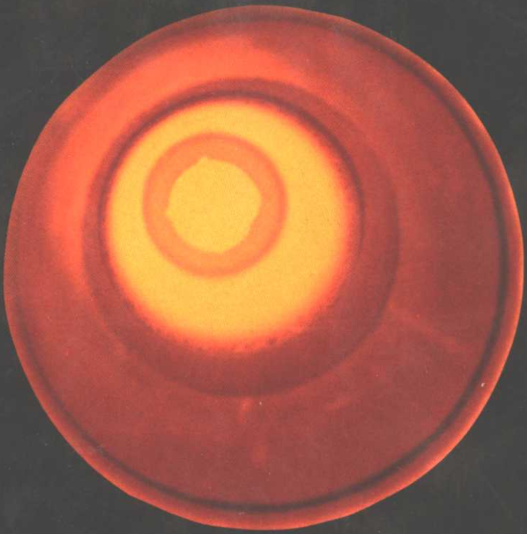
RADIATIVE HEAT TRANSFER



MICHAEL F. MODEST

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The Pennsylvania State University

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ABOUT THE AUTHOR

Michael F. Modest was born in Berlin and spent the first half of his life in Germany. After receiving his Dipl.-Ing. degree from the Technical University in Munich, he came to the United States, and in 1972 obtained his M.S. and Ph.D. in Mechanical Engineering from the University of California at Berkeley, where he was first introduced to theory and experiment in thermal radiation. Since then, he has carried out many research projects in all areas of radiative heat transfer (measurement of surface, liquid, and gas properties; theoretical modeling for surface transport and within participating media). Since many laser beams are a form of thermal radiation, his work also encompasses the heat transfer aspects in the field of laser processing of materials.

For several years he has taught at Rensselaer Polytechnic Institute and the University of Southern California, and he is currently Professor of Mechanical Engineering at the Pennsylvania State University at University Park, PA. He is a fellow of the American Society of Mechanical Engineers and a member of the American Institute of Aeronautics and Astronautics and the Laser Institute of America. Dr. Modest resides in Boalsburg, PA, with his wife, two children, a dog, a cat, and an assortment of other animals.

To the m&m's in my life,
Monika, Mara, *and* Michelle

PREFACE

Over the past 30 years the analysis of radiative heat transfer has received increasing attention. This was first due to the advent of the space age, which made it necessary to develop tools to predict heat transfer rates in such high-temperature applications as rocket nozzles and space vehicle reentry, and in vacuum applications for space craft in outer space. Following a lull during the 1970s and early 1980s, interest in radiative heat transfer has recently increased again because of the need to predict and measure heat transfer rates in ever higher temperature applications in furnaces, engines, MHD generators, and the like.

Today a course on radiative heat transfer is offered by virtually every university in the country with a graduate program in mechanical and/or aeronautical engineering; indeed, it is a required core course for most thermal science majors. A few universities, such as The Pennsylvania State University, with large programs in the fields of combustion, propulsion, etc., sometimes offer a second course on thermal radiation, covering a number of special topics.

The objectives of this book are more extensive than to provide a standard textbook for a one-semester core course on thermal radiation, since it does not appear possible to cover all important topics in the field of radiative heat transfer in a single graduate course. A number of important areas that would not be part of a "standard" one-semester course have been treated in some detail. It is anticipated that the engineer who may have used this book as his or her graduate textbook will be able to master these advanced topics through self-study. By including all important advanced topics, as well as a large number of references for further reading, the book may also be used as a reference book by the practicing engineer.

In each chapter all analytical methods are developed in substantial detail, containing a number of examples to show how the developed relations may be applied to practical problems. At the end of each chapter a number of exercises are included to give the student additional opportunity to familiarize him- or herself with the application of analytical methods developed in the preceding sections. The breadth of the description of analytical developments is such that any scientist with a satisfactory background in calculus and differential equations will be able to grasp the subject through self-study—for example, the heat transfer engineer involved in furnace calculations, the architectural engineer interested in lighting calculations, the oceanographer concerned with solar penetration into the ocean, or the meteorologist who studies atmospheric radiation problems.

