INTRODUCTORY CIRCUIT ANALYSIS

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Preface

This text is intended for the first course in electric circuit analysis. It presumes that the student is proficient in algebra and trigonometry and has had some experience with complex numbers. The student should have had an introduction to differential and integral calculus, and should be continuing his studies in calculus. This text develops the solution of linear differential equations with constant coefficients by both the classical and Laplace transform methods on the basis that the student has not yet encountered these in his mathematics courses.

In order that the student may verify some of the deductions from the circuit model, and gain some appreciation for its shortcomings, a concurrent laboratory course is considered essential. Also, a college physics course prior to, or concurrent with, this course is considered highly desirable.

A new text in an established discipline usually comes into being because some teachers have felt strongly that a change in sequence or emphasis would improve the learning process. If such feelings lead to the creation of class notes and if such notes survive several years of class use with the resulting revisions, these notes may appear in book form. Such is the case for this text.

We had found that our students did not properly relate the time domain, the phasor domain, and the Laplace transform domain. The reason seemed to be that these subjects were often studied in isolation, the student not being required to relate these in a given situation. Also there was considerable confusion concerning the relationships, if any, between the symbol s of the Laplace transform, the symbols p or D used as differential operators, and the symbol s used as the exponent for the complex exponential driving function, Ae^{st}. Also students did not usually realize the close relationship between circuit theory and electromagnetic field theory. The latter condition arises because most texts on circuit theory do not stress this relationship and, unfortunately, neither do most texts on field theory.

The sequence of material in this text was selected with the hope that an improvement in the learning process would result. We have the feeling that this is so.

Chapter 1 is a short chapter whose purpose is to state rather concisely the important relationships in field theory on which circuit theory is based. We urge the student to read this chapter carefully, but not to be concerned if he feels a lack of understanding of the mathematical expressions known as Maxwell's equations. He will become more familiar with these equations in courses on electromagnetic field theory. In Chapter 2 circuit elements are defined and laws and conventions are stated. The dependence of these definitions and laws on electromagnetic field theory is shown whenever this is possible.

From a mathematical point of view, linear circuit theory requires the solution of linear differential equations with constant coefficients. There are two important methods of solution: the classical method and the Laplace transform method. The classical method has the advantage of requiring (and giving) physical insight; the Laplace transform method has advantages which arise from the facts that the differential equations are transformed into algebraic equations and that initial conditions are immediately incorporated into these equations. In order that the student may acquire both the physical insight of the phenomena and a mastery of the techniques in arriving at correct solutions, he is introduced to both methods as early as possible. He is then required to solve many problems by both methods over an extended period of time. Chapter 3 introduces the student to the two methods and gives some practice in solving simple problems by both methods. Additional practice in obtaining complete solutions by use of the two methods is provided in Chapters 9 and 11.

The early introduction of the Laplace transform permits the early presentation of the transform circuit and the definition of transform network functions. These allow the presentation of general circuit reduction methods and theorems, thus avoiding the learning of definitions and techniques which must later be modified because they were not general enough. These methods and theorems are presented in Chapter 4.

Chapter 5 is concerned with the development of the phasor domain and some of its applications. The main development is from the classical method, although the development from the transform domain is also shown. Chapter 6 deals with the loci of phasor network functions for a variable element and with the frequency characteristics of network functions. Because of the large amount of nomenclature and techniques in circuit theory, the driving functions have been limited thus far to sinusoids. In Chapter 7 these are extended to general periodic functions by use of Fourier series; however, in this chapter we limit the solutions to steady-state solutions. Chapter 8 is devoted to three-phase circuits under steady-state conditions, the previous introduction to Fourier series permitting the consideration of harmonic voltages and currents. The study of three-phase circuits causes the student to appreciate the usefulness of phasor diagrams.

In Chapter 9 complete solutions are obtained for pulses of various types by both classical and transform methods. Also the subject of impulses is presented primarily from a physical stand-point.

Analogues are not introduced until the last chapter because our experience indicates that the student needs to have a good comprehension of the language and techniques of one discipline if he is to relate these to a new discipline. This is particularly true if one expects the student to label variables, write equilibrium equations, and determine initial conditions in this new discipline.

We have tried to use standard symbols wherever possible. We have not used boldface to represent complex quantities, since this cannot be used by instructor or student. A complex quantity is designated by a caret ($\hat{}$) over the quantity. Thus we use $i = I\cos(\omega t + \alpha) = \text{Re}(\hat{I}e^{j\omega t})$, in which $\hat{I} = Ie^{j\alpha}$. A detailed list of symbols is given at the beginning of the book.

Boulder, Colorado, 1965

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S. I. P.
G. J. M.

