CONTROL AND SYSTEMS THEORY Volume 5

An Introduction to Linear Control Systems

Thomas E. Fortmann
Konrad L. Hitz

An Introduction to Linear Control Systems

Library of Congress Cataloging in Publication Data

Fortmann, Thomas E.
An introduction to linear control systems

(Control and systems theory ; v. 5)
Bibliography: p.
Includes index.
1. Automatic control. 2. Control theory.
I. Hitz, Konrad L., joint author. II. Title.
TJ213.F62 629.8'32 76-29499
ISBN 0-8247-6512-5

MARCEL DEKKER, INC.

270 Madison Avenue, New York, New York 10016

Current Printing (last digit):

10 9 8 7 6 5 4 3 2 1

PRINTED IN THE UNITED STATES OF AMERICA

CONTROL AND SYSTEMS THEORY

A Series of Monographs and Textbooks

Volume 1: Discrete Techniques of Parameter Estimation: The Equation Error Formulation, JERRY M. MENDEL

Volume 2: Mathematical Description of Linear Systems, WILSON J. RUGH

Volume 3: The Qualitative Theory of Optimal Processes, R. GABASOV *and* F. KIRILLOVA

Volume 4: Self-Organizing Control of Stochastic Systems, GEORGE N. SARIDIS

Volume 5: An Introduction to Linear Control Systems, THOMAS E. FORTMANN *and* KONRAD L. HITZ

OTHER VOLUMES IN PREPARATION

PREFACE

This book is intended for a two-semester introductory course in control systems at the advanced undergraduate or graduate level. It assumes as a prerequisite some familiarity with differential equations, linear algebra, and Laplace transforms. With appropriate omissions it is also suitable for shorter courses, and it is sufficiently self-contained to be used for self-study. Students in all branches of engineering and in mathematics should feel equally comfortable with it. At the University of Newcastle, for example, it forms the basis for a two-semester junior-year engineering sequence in which students may choose to take one or both semesters. It is also being used in draft form at M.I.T. for a senior/graduate level course.

One author (K. L. H.) combines several years' experience in the design and implementation of classical control systems with current research interests involving modern control and optimization theory. The other (T. E. F.) has a more theoretical background spanning modern control, optimization, estimation, and computer systems. This text grew out of two jointly held convictions about the teaching of control.

First, the linear theory and many of the basic analysis and synthesis techniques underlying both modern and classical control systems can and should be integrated and taught jointly. Moreover, much of what has been called "advanced" modern state-space analysis is no more mathematically advanced than the traditional transform approaches and can be safely included in an introductory course (a rigorous proof of Nyquist's theorem, for example, requires mathematics every bit as sophisticated as does a derivation of the Jordan canonical form). Accordingly, the different points of view — time-domain and transform methods, state-space and input-output models — should be developed side by side and linked together as closely as possible.

Second, an introduction to control systems should provide the student with a solid understanding of the fundamental mathematical principles, a clear perception of the physical problems that motivate the theory, and a familiarity with the most important analysis and design techniques, whether his ultimate objectives lie in practical hardware design, theoretical research, or somewhere in between. In addition, it should point out extensions, advanced topics, and research areas that lie beyond the introductory material.

The book's organization and content reflect these convictions. Part I deals principally with the mathematical description of physical systems. After a brief history of control systems and an example to motivate the use of feedback in Chapter 1, differential equations in both state-space and input-output form are introduced in Chapter 2. The modeling of physical devices and systems with such equations, including linearization of nonlinear models, is explored using various examples. These and other examples are developed further in order to illustrate additional material in subsequent chapters.

Part II concerns the solution of linear, time-invariant, differential equations and those properties of solutions which are relevant to control systems. Time-domain and Laplace transform solutions are derived in Chapters 3 and 4, respectively, and their equivalence is carefully pointed out. Chapters 5 and 6 are complementary: one deals with stability and other aspects of transient solutions, the other with steady-state and frequency response. Solution of differential equations by means of analog and digital computation is covered in Chapter 7.

Part III explores feedback control in some depth. Chapter 8 covers single-loop feedback control: steady-state accuracy, root-locus analysis, and Nyquist's stability criterion. Chapter 9 discusses performance specifications for single-loop systems and then introduces compensation as a means of improving performance. Multivariable systems and their structural properties are the subject of Chapter 10, including controllability and observability, canonical forms, realizations, and the significance of pole-zero cancellations. Chapter 11 covers multi-loop control: pole placement using state feedback, steady-state accuracy, and decoupling. Observers are introduced in Chapter 12, including use of the state estimate in a feedback control law and a short digression to discuss the Kalman-Bucy filter as a particular example of an observer.

Part IV contains relatively brief excursions into linear optimal control, time-varying systems, and discrete-time systems (Chapters 13-15), followed by a survey of more advanced topics and research areas in Chapter 16. The bibliography is extensive and includes articles appearing through mid-1977.

