BURTON

Vibration and Impact

VIBRATION
AND
IMPACT

PREFACE

This text, designed for senior and introductory graduate courses, is a broad introduction to the subject. It should serve as a stepping stone between introductory dynamics and the more advanced fields of engineering analysis. The manner of presentation reflects the writer's view that an understanding of basic principles of vibration (and oscillatory phenomena in general) is needed by engineering students, regardless of what their special interests may be. Because of the wide range of subject matter and the many occasions for application of mathematical analysis and physical reasoning, important educational values are inherent in the study of vibration.

The development of the theory as a carefully planned sequence of concepts makes possible the inclusion of much material not ordinarily dealt with in an introductory course. Discussion of such topics as transient phenomena, self-excited vibration, and control systems is made without sacrifice of problem-solving at every step of the way. A large number of worked and unworked problems are distributed throughout the text, ranging from simple applications of theory to derivations which require constructive thought on the part of the student. These problems are directed toward the application of theory to practical situations and also to the clarification of theoretical distinctions.

The writer has simplified the presentation wherever possible and has tried to make the text thought-provoking and stimulating to the student and to others who may be interested in the subject.

Undoubtedly there are errors in the text. It is hoped that the reader who finds errors will inform the publisher so that later printings may be improved. Also, since much of the material came to the author in an informal way, through conversation and observation, it is possible that credit to the proper sources may sometimes be omitted. This will be quickly rectified if such omissions are pointed out.

Many persons have contributed to the preparation of this book. Among those to whom I am especially grateful are: Prof. John Lee, Mr. Everett Reed, Mrs. Olga Crawford, Dr. Paul Paslay, Mrs. Mary Regnier, and Mrs. Thomas Bell. In addition I should note that my interest in this field was initially stimulated by the lectures of Dr. Dana Young, Dr. M. V. Barton, Dr. R. P. Felgar, and Dr. W. J. Carter.

R. B.

Columbia, Missouri
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CHAPTER 1

INTRODUCTION

The study of mechanical vibration is concerned with *systems in oscillatory motion and systems subjected to pulsating forces.* There is little to be gained from a more specific definition than this. On the contrary, there is positive value in leaving the boundaries weakly defined, letting vibration analysis extend as far as it may usefully go into other fields. This point of view will become clearer as we review the history of the science and examine its position in present-day engineering.

1-1 Knowledge of vibration in earliest history. In searching for the beginnings of man's interest in vibration, we might realistically date the first vestiges of a "scientific interest" from the discovery of musical instruments. Once the first instrument (probably a whistle or a drum) came into existence, ingenuity and critical investigation must have been applied to successive improvements. If a man hears the sound of a plucked string, for example, he does not automatically know how to construct a lyre with many carefully matched strings; yet lyres were developed over 4600 years ago.

Early musicians and philosophers sought out the rules and laws of sound production, applied them to instrument design, and passed them on from generation to generation. Through the centuries, they built up a considerable backlog of information. They knew the relationship between small size and high pitch, and between large size and low pitch; they understood the physics of the vibrating string well enough to design a variety of bowed and plucked instruments; and they understood the principle of the sounding board that amplifies a tone from a string attached to it.

Musical scales must also have evolved from a series of discrete experimental discoveries. Although notes were incorporated into scales simply because they "sounded right," even very early studies showed that these scales embody precise mathematical relationships. For example, the Greek philosopher and mathematician Pythagoras (582–507 B.C.) conducted experiments (Fig. 1–1) showing among other things that *if two like strings are subjected to equal tension, and if one is half the length of the other, the tones they produce will be an octave apart.* This is one of the many precise relationships governing the vibrational origin of music. It is interesting to note that such a highly subjective and personal art as music is closely regulated by mathematical laws.