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1 *Electromagnetic Field Theory*[1]

1.1 INTRODUCTION

The student's background in physics or chemistry, as well as his experiences in this age of science and engineering, have given him certain concepts of electricity and magnetism. We shall start with these concepts, expand on them, and then introduce the laws of electromagnetism which are known as Maxwell's equations. It is on these equations that electric circuit theory is based; we should have at least a nodding acquaintance with them if we are to have a good relationship with circuit theory.

To most of us the concept of electric charge is basic to the structure of the atom. Each of the building blocks of matter, such as protons, neutrons, electtons, mesons, etc., is characterized by two properties: its "rest mass" and its electric charge. This charge may be of two kinds which are arbitrarily called positive and negative, the charge of the electron being negative. Charge is quantized; that is, charge occurs in packets, the smallest one of which is equal to the charge on an electron. The effect of these atomic charges is extremely important in determining the mechanical, chemical, and electrical properties of materials.

Forces exist between charges; in fact it is because such forces were observed that the concept of charge was postulated. Probably all of us have seen the demonstration of force between charged pith balls

[1] It is intended that this chapter give a qualitative insight to field theory so that the student may find that a definite and significant relationship exists between field theory and circuit theory and that circuit theory is an approximation to field theory. It is not intended that this material be construed as an introductory course in field theory. Consequently, it is anticipated that only one, or possibly two, class periods be devoted to its consideration.

and noticed that like charges repel, unlike charges attract. Such forces are called electric forces. If the charges are in motion relative to each other and to the observer, additional forces are observed. These forces are called magnetic forces, magnetism thus being associated with moving electric charges or electric fields changing with time.

There is a mystery about forces which "act at a distance" such as those of electricity and magnetism. Such phenomena seem less mysterious if it is postulated that a field is associated with each charge and that forces are the result of reaction between charge and field. Such fields were first proposed by Faraday and were then adopted by Maxwell in his electromagnetic theory. We are accustomed to some terms involving fields; in particular, we speak of the earth's magnetic field and are aware that a compass will point toward the north magnetic pole.

With regard to the student's experience with electricity and magnetism, he has surely received a shock after walking on a rug and learned that charges were separated by friction. He has observed lightning which is believed to be a similar phenomenon on a grand scale, the separation of charge here being caused by relative motion of water particles and air. He knows that energy can be transmitted most efficiently in the form of electromagnetic energy; that power-generating stations are able to convert the energy of combustion of fossil fuels or the energy liberated by nuclear fission into electromagnetic energy, and that somehow this energy can be guided by transmission lines to industries and homes. This transmission usually occurs with voltages and currents varying sinusoidally with time at 60 cycles per second (cps), transformers being used to change the voltage level so that the voltage level in the homes will be relatively safe while high voltages are used for economic transmission of large amounts of power.

Electromagnetism also has a major role in communications. Telephone, telegraphy, radio, and television are all examples of the application of electromagnetism. In some of these instances, wires are used to guide the signal transmission. In others, such as radio or television, no wires are needed to guide the energy; the energy radiates in a fashion similar to that of the light radiated from a candle or electric lamp. In fact the student may know that light is simply that portion of the electromagnetic frequency spectrum to which our eyes are sensitive. Thus electromagnetism covers all the frequency spectrum from zero frequency or direct current through radio frequencies, light frequencies, x-rays, and gamma rays.

Electromagnetism has many applications in industry, in controls, in instrumentation, and in computers. In fact it is hard to find a segment of modern life in which electromagnetism is not playing an important role.

1.2 A DISCUSSION OF LAWS AND UNITS

What are the laws of a science which deals with such a wide range of phenomena, from the interaction of atomic particles to communication between distant points of the earth, or to radiation from distant galaxies? What do we mean by laws in the first place?

In a physical science we think of the laws as a group of relationships which are consistent among themselves and which lead to conclusions that are in accord with experimental evidence. Also we use the term "fundamentals" or fundamental laws as a minimum group of relations which may be considered basic and from which other relations or laws may be derived. Thus what one person calls fundamental need not be the same as what another person calls fundamental as long as there is agreement on the total group of relationships. For example, in mechanics one person may start with force relations as fundamental and then "derive" energy relations; while another person may start with energy relations and then derive the force relations. A third person may well observe that the first two are simply proving that their total groups of relations are consistent.

Since our measuring abilities improve with time, we would expect that physical laws need to be modified as time progresses. A good illustration of this modification is again in the area of mechanics in which Newton's laws have been dominant for many years. However, it has been found necessary to modify Newton's laws in accord with relativistic principles when relative velocities are not negligible in comparison with the velocity of light. Also it has been found necessary to use quantum mechanics to predict phenomena in the case of small magnitudes corresponding to the atomic and nuclear domains. These modifications do not invalidate Newton's laws for a large group of practical cases; in fact both relativistic mechanics and quantum mechanics are in accord with Newton's mechanics for these cases.