The book is divided into 20 chapters, covering the four major areas in the field of radiative heat transfer. After the Introduction, there are two chapters dealing with theoretical and practical aspects of radiative properties of opaque surfaces, including a brief discussion of experimental methods. These are followed by four chapters dealing with radiative exchange between surfaces in an enclosure without a "radiatively participating" medium. The rest of the book deals with radiative transfer through absorbing, emitting, and scattering media (or "participating media"). After a detailed development of the equation of radiative transfer, radiative properties of gases, particulates, and semitransparent media are discussed, again including brief descriptions of experimental methods. Finally, over the last nine chapters the theory of radiative heat transfer through participating media is covered, separated into a number of basic problem areas and solution methods.

I have attempted to write the book in a modular fashion as much as possible. Chapter 2 is a fairly detailed (albeit concise) treatment of electromagnetic wave theory, which can (and will) be skipped by most instructors for a first course in radiative heat transfer. The chapter on opaque surface properties is self-contained and is not required reading for the rest of the book. The four chapters on surface transport (Chapters 4 through 7) are also self-contained and not required for the study of radiation in participating media. Similarly, the treatment of participating medium properties is not a prerequisite to studying the solution methods. Finally, any of the different solution aspects and methods discussed in Chapters 12 through 19 may be studied in any sequence.

I have not tried to mark those parts of the book that should be included in a one-semester course on thermal radiation, since I feel that different instructors will, and should, have different opinions on that matter. Indeed, the relative importance of different subjects may not only vary with different instructors, but also depend on student background, location, or the year of instruction. My personal opinion is that a one-semester course should touch on all four major areas (surface properties, surface transport, properties of participating media, and transfer through participating media) in a balanced way. For the average U.S. student who has had very little exposure to thermal radiation during his or her undergraduate heat transfer experience, I suggest that about half the course be devoted to Chapters 1, 3, 4, 5, and 6 (leaving out the more advanced features). The second half should be devoted to Chapters 8, 9, and 10 (again omitting less important features); some coverage of Chapter 12; and a thorough discussion of Chapter 13. If time permits (primarily, if surface properties and surface transport are treated in less detail than suggested above), I suggest to cover the P_1-approximation (which may be studied by itself, as outlined in the beginning of Chapter 14) and a portion of Chapter 17 (solution methods for nongray media). A second, special-topic course could include detailed discussions of radiative properties (Chapters 2, 3, 9, 10, and 11) and/or special solution methods (such as P_N and S_N approximations, nongray media, or zonal and Monte Carlo methods).

At this point the eager beaver is ready to embark on his or her journey through this book. For the inquisitive of mind and/or procrastinator at heart, here is a short account of *why* and *how* this book was written. Having taught a graduate course on radiative heat transfer every year for many years, I have always lamented the fact that there appeared to be no book that adequately treats all the subjects I feel belong in such a course. In particular, good discussions of radiative properties of, and heat transfer in, participating media are difficult to find. This provided the desire, but not the fire, for this book. The fire came in early 1987, when I was exposed to TEX, a mathematical typesetting language that may be familiar to a large number of readers. Although I began as just a user, TEX quickly became an obsession leading to many homemade macros (which are used throughout this book) and, finally, the development of a preprocessor that allows the typing of text and equations in near-WYSIWIG (what-you-see-is-what-you-get) fashion.

McGraw-Hill and I would like to thank the following reviewers for their many helpful comments and suggestions: D. K. Edwards, University of California–Irvine; James D. Felske, State University of New York–Buffalo; Woodrow A. Fiveland, Babcock and Wilcox; John R. Lloyd, Michigan State University; Theodore Smith, The University of Iowa; and Timothy Tong, Arizona State University.

My thanks go to Ms. Eileen Stephenson, who typed almost all of the manuscript, becoming quite a TEXpert in her own right, hammering out more equations during one day than I could proofread in one evening. Thanks also go to my colleague, Stefan Thynell, who patiently read and criticized a large part of the manuscript. Finally, last but not least, I am grateful to my family, who endured losing their husband and daddy for five long years.

