

† For convenience in writing, the term *potential* will be used instead of the term *potential difference*. Potential difference is always implied by the term potential.

maintained at the terminals independently of the character of the network attached to these terminals, as long as a finite potential is prescribed. The reference current direction is denoted by the arrow ↑, and when the symbol which represents the current is positive, the direction of the current is in the reference direction. In general, currents are specified in the mks unit, the *ampere*.

As the dual of the statement for ideal potential sources, a number of concurring ideal current sources at a junction may impose a constraint on the system which will invalidate the basic constraint of one of the ideal current sources. It is also noted that the occurrence of ideal current and ideal potential sources together in a network may invalidate the basic constraints.†

Circuit Element. The circuit elements are the individual components which make up the circuit. A circuit element possesses two terminals.

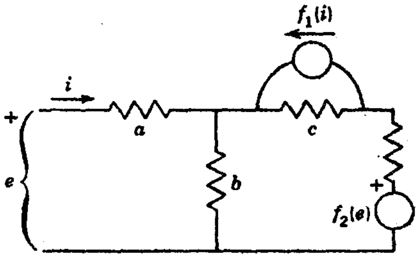
Network. A network is a group of interconnected circuit elements. The term *network* is often used synonymously with circuit, although it often has a somewhat more general connotation than the term *circuit.*

Lumped Network. A lumped network is one which may be represented as an arrangement of physically separate resistors, capacitors, and inductors.

Distributed Network. A distributed network is one in which the resistive, capacitive, and inductive effects are inseparable for analytical purposes. A transmission line is an example of a distributed network.

Branch. A branch is any group of elements and sources which may be combined to form a device with two terminals. Clearly, a branch may contain a number of circuit elements and perhaps sources connected together. Often when the internal arrangement of elements is of no immediate importance, the branch may be denoted by a rectangle, with only the two terminals being shown.

† It is noted that the above ideal sources are assumed to be independent sources. However, it is possible to have dependent ideal sources in a circuit. For example, in



the circuit shown, the source $f_1(i)$ is a current source whose value depends on the current i. Similarly, the source $f_2(e)$ is a potential source whose value depends on the excitation potential e.

Passive Branch, Element. A branch in which there are no sources is called a passive branch. An element which is not a source is a passive element.

Active Branch, Element. A branch in which there are one or more sources is an active branch. There may be passive elements in an active branch. An independent potential or current source is an active element. Vacuum tubes and transistors are also active elements, but these are not ideal sources, as described above. In fact, these form a very important class of two-port or four-terminal devices containing so-called dependent sources. In these devices the behavior of the internal source depends on the currents or potentials in some part of the device due to independent exterior sources.

TABLE 1-1. THE LUMPED CIRCUIT ELEMENTS AND THE PARAMETERS

Element		Parameter		Unit
Symbol	Name	Symbol	Name	
	Resistor	R	Resistance	Ohm
	Inductor	L	Inductance	Henry
	Capacitor	C	Capacitance	Farad
	Potential source	e	Potential	Volt
	Current source	i	Current	Ampere

Linear Element, Branch. The term *linear* may be applied to either an element or a branch. A linear device is one which is governed by a linear differential equation for all values of applied stimulus and expected response.

Bilateral and Unilateral Elements, Branches. Bilateral elements are those which pass current equally well in either direction. Unilateral elements are those which have different laws relating potential and current for different directions of current. In general, elements which are made of the high-conductivity materials are bilateral. Vacuum tubes, crystal rectifiers, certain semiconducting assemblies (special combinations of copper and copper oxide, iron and selenium) are unilateral.

It is interesting to note, following the above definitions, that a linear element must be bilateral. However, a bilateral element is not necessarily linear. For example, an iron-core inductor is bilateral, but it is nonlinear.

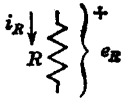
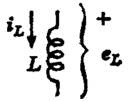
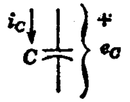
Node (Junction). The connection of two or more branches to a common point forms a node or junction.

