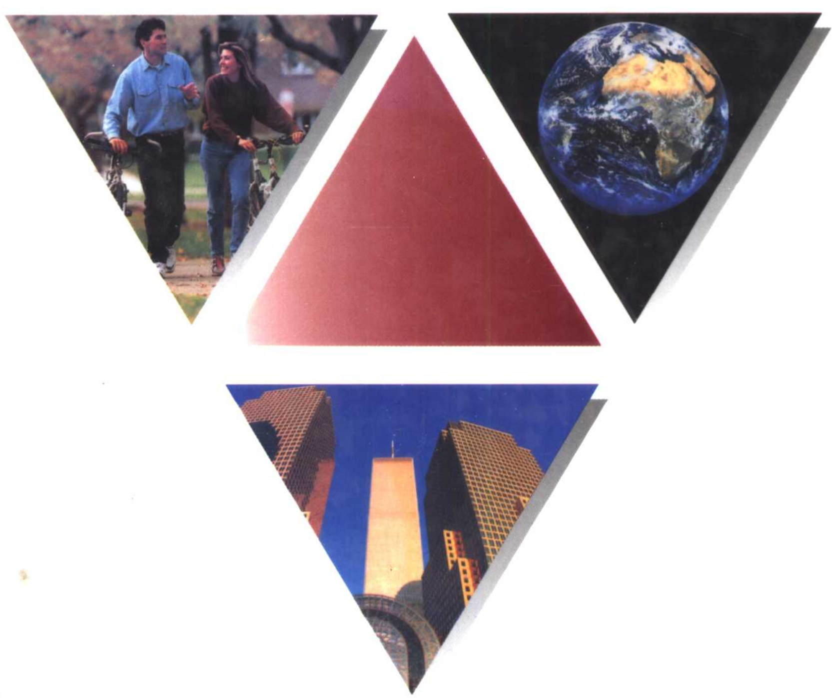
MATHEMATICS IN LIFE, SOCIETY, & THE WORLD



HAROLD PARKS / GARY MUSSER
ROBERT BURTON / WILLIAM SIEBLER

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PREFACE

Traditionally, mathematics departments throughout the country have devoted much of their undergraduate teaching effort to teaching calculus and calculus preparation. However, in recent years, mathematicians have a renewed commitment to students not majoring in science and engineering. This book is designed to be the textbook in such a course. In writing this book, we have kept before us certain goals.

Relevance. The title of the book ***"Mathematics in Life, Society, and the World"*** sums it up. We cover topics that play an important role in everyday life (for example Chapter 6-*Consumer Mathematics*), in civic life (for example Chapter 9 - *Voting and Apportionment*), or in their general appreciation of the world (for example Chapter 11 - *Growth and Scaling*). In class testing the material in this book, we have noticed that students pick up on the relevance of the material and the question "What is this good for?" is seldom heard.

Accessibility. In writing the text we have sought to make the material accessible by developing topics in the most logical and compelling manner. This involves isolating the truly important points and presenting them without needless technical complication. We have avoided writing any impressively complicated derivations, and have tried to make it possible to solve everything with an inexpensive calculator (that can compute x^y), some graph paper, and working knowledge of high school algebra.

Pedagogy. All the material in this book has been class tested many times. The book contains more exercises and problems than most texts of its kind. The exercises and problems, including the applied problems, have a wide range of difficulty so instructors can tailor the assessments to their classroom needs.

OUTLINE

The text is organized into three Parts (12 Chapters). The material in Chapter 2, "*Numbers and Numeracy*" is used throughout later chapters of the book, but the other chapters are relatively independent. Review topics are included near the end of the book for students who need a brief refresher.

Part I–Mathematics in Life. These chapters consist of the mathematics a student will encounter on a regular basis. Chapter 1, *Critical Thinking, Logical Reasoning, and Problem Solving,* sets the tone for this part by showing how mathematics appears in the media (often incorrectly). Then, reasoning patterns are introduced so that students can learn to analyze and construct sound arguments and to recognize incorrect reasoning. Problem solving strategies are presented to aid students in solving problems. Chapter 2, *Numbers and Numeracy,* develops the number systems that are interwoven throughout the book. Our approach to this topic is new in that numeracy is integrated throughout, especially in the coverage of mental math and estimation. Chapters 3-5 provide rich contexts in which statistics and probability are developed. These chapters contain many everyday uses of mathematics. Chapter 6, *Consumer Mathematics,* contains most of the mathematics that students will typically use in their personal financial dealings.