Michael F. Modest

LIST OF SYMBOLS

The following is a list of symbols used frequently in this book. A number of symbols have been used for several different purposes. Alas, the Roman alphabet has only 26 lowercase and another 26 uppercase letters, and the Greek alphabet provides 34 more different ones, for a total of 86, which is, unfortunately, not nearly enough. Hopefully, the context will always make it clear which meaning of the symbols is to be used. I have used what I hope is a simple and uncluttered set of variable names. This usage, of course, comes at a price. For example, the subscript "λ" is often dropped (meaning "at a given wavelength," or "per unit wavelength"), assuming that the reader recognizes the variable as a spectral quantity from the context. Whenever applicable, units have been attached to the variables in the following table. Variables without indicated units have multiple sets of units. For example, the units for total band absorptance depend on the spectral variable used (λ, η, or ν), and on the absorption coefficient (linear, density, or pressure based), for a total of nine different possibilities.

a	semimajor axis of polarization ellipse, [N/C]
a	plane-polarized component of electric field, [N/C]
a	particle radius, [cm]
a_k	weight factors for sum-of-gray-gases, [$-$]
a_n, b_n	Mie scattering coefficients, [$-$]
A	total band absorptance (or effective band width)
A^*	nondimensional band absorptance $= A/\omega$, [$-$]
A, A_n	slab absorptivity (of n parallel sheets), [$-$]
A, A_p	area, projected area, [cm^2]
A_m	scattering phase function coefficients, [$-$]

A_{ij}, B_{ij}	Einstein coefficients
b	line half-width
b	self-broadening coefficient, [−]
b	semiminor axis of polarization ellipse, [N/C]
B	rotational constant
Bo	convection-to-radiation parameter, (Boltzmann number), [−]
c, c_0	speed of light, (in vacuum), [cm/s]
c	specific heat, [kJ/kg K]
C_1, C_2, C_3	constants for Planck function and Wien's displacement law
C_1, C_2, C_3	wide band parameters for outdated model
d	line spacing
D	diameter [cm]
D, D^*	detectivity (normalized), [1/W] ([cm $\text{Hz}^{1/2}$/W])
$\hat{\mathbf{e}}$	unit vector into local coordinate direction, [−]
E, E_b	emissive power, blackbody emissive power
$\mathbf{E}$	electric field vector, [N/C]
E_n	exponential integral of order n, [−]
f_v, f_s, f_l	volume, solid, liquid fractions, [−]
$f(n\lambda T)$	fractional blackbody emissive power, [−]
F_{i-j}	(diffuse) view factor, [−]
F^s_{i-j}	specular view factor, [−]
$\mathcal{F}_{i\to j}$	radiation exchange factor, [−]
g_k	degeneracy, [−]
g	nondimensional incident radiation, [−]
$\overline{g_i s_j}, \overline{g_i g_k}$	direct exchange areas in zonal method, [cm^2]
$\overline{\mathbf{gs}}, \overline{\mathbf{gg}}$	direct exchange area matrix, [cm^2]
G	incident radiation = direction-integrated intensity
$\overline{G_i S_j}, \overline{G_i G_k}$	total exchange areas in zonal method, [cm^2]
$\overline{\mathbf{GS}}, \overline{\mathbf{GG}}$	total exchange area matrix, [cm^2]
h	Planck's constant, $= 6.6262 \times 10^{-34}$ J s
h	convective heat transfer coefficient, [$\text{W/cm}^2\text{K}$]
H	irradiation onto a surface
H	Heaviside's unit step function, [−]
H	nondimensional heat transfer coefficient, [−]
$\mathcal{H}$	nondimensional irradiation onto a surface, [−]
$\mathbf{H}$	magnetic field vector, [C/cm s]
$\hat{\mathbf{i}}$	unit vector into the x-direction, [−]
I	intensity of radiation
I	first Stokes' parameter for polarization, [N^2/C^2]
I	moment of inertia, [kg cm^2]
I_b	blackbody intensity (Planck function)