Loop (Mesh). Any closed circuit of branches is a loop.

Parameters. Circuit elements are involved in networks; their effects are reckoned in terms of defining mathematical relations. The symbolic representation of the physical device is called the circuit parameter. The relation between the circuit element, the parameter, and the appropriate unit is illustrated in Table 1-1.

1-3. Basic Ideal Lumped Linear Elements. There are three basic linear elements: resistors, inductors, and capacitors. These could be lumped or distributed, but they are considered to be lumped elements in this section. The symbols for the basic elements and the behavior of these are defined by the tabulation in Table 1-2, which relates the currents

TABLE 1-2. THE CIRCUIT NOTATION FOR IDEAL LUMPED ELEMENTS†

Element	Circuit designation	Analytic relation
Resistor................		$e_R = Ri_R$
Inductor...............		$e_L = L\frac{di_L}{dt}$
Capacitor..............		$e_C = \frac{1}{C}\int_0^t i_C\,dt$

† The analytic relations given here apply for initially relaxed performance. Otherwise, terms representing the currents or potentials at the time of application would have to be included. The time $t = 0$ represents the reference time for the application of the excitation.

in and the potentials across each element. Attention is called to the fact that the reference polarity and reference current direction are essential parts of the circuit designation.

Example 1-3.1. An ideal potential source which yields an output as shown in Fig. E 1-3.1*a* is applied to the parallel network. Determine graphically:

(*a*) The current in the ideal resistor R of 3 ohms.

(*b*) The current in the ideal inductor L of 2 henrys.

(*c*) The total current supplied by the source.

Solution. (*a*) From the general relation $e = Ri$, we have $i = e/R$, whence the result is that illustrated in Fig. E 1-3.1*b* and consists of a change in scale without affecting the shape of the curve.