Maxwell's equations have a similar relation to electromagnetic field theory that Newton's laws have to mechanics. Newton's laws do not apply to atomic or nuclear realms because the laws do not recognize that mass (energy) occurs in discrete amounts (quanta); Maxwell's equations similarly do not apply in these realms because the equations do not recognize that charge occurs in discrete amounts.

We shall not follow the historical or experimental approach to electromagnetism. Such an approach would possibly start with Coulomb's law of force between point charges and the law of force between two current carrying conductors and then proceed to the observations of Ampere, Kirchhoff, Gauss, Faraday, and others. Maxwell[2], in

[2] *A Treatise on Electricity and Magnetism*, by Clerk Maxwell, Unabridged Third Edition, reprinted by Dover Publications, New York, 1954.

considering the "fundamental" laws of electricity and magnetism as they were known in his day, observed that these laws were not consistent mathematically, added a term to make them consistent and deduced that electromagnetism was a wave phenomenon having the velocity corresponding to that of light. His brilliant deductions have been confirmed and his contribution is considered such a "breakthrough" that the modified laws are known as Maxwell's equations. Thus it seems appropriate to start with Maxwell's equations as basic postulates, for in a real sense they are not derived. There are also energy relations in electromagnetism; however, we shall not try to prove that these energy relations are consistent with Maxwell's equations and the concepts of conservation of charge and energy. Such proofs properly belong in a course on electromagnetism. We shall simply accept all these relations as representing a consistent group of laws.

Maxwell's equations are expressed in terms of field quantities. The electric field is postulated to have a flux, ψ, a flux density $\bar{D}$,[3] and a field intensity $\bar{E}$. The magnetic field is postulated to have a flux, ϕ, a flux density $\bar{B}$, and a field intensity $\bar{H}$. If the medium is isotropic, $\bar{D}$ and $\bar{E}$ are in the same direction; that is, $\bar{D} = \epsilon\bar{E}$, in which ϵ is a scalar quantity called permittivity. Similarly, for an isotropic medium, $\bar{B} = \mu\bar{H}$, in which μ is permeability. If ϵ and μ are constants, independent of the magnitude or direction of the field quantities, the medium is said to be linear. If ϵ and μ are independent of location in space, the medium is said to be homogeneous. The concept of flux and field intensity may seem strange and a person may be prompted to ask whether these quantities are real. They are as real as other useful concepts. We cannot see mass, nor do we know what it is, but yet we willingly accept the concept of mass because this is helpful in predicting the behavior of material objects. The same observation may be made for the concept of charge. And so we should accept these postulates of flux and field intensity as useful concepts in predicting electromagnetic behavior.

A consistent set of units is very important. We shall use only the rationalized meter-kilogram-second (mks) system of units. Occasionally, dimensions may be given in English units; these should be changed to meters before insertion into equations. The mks system of units has been adopted internationally and has many advantages over other systems. One of these advantages is that many practical units such as ampere, volt, watt, and joule are units also in the mks system. Essentially all modern literature of significance in circuit theory is in the mks system of units. Recent literature in electromagnetic field theory is

[3] The bar over the quantity indicates that the quantity is a vector.

primarily in the mks rationalized system of units. The word "rationalized" implies that Maxwell's equations have no constants other than unity; the word "unrationalized" implies that a factor of 4π appears in two of Maxwell's equations. There are only a few quantities that have different magnitudes in these two systems of units. These are $\bar{D}$, $\bar{H}$, ϵ, and μ; these differ by the factor 4π. For all circuit quantities there is no difference between rationalized and unrationalized systems of units.

We sometimes consider a particular group of units to be a fundamental set, the other units being expressed in terms of these fundamental units. In mechanics it is customary to consider mass, length, and time as fundamental units and then express other quantities such as force, velocity, power, and energy in terms of these units. A dimensional check of an expression or equation is then sometimes useful in detecting an error. In electromagnetism, three fundamental units are not enough; it is necessary to add a fourth unit. This unit is frequently charge; one sometimes sees reference to the mksq system of units. However we seldom make a dimensional check in terms of these units; it is usually easier to assure ourselves that each term of a given expression has the same units, such as volts, amperes, or ohms.

1.3 THE LAWS OF ELECTROMAGNETISM (MAXWELL'S EQUATIONS)

There are only four relations which are considered to be the laws of electromagnetism and known as Maxwell's equations. The first two are concepts about magnetic and electric flux, the last two are concerned with the closed line integrals of field intensities. These laws may be expressed as follows.