Part II–Mathematics in Society. Composed of three modern topics whose mathematics underlies much of the social structure and interactions around us. Chapter 7, *Game Theory,* shows how games of a social nature can be analyzed and how optimal winning strategies can be developed. Chapter 8, *Management Mathematics,* provides many interesting applications where the techniques involving linear programming and networks can be used to solve problems, especially in the business world. Chapter 9, *Voting and Apportionment,* contains an analysis of a variety of strategies that may be used by politicians to bring "fairness' into a democratic system.

Part III–Mathematics in the World. These three chapters are geometrical in nature. Since much of the world, most notably in the large-scale structure of objects and living beings, is governed and described by geometry in one way or another. Chapter 10, *Geometry,* develops the mathematics behind a variety of patterns in the world, including the analysis of tilings. Conic sections are also studied due to their underlying importance in naturally occurring phenomena. Chapter 11, *Growth and Scaling,* provides many real world applications which illustrate the importance that growth and decay have in Chapter 12, *Recursion and Fractals,* provides insight into these modern topics by showing how they occur in the world.

Throughout the book, we have sought to be faithful to recommendations of our professional organizations such as the MAA, the AMATYC, and NCTM.

ACKNOWLEDGMENTS

A leading force in the development of the type of course for which this textbook is intended has been the Consortium for Mathematics and Its Applications, and we acknowledge their valuable pioneering efforts. We have benefitted from student feedback as we have tested these materials, and we thank those students for their help and advice. We also thank the Oregon State University Department of Mathematics and its Chair, Professor Francis Flaherty, who have been very supportive of our efforts.

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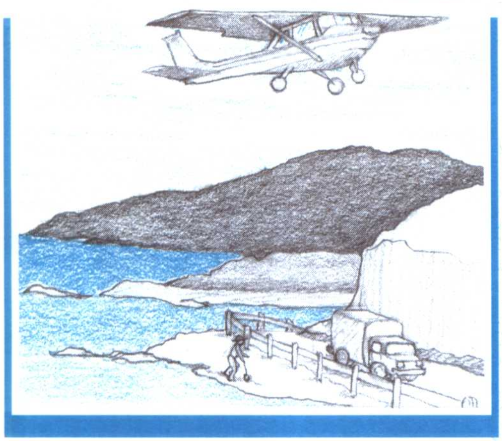
CHAPTER 12

RECURSION AND FRACTALS



MARINE BIOLOGIST COMPLETES 3000 MILE TREK ON OREGON COAST

In what may be the most ambitious study of the Oregon coast ever undertaken, Hal Walker, Ph.D., has just completed the first inventory of all accessible tide pools on the Oregon coast. Dr. Walker covered the entire coastline of Oregon by foot—from the California state line, near Brookings, to Astoria at the mouth of the Columbia River. In following the coastline, the scientist tried to stay within one yard of the water's edge except for impassable stretches and dangerous rocks. Bridges were used only when rivers and bays were too deep to be waded. To measure the distance covered, Dr. Walker used a pedometer, recording 2,928.5 miles by the trip's end. For the return trip from Astoria to Brookings, he used a small plane which hugged the coastline at low level. The scientist said the 317 mile air trip was the most beautiful he had ever taken. The support team from the university used a van to return to Brookings on Highway 101, a distance of 339 miles.

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Chapter opening anecdotes offer realistic media stories to illustrate how mathematics can be used to solve real problems.

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Chapter Goals: Summarize the main ideas students should be learning within the chapter.

CHAPTER GOALS

1. Solve problems involving similar triangles.
2. Transform geometric objects into larger or smaller objects of similar shape through scaling.
3. Compute quantities such as perimeter, area, volume, and weight of scaled objects.
4. Model population growth, radioactive decay, and other quantities that change in proportion to their size.
5. Model physical objects and compute some related quantities.

Initial Problem: Each Section within a chapter begins with a problem which draws upon the skills a student will learn within the section.

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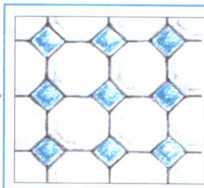
10.1 TILINGS

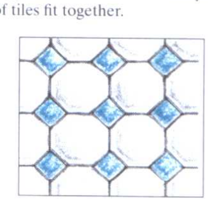
INITIAL PROBLEM



A ceramic tile wall composed of two different shaped tiles is shown. Explain why these two types of tiles fit together.

