Continuous and Discrete Signal and System Analysis

second edition

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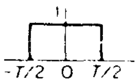
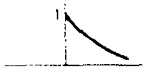
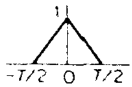
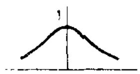
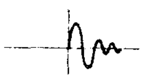
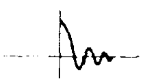
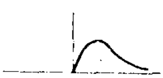
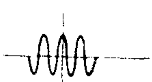
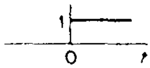
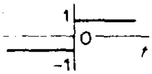
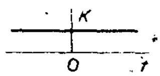
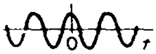
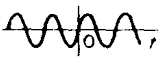
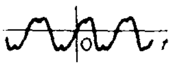
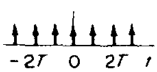
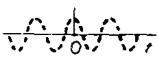
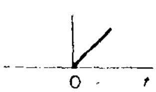
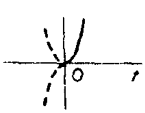
Fourier Transforms of Energy Signals

	$x(t)$	$X(\omega)$	$X(f)$
 Rectangular pulse	$u\left(t+\frac{T}{2}\right)-u\left(t-\frac{T}{2}\right)$	$T\,\frac{\sin(\omega T/2)}{\omega T/2}$	$T\,\frac{\sin \pi Tf}{\pi Tf}$
 Exponential	$e^{-\alpha t}u(t)$	$\frac{1}{j\omega+\alpha}$	$\frac{1}{j2\pi f+\alpha}$
 Triangular	$1-2\frac{\lvert t\rvert}{T},\ \lvert t\rvert<\frac{T}{2}$; 0 elsewhere	$\frac{T}{2}\left[\frac{\sin(\omega T/4)}{\omega T/4}\right]^2$	$\frac{T}{2}\,\mathrm{sinc}^2(Tf/2)$
 Gaussian	$e^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha}\,e^{-(\omega^2/4\alpha^2)}$	$\frac{\sqrt{\pi}}{\alpha}\,e^{-(\pi^2 f^2/\alpha^2)}$
 Double exponential	$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\frac{2\alpha}{\alpha^2+4\pi^2 f^2}$
 Damped sine	$e^{-\alpha t}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(\alpha+j\omega)^2+\omega_0^2}$	$\frac{\omega_0}{(\alpha+j2\pi f)^2+\omega_0^2}$
 Damped cosine	$e^{-\alpha t}\cos(\omega_0 t)u(t)$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_0^2}$	$\frac{\alpha+j2\pi f}{(\alpha+j2\pi f)^2+\omega_0^2}$
	$\frac{1}{\beta-\alpha}\left[e^{-\alpha t}-e^{-\beta t}\right]u(t)$	$\frac{1}{(j\omega+\alpha)(j\omega+\beta)}$	$\frac{1}{(j2\pi f+\alpha)(j2\pi f+\beta)}$
 Cosine pulse	$\cos\omega_0 t\left[u\left(t+\frac{T}{2}\right)-u\left(t-\frac{T}{2}\right)\right]$	$\frac{T}{2}\left[\frac{\sin(\omega-\omega_0)T/2}{(\omega-\omega_0)T/2}+\frac{\sin(\omega+\omega_0)T/2}{(\omega+\omega_0)T/2}\right]$	$\frac{T}{2}\left[\frac{\sin \pi T(f-f_0)}{\pi T(f-f_0)}+\frac{\sin \pi T(f+f_0)}{\pi T(f+f_0)}\right]$

