HEAT CONDUCTION

Second Edition

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HEMISPHERE PUBLISHING CORPORATION
Washington New York London

DISTRIBUTION OUTSIDE NORTH AMERICA
SPRINGER-VERLAG
Berlin Heidelberg New York Tokyo

HEAT CONDUCTION: Second Edition

The first edition of this book was published under the title *Heat Conduction* by the Middle East Technical University.

1 2 3 4 5 6 7 8 9 0 B C B C 8 9 8 7 6 5

Library of Congress Cataloging in Publication Data

Kakaç, S. (Sadik)
Heat conduction.

Includes bibliographies and index.
1. Heat—Transmission. 2. Heat—Conduction.
I. Yener, Yaman, date. II. Title.
QC320.K35 1985 621.402'2 84-27975
ISBN 0-89116-391-3 Hemisphere Publishing Corporation

DISTRIBUTION OUTSIDE NORTH AMERICA:
ISBN 3-540-15244-X Springer-Verlag Berlin

Preface

The study of heat transfer is one of the important fields of engineering science. Heat transfer problems are of great practical significance in many branches of engineering. Although it is generally regarded as most closely related to mechanical engineering, much work in this field has also been done in nuclear, chemical, metallurgical, and electrical engineering where heat transfer problems are equally important.

In this book, various problems of heat conduction in rectangular, cylindrical, and spherical coordinates are solved by the method of separation of variables, integral and Laplace transforms, numerical techniques, and the integral and variational methods. The material presented has evolved from a series of lecture notes developed by the authors when teaching a graduate course in heat conduction over a period of years.

This book is written for both engineering students and engineers practicing in areas involving the applications of heat diffusion problems. In general, the problems at the end of each chapter, in SI units, are designed to clarify the physically and/or theoretically important points, and to supplement the text. The authors have a strong conviction that this is essential for a clear understanding of the application of the theoretical results. Special attention has also been given to the derivation of basic equations and their solutions in sufficient detail to help the student clearly understand the subject matter.

In Chapter 1, the basic concepts and fundamentals of heat transfer are presented. Chapter 2 is devoted to the derivation of general forms of the heat conduction equation. Solutions of steady-state heat conduction problems in one-dimensional bodies with and without internal energy generation together with an extensive treatment of extended surfaces are given in Chapter 3. The mathematical techniques presented in this text involve the use of orthogonal functions, Fourier expansions, integral transforms, etc. Since these topics may be beyond the mathematical experience of some students, important aspects of these special subjects are introduced in Chapter 4. In Chapters 5 and 6, solutions of multi-dimensional steady- and unsteady-state heat conduction problems by the method of separation of variables are given, and in Chapters 7 and 8 the method of solution with integral and Laplace transforms are presented, respectively. Chapter 9 is included to introduce the reader to the finite-difference method. This method may provide the only means of obtaining solutions to some heat conduction problems. An emphasis is given to the finite-difference formulation of heat conduction problems, which can be adapted by an individual to his or her own computer facility. Chapter 10 includes miscellaneous topics of formulation and solution, such as the Duhamel method, the integral method, the variational method, and an introduction to phase-change problems.

In the second edition, the authors have retained the basic objective of the first edition. As before, not all the topics and solution methods of heat conduction could be covered. The topics chosen and the order and the depth of the coverage represent the personal judgement of the authors as to what is of first importance for, and what can be practically taught in a one semester course to the beginning engineering graduate students. The authors feel that a study of heat conduction in the logical sequence that is followed in this book provides the best way for a first-year graduate student to develop an understanding of heat conduction.

With few exceptions, no more engineering background than the usual undergraduate courses in thermodynamics, heat transfer, and advanced calculus is required of the reader.

The authors wish to express their sincere thanks to the Hemisphere Publishing Corporation, whose competent work made this publication possible.

The authors also wish to thank Ms. Donna Pressley for typing the manuscript and Mr. Ali Akgüneş who prepared the drawings at the Middle East Technical University, Ankara, Turkey.

We acknowledge with appreciation the suggestions and encouragement from many colleagues, who have been very helpful to us during the writing of both the first and the second editions. Our graduate students and assistants have also contributed to this book; their perceptive questions and comments have often forced us to review several portions of the manuscript.

Finally, this book could never have been written if it were not for the support of our wives. Their encouragement and patience has been an invaluable contribution.