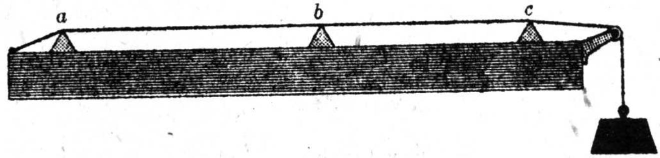

FIG. 1-1. Monochord (or sonometer), invented by Pythagoras and used in his studies of the vibrating string. Bridges *a* and *c* are fixed, and bridge *b* is movable. The weight holds constant tension in the string.

1-2 Early experiments. It had long been understood that sound was related to the vibration of a mechanical system, but not until Galileo Galilei (1564–1642) attacked the problem was it clear that *pitch* is determined by the *frequency* of vibration. This early physicist was also interested in other aspects of mechanical vibration. He discovered the relationship between the length of a pendulum and its frequency, and he was familiar with the isochronism of the simple pendulum, applying this to time measurement. He was aware of the sympathetic vibration (resonance) of two bodies tuned to the same natural frequency and connected by some energy transfer medium.

When Galileo considered the problem of relating pitch and vibrational frequency, he originated a simple and ingenious experiment. He contrived to scrape a brass plate with a chisel in such a way that when the plate emitted a tone, a dashed line was cut on it by the chisel point. In comparing the lines produced by high-pitched and low-pitched tones, he noted that the dashes were shorter and closer together for the high-pitched tones. Furthermore, when he developed a way to estimate the musical interval between these tones, he found that for the interval called a *fifth*, the ratio of frequencies was 3:2. (For other frequency relationships in the scale, see Table 1-1.)

TABLE 1-1

FREQUENCY RELATIONS IN THE MAJOR SCALE

C	D	E	F	G	A	B	C
$\frac{1}{1}$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$

Another investigator, Robert Hooke (1635–1648), showed the relationship between frequency and pitch by engaging a card with a turning cogwheel and observing the tone produced by the teeth. Elegant experiments on this same subject were also conducted by Joseph Sauveur (1653–1716). This experimenter made use of the phenomenon known as *beating*, where two tones sounded together produce a third tone of lower pitch. This low frequency (6 cycles/sec in his experiment) could easily be observed, and from it information about the experimental tones could be obtained.

1-3 The vibrating string. The study of the vibrating string has formed the transition between music and the field of mechanical vibrations as we know it today. Investigation of this relatively simple vibratory system was a fortunate choice, since the string embodies several general principles in such a way that they were easily discovered. The more man learned about the vibrating string, the more he learned about vibration in general and about mathematics itself! Galileo understood the interrelationships among density, tension, length, and frequency for the vibrating string. Marin Mersenne (1588-1703) is credited with having anticipated Galileo's writings on the subject when he published in 1636 a valid account of the vibrating string; even long before Mersenne's time the lyre makers obviously had a working knowledge of these variables.

Mersenne's contemporary, Sauveur, is credited with the first attempt at actually computing the frequency of the vibrating string, but not until Brook Taylor (1685-1731) achieved this goal, *circa* 1713, did computed results agree with the experiments of Galileo and Mersenne. The mathematician Taylor propounded *Taylor's theorem*, an application of infinite series which we shall have occasion to use within this text. Other outstanding mathematicians whose contributions to the understanding of the vibrating string will be further discussed are Daniel Bernoulli (1700-1782), Jean d'Alembert (1717-1783), Leonard Euler (1707-1783), Joseph Lagrange (1736-1813), and J. B. J. Fourier (1768-1830).

1-4 Harmonics. A highly important development arose from the observation that a string *does not vibrate in a simple manner with a single natural frequency*. A second type of vibration can be observed when portions of the string stand still while other portions are in active motion. The stationary points are called *nodes*, and the vibrational configurations corresponding to the several natural frequencies are called *modes* (Fig. 1-2). The *first* or *fundamental* mode has, by definition, the lowest frequency. For a string, this consists of motion in a half-sine-wave configuration, with the ends stationary. The second mode has a node at the halfway point, vibrating as a full sine wave. Fortuitously, this mode has a natural frequency exactly twice that of the first mode or, in other words, one octave above the fundamental. John Wallis (1616-1703) and Sauveur are independently credited with first observing the phenomenon of modes; Sauveur coined the name "fundamental" for the lowest frequency and "harmonics" for the others. Investigators of these modes of motion soon recognized the string to be a system with an *infinite number of degrees of freedom*, having an *infinite number of natural frequencies*.