Fourier Transforms of Power Signals

DR96/07

	$x(t)$	$X(\omega)$	$X(f)$
 Unit impulse	$\delta(t)$	1	1
 Unit step	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
 Signum function	$\operatorname{sgn} t=\frac{t}{\lvert t\rvert}$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
 Constant	K	$2\pi K\delta(\omega)$	$K\,\delta(f)$
 Cosine wave	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$
 Sine wave	$\sin\omega_0 t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{-j}{2}[\delta(f-f_0)-\delta(f+f_0)]$
 Periodic wave	$\left.\begin{array}{l}\sum_{n=-\infty}^{\infty} x_T(t-nT)\\ \sum_{n=-\infty}^{\infty}\alpha_n e^{j2\pi nt/T}\end{array}\right\}$	$\left[\frac{2\pi}{T}\sum_{n=-\infty}^{\infty} X_T\left(\frac{2\pi n}{T}\right)\cdot\delta\left(\omega-\frac{2\pi n}{T}\right)\right]$	$\sum_{n=-\infty}^{\infty}\alpha_n\delta\left(f-\frac{n}{T}\right)$
 Impulse train	$\Sigma\,\delta(t-nT)$	$\frac{2\pi}{T}\Sigma\,\delta\left(\omega-\frac{2\pi n}{T}\right)$	$\frac{1}{T}\Sigma\,\delta\left(f-\frac{n}{T}\right)$
 Complex sinusoid	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
 Unit ramp	$tu(t)$	$j\pi\delta'(\omega)-\frac{1}{\omega^2}$	$\frac{j}{4\pi}\delta'(f)-\frac{1}{4\pi^2 f^2}$
	t^n	$2\pi(j)^n\delta^{(n)}(\omega)$	$\left(\frac{j}{2\pi}\right)^n\delta^{(n)}(f)$

Preface

The purpose of this book, as expressed in the Preface to the first edition, is to present the most widely used techniques of signal and system analysis in a manner appropriate for instruction at the junior or senior level in electrical engineering. In this second edition, some changes in organization of the material have been made, a number of new topics have been added, several computer programs appropriate for numerical computation have been incorporated into the text, and most of the exercises and problems have been revised. Although the techniques discussed in this text are mathematical in nature, the discussion tends to be heuristic rather than rigorous and includes many examples to illustrate important features. The typical undergraduate courses in calculus provide sufficient mathematical background, and the only new mathematics introduced is that necessary to permit a logical development of the various new concepts. It is assumed that the student is familiar with the elements of circuit theory and can write loop and node equations for both passive and active circuits.

The various derivations and examples presented in the text include enough of the detailed steps so that the entire procedure can be followed by the student. An important adjunct to mastering the text material are the exercises included at the ends of most sections. These exercises are relatively straightforward applications of the material immediately preceding them. Their purpose is to reinforce the learning process by permitting an immediate application of this material before proceeding to a new topic. It is intended that all of the exercises be completed as an integral part of the text material. The answers to most exercises are given, although their order may be scrambled relative to the various parts of the exercise in order to provide an additional challenge.

In the second edition, the first three chapters of the first edition are combined into a single chapter to permit coverage of this introductory material with less classroom time. This chapter provides a broad overview of the problems of signal and system analysis and introduces terminology that will be used in subsequent discussions.

Chapter 2, Convolution, begins the development of the basic material of the text. In contrast to many other texts, the concept of convolution is introduced independent of its relationship to transform methods of analysis. There are several reasons why the authors feel that this sequence is preferable. First, the convolution integral is of fundamental importance in the study of linear systems, and an ability to graphically visualize convolution aids greatly in the interpretation of certain transform operations. Second, convolution provides an excellent opportunity for representing signals in terms of elementary functions and provides an excellent way to familiarize the student with the manipulation of singularity functions. Third, convolution is extended readily to cover discrete systems and time-varying systems and, thus, provides a simple and direct introduction to this area of analysis. The discussion of numerical convolution has been expanded, and a FORTRAN program for carrying out numerical convolution is included.

The treatment for Fourier transforms in Chapter 3 is more extensive than is usually found in undergraduate texts. This is because of their great importance in signal theory and the fact that there is a readily understood physical interpretation that can be associated with them. The first sections of this chapter relate to Fourier series and are included primarily for completeness. This material normally is covered in mathematics courses or in earlier engineering courses and may be omitted without loss of continuity in the development.

Discrete-time and sampled continuous-time signals and systems using Fourier methods are considered in Chapter 4. In this chapter, the discrete Fourier transform is discussed along with fast Fourier transform (FFT) techniques for its calculation. A FORTRAN program for computing the FFT is included along with illustrations of its use. Considerable stress is given to proper interpretation of the FFT, its relation to the continuous Fourier transform and its use in signal and system analysis.