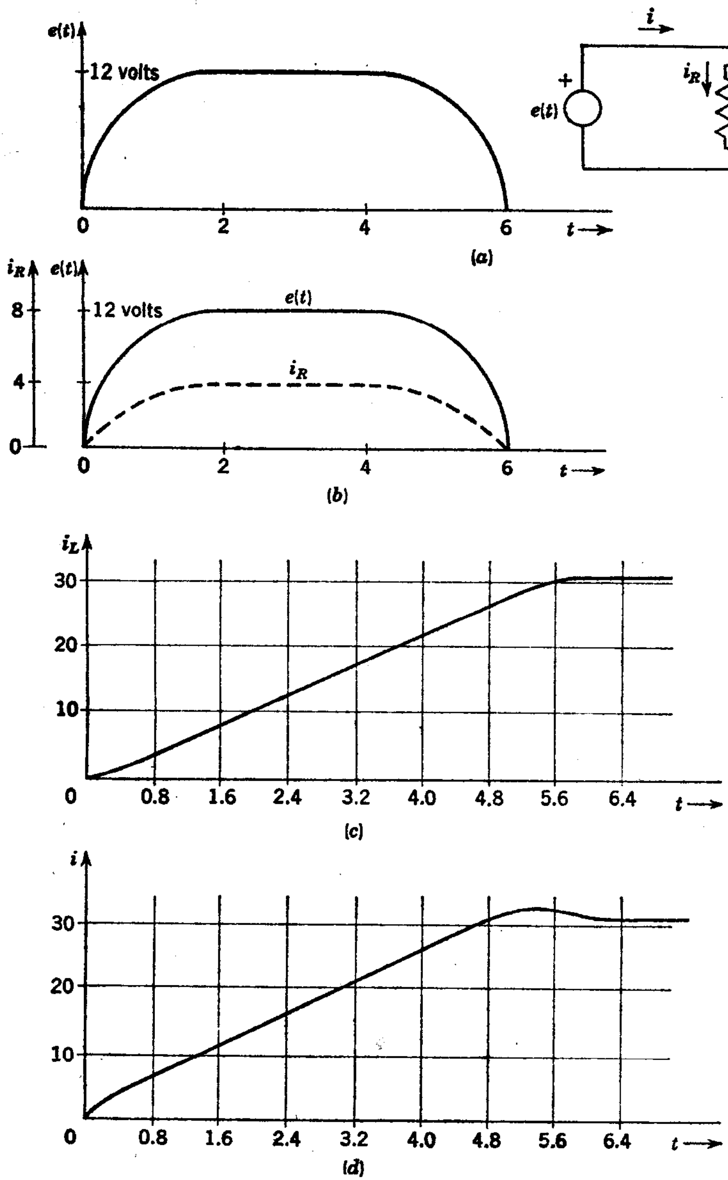

e(t)
12 volts
0
2
4
6
t
i
+
e(t)
i_R
3Ω
i_L
2h
(a)
i_R
e(t)
8
4
0
12 volts
e(t)
i_R
0
2
4
6
t
(b)
i_L
30
20
10
0
0.8
1.6
2.4
3.2
4.0
4.8
5.6
6.4
t
(c)
i
30
20
10
0
0.8
1.6
2.4
3.2
4.0
4.8
5.6
6.4
t
(d)

FIG. E 1-3.1

(*b*) The current in the inductor is related to the potential across it by the expression

$$i_L = \frac{1}{L}\int_0^t e_L\,dt = \tfrac{1}{2}\int_0^t e_L\,dt$$

This integral may be evaluated by graphical integration of the function $e(t)$. This necessitates finding the area under the $e(t)$ curve. The result has the form illustrated in Fig. E 1-3.1*c*.

(*c*) The total current is obtained by a simple application of the Kirchhoff current law at the junction of the source, resistor, and inductor. This requires that

$$i(t) = i_L + i_R$$

This result is obtained by combining i_R deduced in part *a* with i_L deduced in part *b*. The result is shown graphically in Fig. E 1-3.1*d*.

Example 1-3.2. An ideal current source is connected to an ideal capacitor, as illustrated in Fig. E 1-3.2*a*. The waveform of the applied current has the form specified. Sketch the potential across the capacitor.

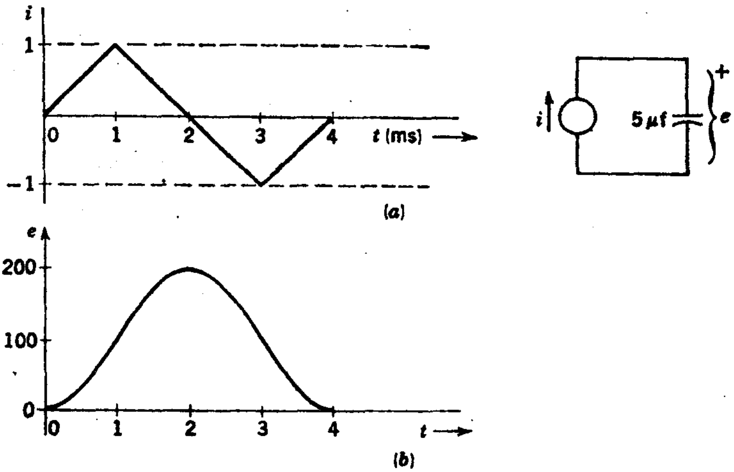


FIG. E 1-3.2

Solution. The potential across the capacitor is related to the current through it by the relation

$$e(t) = \frac{1}{C}\int_0^t i\,dt = 2 \times 10^5 \int_0^t i\,dt$$

In the present case, the value of the integral may be deduced either analytically or graphically. The result has the form shown in Fig. E 1-3.2*b*.

The fundamental law of linear systems must apply for the basic linear passive lumped bilateral elements. This requires, as noted in Table 1-2, that the three linear elements can be expressed by relations of the form

$$e = f(i) \tag{1-1}$$