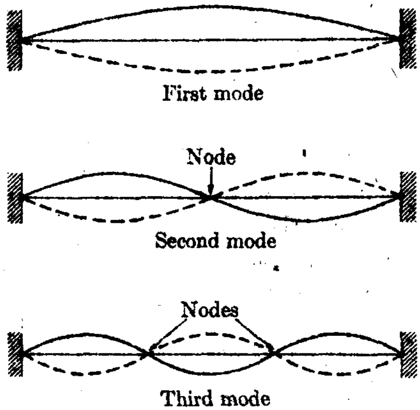

FIG. 1–2. Modes of a vibrating string. The frequency of the second mode is twice that of the first, and the frequency of the third mode is three times that of the first.

The mathematician Daniel Bernoulli first proposed the principle of *linear superposition of harmonics*. This holds that in any general configuration of free vibration, the contour assumed by the string is made up of the individual harmonics acting collectively, independently, and in varying strengths. Not long after this concept became known, d'Alembert and Euler foresaw that it led to the expression of almost any type of curve or function as a series of harmonics.* Fourier finally established the method of *harmonic series* and in 1822 he presented it in his *Analytical Theory of Heat*. The very fact that he used this method in the study of heat flow rather than vibration illustrates its generality. The Fourier expansion is nowadays held to be a singularly important tool of mathematics; yet its introduction was strongly resisted by leading mathematicians.

It should be pointed out that the vibrations of bars, plates, shafts, and other systems with which the engineer must contend are not as simple as those of the string. The modes cannot be represented by sine curves, and their frequencies are not usually related as even multiples of the fundamental (in which case they are called "inharmonics"). The basic principles discovered for the string still hold, however, and it is possible to express configurations in terms of the modes by an expansion like that of Fourier, even though the individual modes are not sine curves.

1–5 Waves. By the middle of the 18th century, mathematics had developed to the point where the vibrating string equations could be written much as they are today, making use of partial derivatives. About this

*Strangely enough, they used this observation as a basis for rejecting the principle of linear superposition.

time (1759), Lagrange was able to predict the frequencies of open and closed organ pipes, and it is interesting to note that the equation for the string as set up by d'Alembert (*circa* 1750) and that for the organ pipe are identical. This equation is now termed the *wave equation*, although the wave character of this type of vibration was not recognized at once. In the case of organ pipes it is quite clear that the vibration can be interpreted in terms of waves racing from end to end, and when both the distance the wave must traverse and the velocity of sound in air are known, it is not difficult to compute the natural frequencies of any given pipe. Although the wave interpretation of the vibrating string is not so easily visualized, it is nevertheless valid—and through it mechanical vibration is inextricably tied to wave mechanics and even to supersonic aerodynamics.

1–6 Vibration of structural elements. When Hooke's law of elasticity had been enunciated (1676) it became possible to analyze vibrating beams. Euler (1744) and Bernoulli (1751) are credited with first accomplishing this task, and both made use of the law of conservation of energy as a basis for their calculations. This method was further developed by Lord Rayleigh, and is known to us as *Rayleigh's method*. The study of plates and membranes came much later and claimed the interest of such mathematicians as G. R. Kirchhoff (1824–1887) and S. D. Poisson (1781–1840).

Aurel Stodola (1859–1943) is a notable modern contributor to the study of vibrating beams, plates, and membranes. His principal subject of study was the turbine, and he found that vibrational analysis was necessary for workable turbine designs. Stodola originated a method for analyzing vibrating beams that is also applicable to those turbine blades which resemble cantilever beams.