INITIAL PROBLEM SOLUTION

A ceramic tile wall composed of two different shaped tiles is shown. Explain why these two types of tiles fit together.



SOLUTION The angle measure in a square is 90° and in a regular octagon is 135° (Table 10.1). Each vertex contains a square and two regular octagons, and $90° + 2(135°) = 360°$. Thus the three tiles will fit at each vertex and, as long as their sides are the same length, will tile a wall. This is one of the eight semiregular tilings.

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Initial Problem – Solution: The initial problem is repeated and its solution is fully worked for the student at the end of each section.

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HISTORY: Intriguing commentary of how mathematics was used and/or developed throughout the ages.

HISTORY

The first maritime insurance companies were established in Italy and Holland in the 14th century. These companies carried out calculations of chances since larger risks made for larger insurance premiums. For shipping by sea, premiums amounted to about 12% to 15% of the cost of the goods.

Page 566

TIDBIT: These marginal notes call out interesting facts and information that relate to many various aspects of life.

TIDBIT

The body's metabolism of drugs works like radioactive decay. Among the data available to prescribing physicians is the half-life of particular drugs in the average person's body. There is a lot of variability from person to person; for example, the half-life of caffeine varies from three to seven hours.

THE HUMAN SIDE OF MATHEMATICS


W. S. Gossett

One of the oldest breweries in the world is the Guinness Brewing Company in Dublin. Guinness began as a family business in 1759. Its markets grew worldwide. However, the brewing process was overseen by master brewers using arcane methods handed down from master to apprentice. The Guinness corporation was interested in making this process scientific and constructing an exact recipe that could be used worldwide. This was a novel idea that required new techniques.

W. S. Gossett, born in 1876, studied chemistry at the university in Dublin. He was hired by Guinness as a brewmaster in 1899 to work on the problem of making brewing a science. One quesences found in the data could be accidental or simply due to natural variation. On the other hand, they could be the result of differences in treatment or process and thus lead to better methods of brewing. There was no way to tell which was which. Gossett went to work at the laboratory of the biometrician, Karl Pearson, to study statistics. During this time, Gossett solved the problem of data variation and developed new techniques. Then, in 1907 he returned to Guinness to be brewer-in-charge. Because of his connection with the Guinness company it was decided that he would not publish his ideas under his own name, but rather use the pseudonym, "Student." His

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The Human Side of Mathematics: Mathematics was developed by real people – these short biographical sketches are of people whose work was relevant to each chapter.

PROBLEM SET 1.2

In problems 1 through 4, state the premises (or hypotheses) and conclusion in each of the arguments. Are there any premises that aren't explicitly stated?

1. **(a)** If the room is warm, then I'll be uncomfortable. The room is warm, so I'll be uncomfortable.
   (b) If the weather is bad, I'll go to the movies. It's raining heavily, so I'll go to the movies.
2. **(a)** If the weather is good, I'll want to play golf. If I decide to play golf, I'll see if Marty can go. It's warm and clear outside; I guess I'll call Marty.
   (b) If Dr. Goldberg teaches the course, I'll register for her class. If you take a class from Dr. Goldberg, you can count on having to do a term paper. Dr. Goldberg is going to teach the course, so I'll have to write a term paper.
3. **(a)** If the weather is good, Barry will paint the house. Barry didn't paint the house, so the weather wasn't good.
   (b) If you average at least 90% on the tests, you'll get an A for the term. You didn't get an A for the term, so you didn't average 90% on the tests.
4. **(a)** Jenna is going to go swimming or play tennis. Jenna didn't go swimming, therefore she must have played tennis.
   (b) We're either going to the play or the movie. The tickets to the play are all sold out, so we're going to the movie.

In problems 5 through 18, translate the argument into symbolic form. Determine whether the argument is valid or invalid. You may compare the form of the argument to one of the four standard forms or use a truth table.