I_l, I_l^m	position-dependent intensity functions
I_0, I_1	modified Bessel functions, [−]
$\Im$	imaginary part of complex number,
j	rotational quantum number, [−]
$\hat{\mathbf{j}}$	unit vector into the y-direction, [−]
J	radiosity, [W/cm^2]
$\mathcal{J}$	nondimensional radiosity, [−]
k	thermal conductivity, [W/m K]
k	Boltzmann's constant, $= 1.3806 \times 10^{-23}$ J/K
k	absorptive index in complex index of refraction, [−]
$\hat{\mathbf{k}}$	unit vector into the z-direction, [−]
K	kernel function
K	luminous efficacy, [lm/W]
l, m, n	direction cosines with x-, y-, z-axis, [−]
L	length, [cm]
L	latent heat of fusion, [J/kg]
L	luminous intensity or luminance
L_e	mean beam length, [cm]
L_0, L_m	geometric, or average mean beam length, [cm]
m	mass, [kg]
m	complex index of refraction, [−]
$\dot{m}$	mass flow rate, [kg/s]
n	self-broadening exponent, [−]
n	refractive index, [−]
n	number distribution function particle for particles, [cm^{-4}]
$\hat{\mathbf{n}}$	unit surface normal (pointing away from surface into the medium), [−]
N	conduction-to-radiation parameter, (Stark number), [−]
N_c	conduction-to-radiation parameter, [−]
N_T	number of particles per unit volume, [cm^{-3}]
Nu	Nusselt number, [−]
$\mathcal{O}$	order of magnitude, [−]
p	pressure, [atm]; radiation pressure, [N/m^2]
P	probability function, [−]
P_l, P_l^m	(associated) Legendre polynomials, [−]
Pr	Prandtl number, [−]
q, $\mathbf{q}$	heat flux, heat flux vector [W/cm^2]
q_{lum}	luminous flux, [lm/m^2 = lx]
Q	heat rate, [W]
Q	second Stokes' parameter for polarization, [N^2/C^2]
$\dot{Q}'''$	heat production per unit volume, [W/cm^3]
r	radial coordinate, [cm]

r	reflection coefficient, [−]
$\mathbf{r}$	position vector, [cm]
R	radius, [cm]
R	random number, [−]
R	radiative resistance, $[\text{cm}^{-2}]$
R, R_n	slab reflectivity (of n parallel sheets), [−]
$\Re$	real part of complex number
Re	Reynolds number, [−]
s	geometric path length, [cm]
$\hat{\mathbf{s}}$	unit vector into a given direction, [−]
$\overline{s_i s_j}, \overline{s_i g_k}$	direct exchange areas in zonal method, $[\text{cm}^2]$
$\overline{\mathbf{ss}}, \overline{\mathbf{sg}}$	direct exchange area matrix, $[\text{cm}^2]$
S	distance between two zones, or between points on enclosure surface, [cm]
S	line-integrated absorption coefficient = line intensity
S	radiative source function
$\mathbf{S}$	Poynting vector, $[\text{W/cm}^2]$
St	Stanton number, [−]
Ste	Stefan number, [−]
$\overline{S_i S_j}, \overline{S_i G_k}$	total exchange areas in zonal method, $[\text{cm}^2]$
$\overline{\mathbf{SS}}, \overline{\mathbf{SG}}$	total exchange area matrix, $[\text{cm}^2]$
t	time, [s]
t	transmission coefficient, [−]
t	fin thickness, [cm]
$\hat{\mathbf{t}}$	unit vector in tangential direction, [−]
T	temperature, [K]
T, T_n	slab transmissivity (of n parallel sheets), [−]
u	internal energy, [kJ/kg]
u	radiation energy density
u	velocity, [cm/s]
u_k	nondimensional transition wavenumber, [−]
U	third Stokes' parameter for polarization, $[\text{N}^2/\text{C}^2]$
v	vibrational quantum number, [−]
v	velocity, [cm/s]
$\mathbf{v}$	velocity vector [cm/s]
V	volume, $[\text{cm}^3]$
V	fourth Stokes' parameter for polarization, $[\text{N}^2/\text{C}^2]$
$\mathbf{w}$	wave vector, $[\text{cm}^{-1}]$
w_i	quadrature weights, [−]
W	equivalent line width
x, y, z	Cartesian coordinates, [cm]