For students who need a bit of review or practice in linear algebra or Laplace transform theory, Appendices A and B provide concise but complete summaries of the concepts and results in these areas which are relevant to the main text.

Problems are provided at the end of each chapter, grouped by section; these range from mundane to challenging, and a few are intentionally vague to encourage free thinking. Many of these request completion or extension of material in the text. We recommend reading all of the problems even if solutions are not written out formally. Solutions have been found at one time or another for most of the difficult ones (a solutions manual will be published if demand is sufficient), and only one is known to be a potential thesis topic.

A great many people have influenced and assisted in the preparation of this book. We are especially grateful to all of the following:

- Brian Anderson and Shelly Baron, whose encouragement and support were indispensable.
- Lorraine Pogonoski, who typed and corrected draft after draft with great skill and patience; Pauline Levreault, whose splendid final copy speaks for itself; Peter McLauchlan, who drew many of the figures; and Lorraine King, Ann Sarajian, and Brenda Aighes, who typed various sections of the drafts.
- Tim Johnson, Ian Rhodes, Mike Athans, Vishu Viswanathan, Dick Pew, Nils Sandell, Al Willsky, Rob Evans, Dave Kleinman, Ted Davison, and Jeff Berliner, who provided many helpful comments and criticisms.
- Ramal Muralidharan and Oliver Selfridge, who took on the thankless task of proofreading the final copy.
- Carla Fortmann and Janice Hitz, who provided love and good spirits when they were needed most.

Thomas E. Fortmann
Lexington, Massachusetts

Konrad L. Hitz
Newcastle, Australia

ORGANIZATION AND NOTATION

This book contains 16 chapters and 2 appendixes, each of which is subdivided into sections such as 2.3 and 9.2. Theorems, examples, definitions, and the like are numbered consecutively as they appear within each section. Equations are numbered in the same way, as are figures. Problems appear at the ends of the chapters, numbered separately for each section. References to items within a section use a single number, for example, Theorem 5, Equation (8), Fig. 3, Problem 6. References to items in a different section include its number, e.g., Definition 8.4.3, Equation (4.3-12), Fig. 2.4-3, Problem A.3-9. The ends of theorems, examples, etc. are denoted by ΔΔΔ. References to the bibliography at the end of the text appear in the form [312] or Ref. 86; those below 200 refer to books, those above 200 to papers.

Standard notation is used for proportional $\propto$, approximately equal $\approx$, identically equal $\equiv$, definition $\triangleq$, much less than $\ll$, much greater than $\gg$, implication $\Longrightarrow$, and equivalence $\Longleftrightarrow$. Vectors $\underline{y}$ and matrices $\underline{C}$ are indicated by underscoring; other vector/matrix notation may be found in Appendix A.

Standard set notation is also used. If A and B are sets, then $b \in B$ means that b is a member of the set B, and $A \subseteq B$ means that A is contained in B. If, for example, B is the set all real numbers, then $\{b \in B: b \geq 0\}$ denotes the set of all nonnegative real numbers. The intersection and union of A and B are denoted $A \cap B$ and $A \cup B$, respectively.

If z is a complex number, its conjugate, magnitude, and angle are denoted $\bar{z}$, $|z|$, and $\angle z$, respectively. Derivatives of a function x(t) with respect to time t are denoted as follows,

$$\dot{x} \triangleq \frac{dx}{dt} \qquad \ddot{x} \triangleq \frac{d^2x}{dt^2} \qquad \cdots \qquad x^{(n)} \triangleq \frac{d^nx}{dt^n}$$

When the context is clear, we will assume that the statement $x(t) = y(t)$ includes the implicit proviso, "for all t in the region of interest," and that x(t) refers to a whole function $x(\cdot)$ rather than its value at a single point. The unit step function 1(t) is defined as

$$1(t) \triangleq \begin{cases} 0 & (t \leq 0) \\ 1 & (t > 0) \end{cases}$$

CONTENTS

Part IV: Related Topics

Part 1

DESCRIPTION OF DYNAMIC SYSTEMS

Chapter 1

INTRODUCTION

1.0 INTRODUCTION

A cornerstone in the development of modern technology has been man's capacity to construct devices which automatically control or regulate the operation of an enormous range of machines and processes. This has allowed him to operate plants which would otherwise require an intolerable amount of monotonous effort, and it has allowed him to build and use machines whose control is quite beyond his own physical capacity. Consider transportation: a horse-drawn coach could be quite adequately handled by the coachman, using simple and direct controls. Operation of a modern car is still within the physical capacity of most people, although power-assisted steering and braking has become very common. A large aircraft, on the other hand, simply cannot be flown without the assistance of various devices which automatically regulate the operation of the engines, and which translate the pilot's manual actions into movements of huge control surfaces requiring very large forces.

The design of devices which monitor and regulate the operation of machines and processes has become known as *control engineering*. Like most engineering disciplines, it began as an art, practiced by gifted craftsman-engineers with an unusual amount of common sense and ingenuity. However, the early inventions soon led to