1–7 Vibration of machines. Every major type of prime mover has given rise to vibrational problems. The diesel engine is well known for its ability, in certain instances, to send out ground waves sufficiently powerful to constitute a nuisance in urban areas. Steam locomotives were notorious for pounding the rails that supported them; motion pictures have shown the wheels periodically rising half an inch off the track at high speeds. More recently, turbines have bowed to the destructive power of vibration in the form of spectacular mechanical failures. Almost from the beginning, turbine designers were vibration-conscious, and one of the earliest, C. G. P. De Laval (1845–1913), originated a practical solution to the problem of vibration of an unbalanced rotating disk. When strong steel shafts failed in his speed turbine, he mounted the rotor on a shaft made from a bamboo fishing rod. Not only did this survive up to speeds as high as 100,000 rev/min, but in addition it eliminated the vibratory effects of the unbalanced rotor.

Stodola, in his book *Steam and Gas Turbines,* dealt almost as extensively with vibrations as he did with thermodynamics, and recently, Curt Keller* of Escher Wyss, Ltd., in describing the development of the closed cycle gas turbine, wrote: "Not a single tube in any air heater has been damaged in normal service and no high temperature blading or shaft either. All worries were encountered with so-called normal things such as auxiliaries, bearings, shaft vibrations . . ." This clearly indicates that our understanding of rotating machinery has not developed to the point where vibration problems no longer appear. Failures resulting from blade and disk vibration were reported in engineering journals 30 years ago, and similar failures are reported today. Not only do various parts of the rotor vibrate in many ways, but in some cases the entire rotor has been known to go into a state of axial vibration. Early governor designs also led at times to a phenomenon known as *hunting,* a type of periodic speed fluctuation closely related to vibration.

One might think that the problem of steam turbine vibrations should have been solved by now, but recent growth to new sizes (exceeding 200,000 kw) has perpetuated the difficulties. The gas turbine, which is subject to conditions more strenuous than those affecting the steam turbine, has also played a part in reopening these problems. Then, too, the gas turbine has distinctive problems of its own. One of these, for example, results from the fact that combustion takes place within the unit. As a case in point, the following problem arose during development of the Boeing gas turbine engine:

> "Engine tests brought out the fact that compressor performance on the engine was below that on the component test rig. These tests also demonstrated that the loss of performance was due to burner howl. It was found that the burner howl resulted from an interaction between the characteristic frequency of burner vibration and the compressor-collector volume. Solution of the problem required that the pressure drop across the burner be increased and that the compressor-collector volume be minimized. (The maximum value permissible to assure the elimination of burner howl being established by test.) Generally, when the conditions of stability of operation of the engine were satisfied, component test data were reproduced on engine tests."†

1-8 Circuits and control systems. When the alternating-current electric circuit and radio came into being, scientists and electrical engineers found that the methods of vibrational analysis were also applicable to circuits, and many contributors to the study of vibration were important con-

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*Keller, Curt, "Operating Experience and Design Features of Closed Cycle Gas Turbine Power Plants." A.S.M.E. Paper No. 56—GTP—15.

†Balling, N. R., and V. W. Van Ornum: "Development of a Centrifugal Compressor for a Small Gas-Turbine Engine." ASME Paper No. 56—GTP—2.

tributors to the study of circuits. The parallel development of methodology in the two fields has provided a useful feedback of information to the study of mechanical vibrations. For example, the vibration of *nonlinear systems* (systems where the principle of superposition does not apply) has come to be of interest in recent years, and several significant contributions to this field have been made by radio engineers. More recently, the field of *control* has attracted attention, in that systems such as guided missiles or robot machine tools tend to behave somewhat like vibratory systems in carrying out their functions. The study of these systems has become a field in itself, drawing investigators from many related areas, including those of mechanical vibrations and electrical engineering. It appears that control is the most rapidly growing branch on the tree whose roots are described in the brief history above, although the field of mechanical vibrations *per se* is still vigorous. Several significant articles on this subject appear each month in technical publications, and many of these papers contain major contributions.