Chapter 5 provides a broad coverage of the Laplace transform and its use in system analysis. Both the one-sided and two-sided transforms are included, and careful consideration is given to the relationship between the Laplace transform and the Fourier transform. Near the end of Chapter 5, the inversion of the Laplace transform by means of contour integration in the complex plane is considered. A discussion of this technique of integration is given in Appendix B. A clear picture of the significance of pole locations in rational transforms is presented in terms of the inversion integral. New material relating to negative feedback has been added to this chapter, and its effects on system transfer functions and stability are discussed.

Analysis of discrete signals and systems is considered in Chapter 6 by means of the z transform. The concept of the discrete transfer function is introduced, and an introduction to digital filtering is given. A more extensive discussion of inversion of the z transform is given here than in the previous edition. Also, a detailed discussion has been added concerning the requirements on discrete-time systems for their use in simulation of continuous-time systems. Examples of such simulation are provided.

The discussion of state space methods has been revised to include additional examples and a new section on time-varying systems.

The tables in the text and Appendix A have been extended to include some additional frequently used mathematical expressions, some additional Fourier transforms, and a substantial increase in the number of z transforms of both discrete and sampled time functions. All of the problems have been keyed to the section of the text to which they are most closely related.

There is more material contained in the text than can be covered in a single semester, and so a selection of topics must be made. In all chapters, the basic material is covered in the early sections, and more specialized topics are covered in later sections. Generally, sections near the end of a chapter can be omitted without loss of continuity in the development. A typical three-hour course might include: Chapter 1; the first seven sections of Chapter 2; Section 3-7 through 3-14 of Chapter 3; Chapter 4; Sections 5-1 through 5-15 of Chapter 5; and Sections 6-1 through 6-4 of Chapter 6. In curricula where the Laplace transform or convolution has been covered in previous courses, it should be possible to cover the remaining sections of Chapter 6 and part or all of Chapter 7. Other selections may be made to meet particular requirements.

The authors would like to acknowledge the many helpful suggestions received from faculty and students using the first edition of this text. Their suggestions have been the basis of the corrections and modifications incorporated into the second edition.

Clare D. McGillem
George R. Cooper

Contents

1
Signals and Systems

1-1 Introduction

This book is concerned with the methods of characterizing systems and signals and with the determination of the interaction of systems and signals. Systems can be studied in many ways. The detailed structure of the component parts and subsystems can be examined, and this information can be used to build up a comprehensive description of system operation, including the interactions of all the various internal components. This might properly be called a *microscopic system analysis*, meaning that the fine structure of the system is taken into account in the study. In general this type of analysis is extremely difficult in that it involves almost unlimited numbers of variables and enormous complexity in any mathematical description of the system.

The most useful and most frequently employed type of system analysis is what may be called *macroscopic system analysis*. In this type of analysis a system is characterized in terms of subsystems and components and their interactions with each other, and no account is taken of the details of their internal operations. It is this type of analysis that has proven highly effective in the solution of engineering problems, and it is this type of analysis that is the subject of this text.

Macroscopic system analysis requires that a system be broken down into a number of individual components. The behavior of the various components is then described in sufficient detail and in such a manner that the overall system operation can be predicted from appropriate calculations of the component behavior. The crux of this type of analysis is the description of the component behavior. For system analysis this is done in terms of a mathematical model, that is, a set of mathematical relationships that sufficiently characterizes the component so that its interaction with other components can be computed. For many components of engineering interest the mathematical model is an expression of the response of the component (or subsystem) to some type of forcing function. The forcing function is a variable in the mathematical model, as is the system response. All of these variables are categorized as *signals* in this text, although in an actual system they might be called displacement, voltage, Btu per hour, or they might even be such quantities as the price of a stock or the number of items maintained in an inventory.

Much light is shed on the subject of mathematical models as a direct result of attempting to use them in system analysis. It is vital for a system analyst to have a good understanding of the basis and validity of the models he or she is using in order to avoid serious errors in their application. However, the subject of developing good models is itself very involved and frequently requires study of a component on a microscopic scale to obtain useful results. This subject is most properly deferred until after the subject of system analysis has been covered, so that a clearer understanding of the model requirements is available.