Sadik Kakaç
Yaman Yener

Nomenclature

A	area, m^2
Bi	Biot number
c	specific heat, J/kg.K
c_p	specific heat at constant pressure, J/kg.K
c_v	specific heat at constant volume, J/kg.K
e	energy per unit mass, J/kg
E	energy, J
F_{ij}	radiation shape factor
Fo	Fourier number
g	gravitation acceleration, m/s^2
h	heat transfer coefficient, $W/m^2.K$
$\hat{i},\hat{j},\hat{k}$	unit vectors in the x-, y-, and z-directions
I_ν	modified Bessel function of the first kind of order ν
J_ν	Bessel function of the first kind of order ν
k	thermal conductivity, W/m.K
k_m	mean thermal conductivity, W/m.K
K	kernel
K_ν	modified Bessel function of the second kind of order ν
L	length, m
m	mass, kg
N	normalization integral, Eq. (4.23b)
p	pressure, N/m^2
P	perimeter, m
P_n	Legendre polynomial of order n
q	heat flux, W/m^2
$\dot{q}$	rate of internal energy generation per unit volume, W/m^3
Q	heat, quantity of heat, J

$\dot{Q}$	rate of heat transfer, W
Q_n	Legendre function of the second kind
r	radius
$\vec{r}$	position vector
R_t	thermal resistance, K/W
s	entropy per unit mass, J/kg.K
S	entropy, J/kg
t	time, s
T	thermodynamic temperature, K
u	internal energy per unit mass, J/kg
U	internal energy, J; overall heat transfer coefficient, $W/m^2.K$
v	specific volume, m^3/kg
ν	volume (element)
$\vec{V}$	velocity vector, m/s
W	work, J
$\dot{W}$	power, W
Y_ν	Bessel function of the second kind of order ν

Greek Letters

α	absorptivity; thermal diffusivity, m^2/s
δ	penetration depth, m; velocity boundary layer thickness, m
δ_t	thermal boundary layer thickness, m
ε	emissivity
η_f	fin efficiency
θ	polar angle, rad; temperature difference, K
λ	eigenvalue
ρ	mass density, kg/m^3; reflectivity

σ	Stefan-Boltzmann constant, $W/K^4 \cdot m^2$
τ	transmissivity
ϕ	azimuthal angle, rad; fin effectiveness
χ	reciprocal of the neutron diffusion length, 1/m

Coordinates

(x,y,z)	rectangular coordinates
(r,ϕ,z)	cylindrical coordinates
(r,ϕ,θ)	spherical coordinates

Subscripts

b	base of the fin
c	contact surface, center
cr	critical insulation thickness
e	electrical
f	fin conditions
i	initial condition
m	mean value
r	radiation
t	thermal
x	local condition
∞	ambient condition

Contents

1

Foundations of Heat Transfer

1.1 INTRODUCTORY REMARKS

Heat transfer is the study of energy transfer solely as a result of temperature differences. The laws which govern heat transfer are very important to the engineer in the design, construction, testing and operation of heat exchange devices. Heat transfer problems confront investigators in nearly every branch of engineering. Although it is generally regarded as most closely related to mechanical engineering, much work in this field has also been done in chemical engineering, nuclear engineering, metallurgical engineering, and electrical engineering, where heat transfer problems are equally important. It is probably this fundamental and widespread influence which has brought heat transfer into the category of an engineering science.

In thermodynamics, *heat* is defined as the form of energy that crosses the boundary of a thermodynamic system by virtue of a temperature difference existing between the system and its surroundings. That is, heat is a form of energy in transition and the temperature difference is the driving potential for its propagation. Since heat is energy in transit we can really talk about the transfer of heat. *Heat flow* is vectorial in the sense that heat is transferred in the direction of a negative temperature gradient; that is, from higher toward lower temperatures, which is, in fact, a statement of the second law of thermodynamics.

Thermodynamics is that branch of science which deals with the study of heat and work interactions of a system with its surroundings. The laws of thermodynamics may be used to predict the gross amount of heat transferred during a process in which a system goes from one equilibrium state (including mechanical, and chemical as well as thermal equilibriums) to another. In most instances, however, the overriding consideration may be the length of time over which the transfer of heat occurs or, simply, the time *rate* at which it takes place. The laws of thermodynamics alone are not sufficient to provide such information, neither can they explain the *mechanism* of heat transfer. The science of heat transfer, on the other hand, studies heat transfer mechanisms and extends thermodynamic analysis, through the development of necessary relations, to calculate heat transfer rates.