x	particle size parameter, [−]
x	line strength parameter, [−]
X	optical path length
X	interface location, [cm]
y	mole fraction, [−]
Y_l^m	spherical harmonics, [−]
z	nondimensional spectral variable, [−]
α	absorptance or absorptivity, [−]
α	band-integrated absorption coefficient = band strength parameter
α	opening angle, [rad]
α	thermal diffusivity, [m^2/s]
β	extinction coefficient
β	line overlap parameter, [−]
γ	line overlap parameter for dilute gas, [−]
γ	complex permittivity, [$C^2/N\,m^2$]
γ	azimuthal rotation angle for polarization ellipse, [rad]
γ	oscillation damping factor, [Hz]
γ_E	Euler's constant, $= 0.57221\ldots$
δ	Dirac-delta function, [−]
δ	polarization phase angle, [rad]
δ_{ij}	Kronecker's delta, [−]
δ_k	vibrational transition quantum step $= \Delta v$, [−]
ϵ	emittance or emissivity, [−]
ϵ	energy level, [J]
ϵ	electrical permittivity, [$C^2/N\,m^2$]
ε	complex dielectric function, or relative permittivity, $= \varepsilon' - i\varepsilon''$, [−]
η	wavenumber, [cm^{-1}]
η	direction cosine, [−]
η	nondimensional (similarity) coordinate, [−]
η_{lum}	luminous efficiency, [−]
θ	polar angle, [rad]
θ	nondimensional temperature, [−]
Θ	scattering angle, [rad]
κ	absorption coefficient
λ	wavelength, [μm]
μ	dynamic viscosity, [kg/m s]
μ	magnetic permeability, [$N\,s^2/C^2$]
μ	direction cosine (of polar angle), $= \cos\theta$, [−]
ν	frequency, [Hz]
ν	kinematic viscosity, [m^2/s]
ξ	direction cosine, [−]

ξ	nondimensional coordinate, [−]
ρ	reflectance or reflectivity, [−]
ρ	density, [g/cm^3]
ρ_f	charge density, [C/m^3]
σ	Stefan-Boltzmann constant, $= 5.670 \times 10^{-8}$ W/m^2 K^4
σ_s	scattering coefficient
$\sigma_e, \sigma_{\rm dc}$	electrical conductivity, dc-value, [C^2/N m^2 s $= 1/\Omega$ m]
σ_m	root-mean-square roughness, [cm]
τ	transmittance or transmissivity, [−]
τ	optical coordinate, optical thickness, [−]
ϕ	phase angle, [rad]
Φ	scattering phase function, [sr^{-1}]
Φ	nondimensional medium emissive power function
Φ	temperature function for line overlap β, [−]
Φ	dissipation function, [J/kg m^2]
ψ	azimuthal angle, [rad]
ψ	stream function, [m^2/s]
Ψ	temperature function for band strength α, [−]
Ψ	nondimensional heat flux
ω	single scattering albedo, [−]
ω	angular frequency, [rad/s]
ω	relaxation parameter, [−]
Ω	solid angle, [sr]

Subscripts

0	reference value, or in vacuum, or at length = 0
1, 2	in medium, or at location, "1" or "2"
a	absorbing, or apparent
av	average
b	blackbody value
B	band integrated value
c	at band center, or at cylinder, or critical value, or denoting a complex quantity,
C	collision
D	Doppler, or based on diameter
e	effective value, or at equilibrium
g	gas
i	incoming, or dummy counter
j	at a rotational state, or dummy counter
L	at length $= L$
m	modified Planck value, or medium value, or mean (bulk) value

n	in normal direction
o	outgoing, or from outside
p	related to pressure, or polarizing value, or plasma
P	Planck-mean
r	reflected component
ref	reference value
R	Rosseland-mean, or at $r = R$
s	along path s, or at surface, or at sphere, or at source
sol	solar
t	transmitted component
u	upper limit
v	at a vibrational state, or at constant volume
w	wall value
W	value integrated over spectral windows
x, y, z, r	in a given direction
θ, ψ	in a given direction
η	at a given wavenumber, or per unit wavenumber
λ	at a given wavelength, or per unit wavelength
ν	at a given frequency, or per unit frequency
$\parallel$	polarization component, or situated in plane of incidence
$\perp$	polarization component, or situated in plane perpendicular to plane of incidence

Superscripts

$'$ $''$	real and imaginary parts of complex number, or directional values, or dummy variables
$\frown$	hemispherical value
$*$	complex conjugate, or obtained by P_1-approximation
$+, -$	into "positive" and "negative" directions
d	diffuse
s	specular
$\bar{\ }$	average value
$\tilde{\ }$	complex number, or scaled value (for nonisothermal path), or Favre average
$\hat{\ }$	unit vector

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