5. If the movie is good, the people will go.
   The people will go to the movie.
   Therefore the movie is good
6. If the sun is shining, I'll wear a hat.
   The sun isn't shining.
   Therefore I won't wear a hat.
7. If it's cold outside, my hands will be cold.
   It's freezing outside.
   Therefore my hands will be cold.
8. These shoes are not expensive
   If shoes are expensive, I won't buy them.
   Therefore I won't buy these shoes
9. If we miss the bus, we'll have to walk.
   If we walk, then we'll be late.
   Therefore if we miss the bus, we'll be late.
10. If you do the work, you'll get paid.
    If you get paid, you can go to the show.
    Therefore if you do the work, you can go to the show.
11. If Spike Lee is the director, then the movie should be good. Spike Lee didn't direct the movie, so it probably isn't good.
12. The night is cold and dark. The night is not dark or it is cold. Therefore the night is not cold.
13. If Carlos passes his entrance exam, he will attend college. Carlos will not be attending college; therefore he did not pass his entrance exam.
14. If you arrive on time, you'll get a good seat. If you get a good seat, you'll enjoy the play. Therefore if you didn't enjoy the play, you didn't arrive on time.
15. If I can't go to the movie, then I'll go to the park. I can go to the movie. Therefore, I will not go to the park.
16. If you score at least 90%, then you'll earn an A. If you earn an A, then your parents will be proud. You have proud parents. Therefore, you scored at least 90%.
17. If you work hard, then you will succeed. You do not work hard. Therefore you will not succeed.
18. If it doesn't rain, then the street won't be wet. The street is wet. Therefore it rained.

In problems 19 through 26, identify which form of argument (Modus Ponens, the chain rule, Modus Tollens, or disjunctive syllogism) is being used.

19. If Joe is a professor, then he is well educated. If you are well educated, then you went to college. Joe is a professor, so he went to college.
20. If you have children, then you are an adult. Paul is not an adult, so he has no children.
21. Whenever the weather is bad, I stay inside and work. It's raining heavily outside. I'll stay inside and work.
22. Either I get a raise or I'm going to look for a new job. I didn't get a raise, so I'm going to look for a new job.
23. If I don't have enough money to buy gas, I will ride the bus. When I ride the bus I am always late. I can't afford to buy gas, so I'm going to be late.
24. If you get to the store on time, you can pick up a carton of ice cream. You didn't pick up any ice cream, so you didn't get to the store on time.
25. Kim is going to have her old car painted or buy a new car. Kim didn't buy a new car, so she had her old one painted.
26. If I don't eat breakfast, I'll be hungry by 10:00. If I'm hungry before noon, I always snack before lunch. I skipped breakfast, so I will have a snack before lunch.
27. The *Initial Problem* from section 1.1 was to find a symbolic form for an exercise in logic written by Lewis Carroll. The final form of the argument was:

$$\sim p \Rightarrow \sim q$$
$$\sim r \Rightarrow \sim s$$
$$\sim q \Rightarrow \sim t$$
$$u \Rightarrow \sim p$$
$$\sim t \Rightarrow \sim r$$

Use the antecedent: u (a kitten is green-eyed) and provide a logical conclusion for the argument. State the conclusion in terms of the original statements and show that the argument is valid.

Exercises: Each section has an extensive set of exercises and problems covering all the topics developed in the section. Odd and even numbered problems are matched – with the even problems being slightly more difficult.

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EXTENDED PROBLEMS

43. Investigate the interest rates available on bank credit cards. Is there any reason a person would choose a card with a high interest rate?
44. Compare the terms at a rent-to-own store with the available retail price for an appliance of interest to you. What is the approximate annual percentage rate if the rent-to-own arrangement is treated like a loan with add-on interest?
45. Research the history of consumer credit. Consider such questions as: Is it a recent phenomenon? When did it become institutionalized and regulated? What legal and/or social issues have received special attention?
46. Consult an encyclopedia, almanac, or other suitable reference source for the total amount of consumer debt during the last 25 years. Prepare a bar graph to display the data. Are there any trends or special features to the graph?
47. Research the effect of inflation and deflation on those who owe money. Has this ever had important political ramifications?
48. What are usury laws? Are such laws in force in your state? Were they ever?

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EXTENDED PROBLEMS: Each section has a selection of extended problems. Some of these extended problems introduce additional topics that an instructor may wish to cover, while others offer an opportunity to do open-ended projects.

Chapter Six Problem

You are buying a car for $10,000 from a dealer who will give you a loan for 5 years at 10% interest. As an extra incentive he offers either $1200 cash back on the 10% loan or a 6% interest rate (which is a rate that you could expect to get if you deposited funds in a bank or money market account). Which deal should you choose?