In view of so much recent development the reader may be dismayed to find that as late as Chapter 12 he will be trying to master derivations that were first carried out before 1800. It is a peculiarity of this field that, at an early date, leading thinkers did work which is still of significance today. Lord Rayleigh's *Theory of Sound* (1st ed., 1877) could still serve as a textbook for graduate engineering students.

1–9 Practical examples. Since most of this book will deal with derivations from fundamental laws and the application of these, there is, perhaps, some value in discussing vibrational situations in a less rigorous way in this introductory chapter. When one becomes "vibration oriented," it is surprising how much he sees in nature that can be explained in terms of vibration theory. McLachlan (Ref. 1–4) says that "vibration is ubiquitous." Even social and psychological interactions can advantageously be interpreted from the viewpoint of vibration in certain cases! The following examples will give some idea of the range of applicability.

1. *Structural vibration.* The air-conditioning equipment of an office building was mounted on the concrete floor of an upper story. There was nothing unique about the installation; all the conventional means of vibration isolation were used. But when the equipment was put into operation, a disproportionate amount of noise filled the halls and offices. Hastily suggested remedies ranged from alteration of the equipment to soundproofing the building. All of these appeared to be costly and to require considerable planning for proper execution.

When the real cause of the noise was discovered, the problem was quickly solved. Tests showed that the natural frequency of the floor was exactly twice the cyclic speed of the compressor, and that a *harmonic* of the force produced by the compressor was exciting the floor into vibration. Figure 1-3, a plot of vibration amplitude against forcing frequency, shows the response peak coinciding with the first harmonic from the compressor. When a concrete block of carefully chosen size was cast at the proper location on the floor, the natural frequency was lowered, placing the point of resonance below the first harmonic and above the fundamental. As a result, the vibration was reduced to an acceptable level.

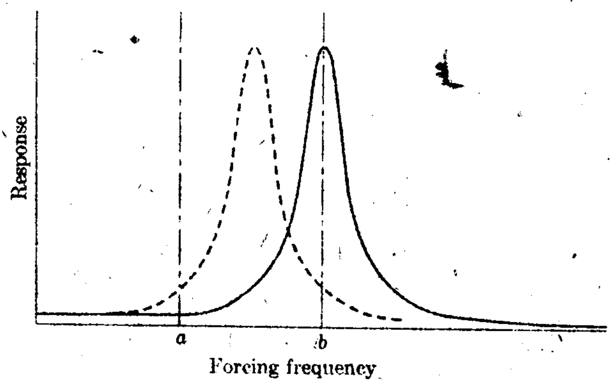

FIG. 1-3. The solid curve is a plot of vibrational response against amplitude for the original floor. The frequencies *a* and *b* are the fundamental and first harmonic produced by the compressor. The broken curve is the response of the floor when altered by addition of a concrete block. With this alteration the amplitude of response in the neighborhood of either of the two forcing frequencies is relatively small compared with the peak.

2. *Water hammer.* Vibration may arise from unexpected sources even in conventional design settings: the water for a hydroelectric station was brought from a snow-fed lake to the turbine by means of a long pipe ending in a valve. The purpose of the valve was to start or stop the flow to the turbine; other valves connected to the governor mechanism controlled the flow rate. To the consternation of the plant operators it was found that when the main valve was left partially open, powerful pressure surges were recorded on a gage in the pipe. The diagnosis rendered by a consulting engineer was that the partially opened valve functioned much like the reed of a musical instrument, and that the system was behaving rather like a clarinet, although at a much lower frequency. With understanding came a remedy (in the form of a slight alteration of the valve), and serious damage to the plant was avoided.

3. *Shaft failure.* Sometimes even a conscientious engineer will encounter vibration difficulties of more or less obvious origin when plans are

hurriedly altered to meet constructional exigencies, or when standard designs are modified for new uses. Typical is the case where the shaft between a motor and a pump was lengthened to permit better orientation of the pump with respect to a reaction vessel. Unexpectedly, this new shaft failed in torsional vibration after a brief period of use. When confronted with the problem the plant engineer found that two solutions were possible: a more flexible shaft or a stiffer one. Either would restore reliability to the system. A slightly larger one was chosen because its ends could quickly be turned down to fit the original couplings with a minimum of lost time. In this instance the fault was quickly remedied, but such a failure of a minor part can sometimes lead to serious consequences.