Systems and system analysis. An engineer's analysis of a system is usually made to determine the response of a system to some input signal or excitation. Such studies are used to establish performance specifications, to aid in selection of components, to uncover system deficiencies, to explain unusual or unexpected behavior, or to meet any of a variety of other needs for quantitative data about system operations. In order to clarify some of the concepts relating to systems and system analysis, it is helpful to consider some specific examples.

The ignition system in an automobile is representative of a small system. It might consist of an energy source made up of the battery and alternator, the ignition switch, the distributor and points, the spark coil, the capacitor, the interference suppressors, the spark plugs, the interconnecting wires, and the general environment within which the system operates. By representing the various components by suitable mathematical models, an engineer could study analytically the behavior of the system under various ambient operating conditions and determine the effects of changing parameters of the components. For example, the power drain from the energy source could be calculated as a function of speed, or the effect of changing inductance on the spark energy could be determined. In many instances a final system design requires a combined analytical and experimental approach. The analytical studies establish the proper direction and ranges of variables for the experimental program and are invaluable in the interpretation of results. For example, in the case of the ignition system, the erosion of the breaker points for different materials and operating conditions could probably best be determined experimentally. Once a satisfactory operating range for the current and voltage has been determined, this can be used as a design specification to be preserved even though other parameters in the system are changed. Such a combined analytical and experimental approach is essential to the successful accomplishment of any engineering problem in which accurate and complete models do not exist.

When the system being considered becomes large, it is necessary to break it up into a number of subsystems, each of which can be analyzed separately. Appropriate models of these subsystems are then used to study the overall system. An example of this type of system is the control and communication system of a small earth-orbiting satellite, such as illustrated by the block diagram in Fig. 1-1. Most of the blocks in this diagram represent functional subsystems and are studied themselves by the methods of system analysis. Their design must be coordinated with overall system requirements, and these are based on an overall system study in which only the external performance characteristics of the subsystems are considered. In carrying out designs of this complexity some type of iterative procedure is always used. A preliminary design is made based on previous experience or engineering judgment. The implications of specifications resulting from this design are then determined by

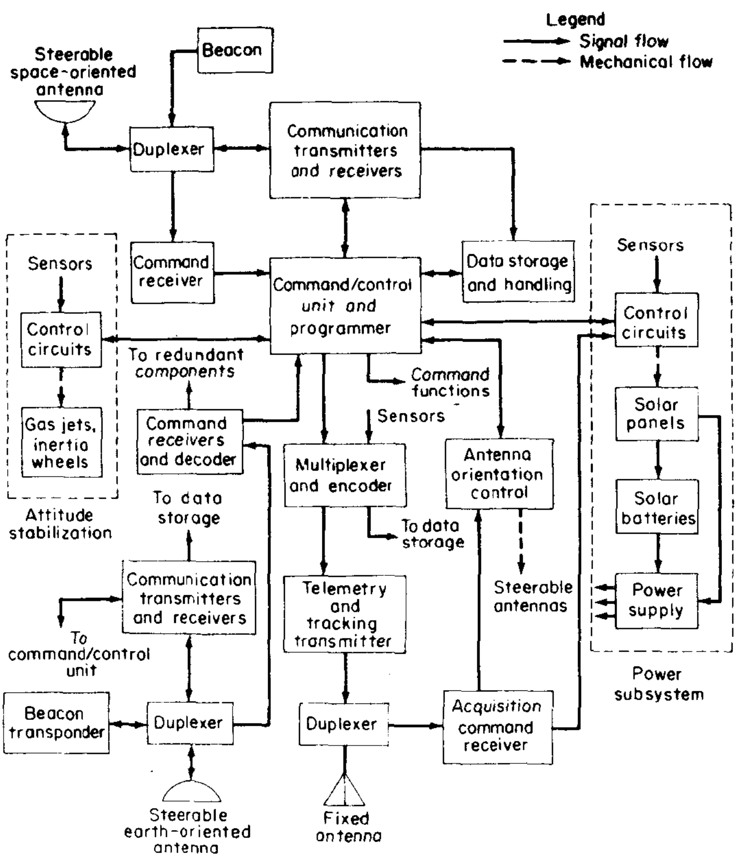


Fig. 1-1. Spacecraft control and communication system.

analysis, and modifications made to improve performance or feasibility. The modified design is then reanalyzed, and further changes are made as required. This refining operation continues throughout the design phase and may extend into the fabrication phase if specifications cannot be met.