SOLUTION

Strategy: Use a Table

One approach to making a rational decision is to compare the total amount you would pay under each option. That means you have to first find the amount of the monthly payments for each choice. From this information, you can compute the total for *all* payments.

Since the interest rate is a whole percent, we can use a table rather than computing the monthly payment from a formula. Table 6.5 can be used for this purpose. The part of the table we are interested in is the following:

Chapter Problem: Each chapter ends with a solved problem illustrating the usefulness of the material covered in the chapter and emphasizing one of the important topics.

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Section 9.1

Plurality Method 424	Plurality with Elimination Method 426	Preference Table 426
Borda Count Method 424	Run-off Election 426	Pairwise Comparison Method 428

Section 9.2

Majority Criterion 435	Monotonicity Criterion 438	Arrow Impossibility Theorem 442
Head-to-Head Criterion 437	Irrelevant Alternatives Criterion 441	

Chapter Review—Vocabulary/Notation: To help organize studying, all key vocabulary words and notation are listed with the sections and page numbers where they can be found.

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Chapter Seven Review Problems

1. Consider the following game matrix.

$$\begin{bmatrix} 3 & -1 & -2 & 4 \\ 1 & 4 & -3 & 1 \\ -3 & 2 & 5 & -1 \\ -1 & 3 & 2 & -4 \end{bmatrix}$$

What will the payoff be if both players use their third strategy?

2. In the matrix in problem 1, what are the most aggressive strategies? What will the payoff be if both players use their most aggressive strategy?
3. In the matrix in problem 1, what are the most conservative strategies? What will the payoff be if both players use their most conservative strategies?
4. Is the game in problem 1 determined or not? Why? If it is determined what is the value of the game?
5. Construct a game matrix for the following game. Alice is on attack and can either pitch or kick. Bob is on defense and can either play tight or loose. If Alice pitches and Bob plays tight, then Alice loses one yard. If Alice pitches and Bob plays loose, then Alice gains 4 yards. If Alice kicks and Bob plays tight, then Alice gains 5 yards. If Alice kicks and Bob plays loose, then Alice loses 10 yards.
6. What is the most aggressive strategy for Alice and Bob in the game in problem 5? Suppose that Alice knows that Bob will choose his most aggressive strategy. What strategy should Alice employ? How many yards will she gain or lose?
7. Consider the following game matrix. Use the box and circle method to decide if the game is determined and, if so, the value of the game.

$$\begin{bmatrix} 0 & 1 & 2 & -1 \\ -3 & 1 & 3 & -2 \\ 1 & 2 & -3 & -1 \\ -1 & 3 & 2 & -4 \end{bmatrix}$$

8. Consider the following game matrix.

$$\begin{bmatrix} -3 & 3 \\ 4 & -2 \end{bmatrix}$$

Suppose that the first player uses a mixed strategy with frequencies $(\frac{1}{2}, \frac{1}{2})$ and the second player uses a mixed strategy with frequencies $(\frac{1}{3}, \frac{2}{3})$. What is the average payoff to the first player of this game?

9. Suppose you are the first player in the game in problem 8 and learn that the second player has a mixed strategy with frequencies $(\frac{1}{3}, \frac{2}{3})$ What is your optimal strategy in this case, and what is the payoff?
10. What is the optimal mixed strategy for each player in problem 8?

Chapter Review—Problems: Each chapter ends with a set of problems which cover the most important topics in the chapter.

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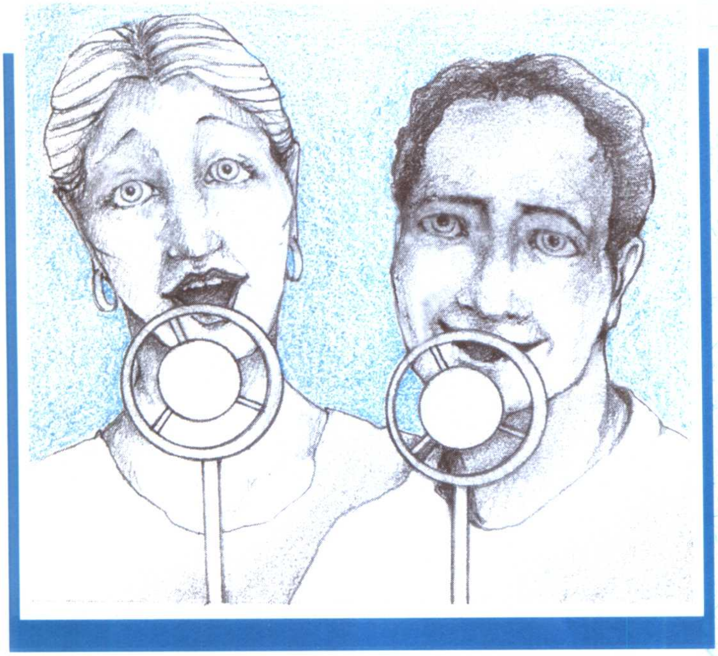
CONTENTS