(a) Magnetic flux is continuous; that is, this flux has no beginning or end. The expression for the magnetic flux, ϕ, passing through an area a may be calculated from the surface integral,

$$\phi = \int \bar{B} \cdot d\bar{a},$$

in which $d\bar{a}$ is a vector differential area and $\bar{B} \cdot d\bar{a}$ is the scalar product of two vectors meaning $Bda \cos \angle \frac{\bar{B}}{d\bar{a}}$, ϕ has the units of webers and $\bar{B}$ has the units of webers per square meter. Fig. 1.1 shows the magnetic field about a long wire carrying a conduction current shown going into the paper. The direction of conduction current is taken to be the direction of motion of positive charge.

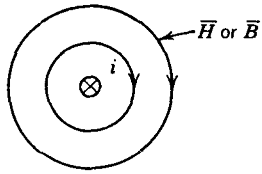

FIG. 1.1 Magnetic field around a current, i.

One way of expressing this law mathematically is to say that the net flux leaving a closed surface must be zero. Thus,

$$\oint \bar{B} \cdot d\bar{a} = 0, \tag{1.1}$$

in which now the direction of $d\bar{a}$ is taken to be outward from the volume enclosed and the circle about the integral sign means that the integration is over a closed surface.

(b) Electric flux may be continuous or may terminate on electric charge. In the latter case, the amount of flux leaving a closed surface is equal to the charge enclosed, which implies that flux leaves a positive charge and enters a negative charge. An example of electric flux being continuous would be the electric field of a dipole antenna at a distance which is large in comparison with a wavelength (see Fig. 1.2). An example of the electric field terminating on charge is the field between the two conductors of a direct current (d-c) transmission line, as shown in Fig. 1.3.

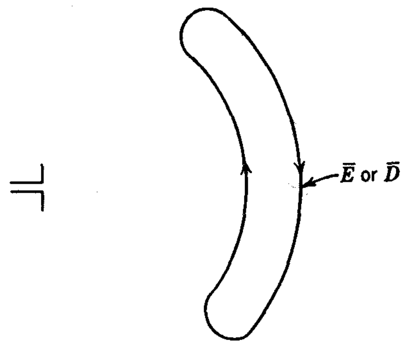

FIG. 1.2 Electric far field of antenna.

Electric flux, ψ, has the units of coulombs and electric flux density, $\bar{D}$, has the units of coulombs per square meter. We may express this law mathematically as

$$\oint \bar{D} \cdot d\bar{a} = q, \tag{1.2}$$

in which q is the charge enclosed by the closed surface.

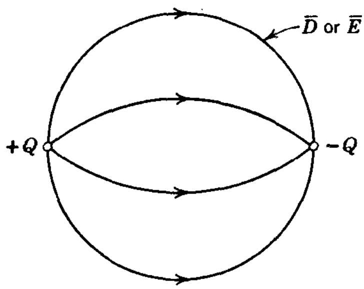

FIG. 1.3 Electric field about two d-c conductors.

This law is known as Gauss's law for electricity or simply as Gauss's law and is treated extensively in most physics texts.[4]

(c) The closed line integral of the electric field intensity, $\oint \bar{E} \cdot d\bar{l}$, is equal to the negative time rate of change of the magnetic flux which links the closed line, the positive direction of magnetic flux being in the direction of the thumb of the right hand if the fingers of this hand are in the direction of the integration. The designation $d\bar{l}$ is a vector differential displacement, and the circle about the integral sign means that the integration path is such that it returns to its starting point. In the sketch shown in Fig. 1.4, if ϕ is increasing with time in the direction shown, we will obtain a negative value for $\oint \bar{E} \cdot d\bar{l}$. The units of E are volts per meter, and thus the units of $\oint \bar{E} \cdot d\bar{l}$ are volts or joules per coulomb.

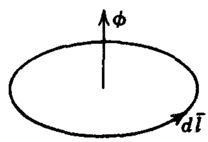

FIG. 1.4 Magnetic flux linking a closed line.

The magnetic flux linked by the closed line may be designated $\lambda' = \int_a \bar{B} \cdot d\bar{a}$, in which the area a is the area bounded by the closed line. We need to distinguish between magnetic flux, ϕ, and magnetic flux linkage, λ', since the linkage is found by integrating flux density over the area bounded by some closed line and since a particular quantity of flux may cross this area more than once. This latter condition is illustrated in Chapter 2, Fig. 2.17, page 41.

This law may then be written mathematically,

$$\oint \bar{E} \cdot d\bar{l} = \sum v = -\frac{d\lambda'}{dt}, \tag{1.3}$$

[4] See, for example, David Halliday and Robert Resnick's, *Physics for Students of Science and Engineering*, Part II, Second Edition, Wiley, New York, 1962, Chapter 28.