Many systems of practical engineering interest cannot be analyzed as a whole. A number of factors lead to this situation, two of the most common being the enormous complexity of the system and lack of satisfactory mathematical models for the components or subsystems. The power system for a large city is a good example of a system that is too large and complex for a comprehensive analysis. Such a system has several inputs from generating plants and coupling with other power systems and has a huge quantity and variety of outputs consisting of users. The nature of the outputs varies with such things as time of day, week, or year; weather; whims of consumers; and installation of new or different equipment at the output terminals. Because it is impossible to determine exactly what the system loading will be at any time, it is usually necessary to rely on past experience and to use probabilistic and statistical methods of analysis to arrive at suitable criteria for selecting system parameters. Other systems such as urban transportation systems involving social, political, and technological problems are even less susceptible to presently known techniques of system analysis. Research is in progress on methods for handling such systems, and in the years to come it seems likely that new tools will be made available to the system analyst for handling such problems.

The system analysis methods discussed in the following chapters have proved remarkably effective in a wide range of system applications. Their direct application, or some kind of modification or extension of them, provides the basis of most engineering system analysis today. Most of the detailed examples of systems considered in later chapters are drawn from the field of electrical engineering. The reason for this is that such examples are simple to visualize and are easily extended to cover a wide variety of problems. It is through studies of electrical engineering problems that some of the most successful techniques of system analysis have evolved. However, even though the examples relate primarily to electrical signals and systems, the techniques are very general and can be applied to any system that can be described by comparable mathematical models.

Signals. In order to study and analyze a system properly, it is necessary to study the means by which energy is propagated through the system. In most systems this can be done by specifying how varying quantities within the system change as functions of time. Such a varying quantity is referred to as a *signal* in this text, even though it may actually be a force, voltage, power, volume per unit time, or any of an almost unlimited number of other variables. These signals measure the excitations and responses of systems and are indispensable in describing the interactions among various components and subsystems making up the complete system.

In complicated systems there are frequently many inputs and many outputs.

The number of inputs and outputs does not have to be the same. For example, in an aircraft control system the motion of a single lever can change several different aerodynamic surfaces in a manner designed to produce an optimum result.

Besides their usefulness in analyzing system performance, signals also have importance in their own right as a means of carrying information from one point to another. Often times a system is designed for the purpose of transmitting or processing signals. In such cases the signals themselves are of primary concern, and the analyst's problem is to determine how a particular component or subsystem affects a signal propagating through the system. The problem of selecting signals for various engineering applications is of major importance and has led to the development of an extensive body of knowledge referred to as *signal theory*. Designing radar or sonar signals or choosing the proper modulation for a communication system are typical applications of signal theory. It is because of the intrinsic importance of signal theory, in addition to its role in system studies, that it is given a prominent place in this text.

System analysis and system design. The principal problem in system analysis is finding the response of a particular system to a specified input or range of inputs. Such results represent an important part of system studies by meeting the following needs:

1. When the system does not physically exist (as in the case of feasibility studies, for example), only mathematical analysis is possible.
2. Experimental evaluation of systems is often more difficult and expensive than analytical studies.
3. In some cases it is necessary to study systems under conditions that are too dangerous for actual experimentation. Examples of this are the operation of a nuclear power plant at fission rates that are too high for safety or the response of an aircraft flight control system under conditions of severe turbulence.

In addition to the direct use of the results of system analysis indicated above, there is the equally or perhaps more important application to system design. The system design problem is that of determining the necessary system characteristics to yield a desired response to a specified input. Frequently system designs are accomplished by means of parametric studies in which system performance is computed for a variety of cases in which parameters of the system are changed over appreciable ranges. From these analytical results specifications are made as to the parameter values required to give the desired performance. This method can generally be used when specific synthesis techniques are not available. It is in the area of design that a system engineer finds some of his most challenging and rewarding work. In order to be successful he or she must be creative, resourceful, and knowledgeable about the great