PART 1

MATHEMATICS *in* LIFE

CHAPTER 1

CRITICAL THINKING, LOGICAL REASONING, AND PROBLEM SOLVING



BIG SPENDER OR RESPONSIBLE LEGISLATOR?

The following 1994 radio advertisement was run by Representative Buck who was a candidate for governor running against former state Senator Doe.

Narrator: "To learn what people think about some of the issues in the governor's race, we called a few. We asked how much they know about Doe's record. Did you know Doe voted for $2.7 billion in higher taxes when he was a state senator?"

Male voice: "I had no idea that he voted for that amount in higher taxes."

Female voice: "It's a typical tax and spend attitude."

Male voice: "This state needs less taxes, not more."

Female voice: "The liberals always have their hands in my pocket."

Although the advertisement places Senator Doe in a bad light, is he really a big spender?

CHAPTER GOALS

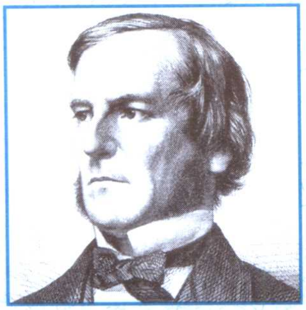
1. Learn about the logical connectives *and, or, if then, and if and only if* and how they are used in reasoning.
2. Learn about the various forms of "if . . ., then" statements and how they are used in constructing valid arguments.
3. Learn to make valid arguments and to recognize invalid argument forms.
4. Learn a problem-solving framework as well as strategies that can be used throughout the book and in life to solve a variety of problems.

aily we are inundated with statements and images that are designed to influence us. In the advertisement above, Representative Buck was trying to undermine his opponent in a state where reducing taxes was in vogue. A reasonable person would expect the tax increase figure of $2.7 billion to refer to one year's tax increase, unless there was a statement to the contrary. However, a statewide newspaper published the following analysis of the ad: "the figure of $2.7 billion in tax increases includes estimates of how much a tax would have been raised over varying lengths of time. For example, in one case, the total includes the 10-year impact of a gas-tax increase. In some cases, it is four years. Also, some of the bills cited are not even tax increases. For example, two of the bills removed the exemption from income taxes for public employee retirement pensions in exchange for higher benefits." Thus, we see that in his advertisement Representative Buck was misusing information to gain votes.

The media is replete with information that helps form our opinions and guide our actions. The purpose of this chapter is to help you develop the skills needed to think critically about a variety of problems and issues, especially those involving quantitative information and relationships. In the chapter, you will increase your sensitivity regarding the information and misinformation you face on a daily basis. In addition to developing logical reasoning patterns that will help you analyze arguments better, you will also learn to organize your own arguments (which we hope will be logical and truthful). Finally, you will be introduced to a framework and strategies for solving problems.

THE HUMAN SIDE OF MATHEMATICS

George Boole (1815–1864) was the son of an Irish cobbler. Boole was self-taught in higher mathematics and never received a degree. His father, an amateur mathematician, began teaching mathematics to Boole at an early age. Unfortunately, the father had no money for his son's formal education. Nonetheless, Boole continued his studies and was able to write many high quality mathematical papers. He received enough recognition for his work to be appointed professor of mathematics at Queen's College in Cork, Ireland.



George Boole

Boole became one of the most influential mathematicians of his time, and his influence carried forward into modern times. Bertrand Russell, British mathematician and philosopher, credits Boole with discovering pure mathematics, referring to work published in a book called *The Laws of Thought,* in 1854. Traditional logic was incomplete and failed to account for many principles of inference employed in even elementary mathematical reasoning. Boole's primary concern was to develop a nonnumerical algebra of logic that would provide precise methods for handling more general and varied types of deductions, or proofs, than were covered by traditional logic. His work laid the foundations for modern algebra and the study of symbolic logic.