4. *Aerodynamic flutter.* Airplanes, ships, and even road vehicles have taught us much about vibration. For instance, a ship, even of the most modern design, is a mass of oscillatory motions of various frequencies. Such motions in an airplane are a cause for still greater concern. Quite regularly, progress in aeronautics is halted until a problem of vibration or flutter can be solved. Aerodynamic flutter of one sort or another has been with us since the invention of sails, and it has become more and more serious as man grows more venturesome in his designs. In a far different field of engineering, the Tacoma Narrows suspension bridge failure, which was due to wind-excited vibration, serves as a reminder that flutter is not a problem for the aeronautical engineer alone.

The simplest type of aerodynamic flutter, *stall flutter*, is discussed in Chapter 4. Other kinds of flutter involve complicated combinations of motions and are considered to be beyond the scope of this introductory text; these, however, are often the most troublesome, because they occur at high speeds and normal flight attitudes, and at times are powerful enough to break off the wings of an aircraft.

5. *Control system.* As a final indication of the scope of the field of mechanical vibrations, let us take a nonmechanical example. A constant-temperature chamber in a research laboratory was designed to produce a steady flow of air at a particular temperature. A blower first passed the air over electrical heating units and then over a thermosensitive element designed to regulate the current in the heating units and thereby correct errors in temperature of the departing gas. When the system was tested, a recorder showed a steady temperature rise followed by cyclic fluctuation. A check of the components of the system showed that *each element was working perfectly,* and that the fluctuation was a result of the way in which they had been made to interact. Several means of correcting this kind of malfunction have been devised. A simple method would be to let the temperature of the air entering the heater regulate the current, thus breaking the feedback loop from the sensing element to the heater and so eliminating the oscillation.

1-10 The role of vibration analysis in design. In the preceding examples we have seen something of the broad range of vibration problems, and how important the effects of vibration can be. What is perhaps most interesting is that slight changes in design often would avoid vibration difficulties altogether. This does not mean that it is easy or inexpensive to remedy vibration in existing systems. It does mean, however, that in many cases anticipation of trouble in the original planning can make possible the avoidance of vibration problems at little cost. Sometimes, of course, the causes of vibration are so subtle that they cannot be predicted, and it is only by considerable detective work that a remedy can be found even after the vibration is observed.

In almost every design it has long been customary to check from the point of view of strength of materials, but in recent years it has become more and more generally recognized that a check should also be made from the point of view of vibrations. It is difficult to say where the necessity for this checking ends. For instance, the vibration of the rear view mirror of an automobile will never lead to mechanical failure of the machine, but it *can* significantly affect customer acceptance. A designer should be aware of the major causes of undesirable vibration and he should know how to suppress it once it appears. To do this he must be able to make the principal types of calculation which will enable him to predict quantitatively the behavior of a given system. The purpose of this book is to aid him in preparing for that task.

1-11 Objectives. Our study will be directed toward several major objectives. *First* we shall develop methods for finding the frequency at which a given system will vibrate when its equilibrium has been upset. This *natural frequency** is perhaps the most important single characteristic of a vibratory system. We shall find that the application of even a weak force at this frequency sometimes can excite the system into violent motion. For example, by carefully pitching his voice to the natural frequency of a thin drinking glass, it is possible for a singer to shatter it.

Another demonstration of this idea is attributed to the engineer Nikola Tesla (1857–1943). In the course of his experiments Tesla attached a variable-frequency, compressed-air-driven vibrator to one of the steel frame members of his laboratory. Unknown to the experimenter, there was wholesale breaking of glass, plaster, and plumbing in the surrounding buildings as the vibrator passed through its range of frequencies. In this case a "damping effect" was provided by the arrival of the police, who suppressed the vibration by stopping the experiment.

*Some systems have more than one natural frequency.