George Boole was a man of goodwill and reputation. Among the poor people of the neighborhood, he was regarded as an innocent who should not be cheated; among the higher classes he was admired as something of a saint, although a bit odd. In 1864, he was caught in the rain one day while hurrying to class. Concerned for his students because he was late, Boole gave his entire lecture while still dripping wet. Boole had suffered from poor health most of his life and came down with a cold that turned to pneumonia. He died shortly thereafter.

Lewis Carroll, one of the most revered names in children's literature, was the pen name of Charles Dodgson (1832–1898), an English mathematician and logician. Born to an upper-class clergyman's family, Dodgson was very religious and was expected to serve God by joining the ministry. Instead, after some agonizing, he thought God might be served as well by a teacher and a writer. After earning his university degree, he took a job as a lecturer in mathematics at Oxford University under the condition that he never marry. He held the job all his life. A stutterer and very hard of hearing, he had difficulty in social situations but did very well with children. He told them stories and then put the stories on paper, eventually having them published. His most famous contributions to children's literature were *Alice's Adventures in Wonderland* and *Through the Looking Glass.*

Lewis Carroll

Lewis Carroll lived quietly, free from the distractions of life, worldly want, and family responsibilities. He produced some of the most enduring works in children's literature as well as a lasting legacy in mathematics. He made significant contributions in Euclidean geometry and in logic, where he invented a method to test whether logical arguments were valid or not. Queen Victoria was said to be so taken by his children's books that she requested copies of every book he had ever written. Imagine her surprise when she received a pile of mathematics books! Lewis Carroll was interested in logical reasoning, especially as it applied to games, puzzles, and pure mathematics. He considered his texts on logic as his most important works. An interesting aspect of his children's books is that they can also be appreciated from a mathematical standpoint. For example, *Through the Looking Glass* is based on a game of chess and *Alice's Adventures in Wonderland* involves size and proportion.

1.1 STATEMENTS AND LOGICAL CONNECTIVES

INITIAL PROBLEM

The following exercise in logic comes from a textbook on the subject by Lewis Carroll, author of *Alice's Adventures in Wonderland.*

No kitten that loves fish is unteachable.
No kitten without a tail will play with a gorilla.
Kittens with whiskers always love fish.
No teachable kitten has green eyes.
No kittens have tails unless they have whiskers.

Is there a way we can make sense of these statements and draw a conclusion?

HISTORY

The Chinese philosopher Mo Ti founded logic in China during the 400s B.C. In the A.D. 300s the Buddhist philosopher Acarya Dignaga invented symbolic logic. His influence in Buddhist Asia was comparable to that of Aristotle in the West. This system spread to China and then Japan in the A.D. 600s.

Although his lasting contributions were to children's literature, the focus of Lewis Carroll's intellectual life was the study and teaching of logical reasoning. As is true with most of mathematics, the study of logic is made easier by the substitution of symbols for words. Yet in the process of logic, the basic tool with which we test all ideas and also solve most of our everyday problems, we are still at the mercy of the inadequacies and clumsiness of words. Some collections of words are meaningful in a mathematical context, and some are not. We begin our study of logic by defining what we mean by statements. Then we will see how statements are combined, modified, and organized to form meaningful arguments. In the second section, we will see how arguments are analyzed for validity and how conclusions are drawn.

STATEMENTS

Our written and spoken language is organized into units called sentences. For the study of logic, the declarative sentence is the most important type since such a sentence makes an assertion, and thus is either true or false. Sentences that can be classified as true or false are called **statements.** Examples of statements are:

1. Based on area, Alaska is the largest state of the United States. (True)
2. Based on population, Texas is the largest state of the United States. (False)
3. $2 + 3 = 5$. (True)
4. $3 < 0$. (False)

It may be impossible to determine if some sentences are true or false. For example, an interrogative sentence or an exclamation is typically neither true nor false. The paradoxical statement "This sentence is false" can be neither true nor false. The following are not statements as defined in logic.

1. My home state is the best state. (Subjective)
2. Help! (An exclamation)
3. Where were you? (A question)