In the study of heat transfer in a medium, attention is directed towards the relations among the temperature distribution and heat flow in the medium, and geometry, dimensions and thermo-physical properties of the medium. Such studies are based upon the foundations comprising both theory and experiment. As in other branches of physical sciences, the theoretical part is constructed from one or more *physical (natural) laws*. Physical laws are statements in terms of concepts that have been found to be true through many years of experimental observations. A physical law is called a *general law* if the application of it is independent of the medium under consideration. Otherwise, it is called a *particular law*. There are, in fact, four general laws upon which all the analyses concerning heat transfer, either directly or indirectly, depend. These are:

(a) the law of conservation of mass,
(b) the first law of thermodynamics,
(c) the second law of thermodynamics, and
(d) Newton's second law of motion.

For most heat conduction problems, the use of the first and second laws of thermodynamics is sufficient. In addition to these general laws, it is usually necessary to bring certain particular laws into an analysis. There are three such particular laws that we will employ in the analysis of conduction heat transfer. These are:

(a) Fourier's law of heat conduction,
(b) Newton's law of cooling, and
(c) Stefan-Boltzmann's law of radiation.

1.2 MODES OF HEAT TRANSFER

The mechanism by which heat is transferred is, in fact, quite complex. There appear, however, to be three rather basic and distinct types or *modes* of heat transfer processes. These are:

(a) conduction,
(b) radiation, and
(c) convection.

Conduction is the process of heat transfer by molecular motion, supplemented in some cases by the flow of free electrons, through a medium (solid, liquid or gaseous) from a region of high temperature to a region of low temperature. When two bodies or materials at different temperatures are in direct contact, heat transfer by conduction also occurs across the interface between these bodies.

The mechanism of heat conduction in liquids and gases has been postulated as the transfer of kinetic energy of the molecular movement. Thermal energy stored in a fluid increases its internal energy by increasing the kinetic energy of its vibrating molecules and is measured by the increase of its temperature. A high temperature measurement would then indicate a high kinetic energy of the molecules, and the heat conduction would imply the transfer of kinetic energy by the more active molecules in the high temperature region to the molecules in the low molecular kinetic energy region by successive collisions. Heat conduction in solids with crystalline structures, on the other hand, depends on the energy transfer by molecular and lattice vibrations,and free electron drift. In general energy transfer by molecular and lattice vibrations is not so large as the energy transfer by free electrons, and it is for this reason that good electrical conductors are almost as good heat conductors, while electrical insulators are usually good heat insulators. In the case of solids with amorphous structure, however, heat conduction depends upon the molecular energy transport only.

Radiation, on the other hand, is a process of heat transfer which takes plac in the form of electromagnetic waves. Thermal energy in the form of electromagnetic waves or, in other words, thermal radiation can pass through certain types of substances and can also pass through a vacuum, whereas for heat conduction to take place a material medium is necessary.

Conduction is the only mechanism by which heat can flow in *opaque* solids. Through certain *transparent* or *semi-transparent* solids, such as glass and quartz, some energy is transmitted by radiation in addition to the energy that can be transmitted by conduction. With gases and liquids, if there is no observable fluid motion, the problem becomes one of simple conduction (and radiation). However, if there is observable fluid motion, energy is transported by temperature gradients (as in conduction) as well as in the form of internal energy due to the movement of the fluid particles. This process of energy transport by the combined action of heat conduction (and radiation), and the motion of fluid particles is referred to as *convection* or *convective heat transfer*. Although in the foregoing classification we have considered convection to be a mode of heat transfer, it is actually conduction (and radiation) in moving fluids.

In reality, temperature distribution in a medium is controlled by the combined effect of these three modes of heat transfer. Therefore, it is not actually possible to entirely isolate one mode from interactions with other modes. However, for simplicity in the analysis these three modes of heat transfer are almost always studied separately. In this book we will study conduction heat transfer only.

1.3 CONTINUUM CONCEPT

Matter, while seemingly continuous, is composed of molecules, atoms and electrons in constant motion and collisions. Since heat conduction is thought to come about through the exchange of kinetic energy among the particles, the most fundamental approach in analyzing the transfer of heat in a substance by conduction is therefore to apply the laws of motion to each individual particle or a statistical group of particles, subsequent to some initial state of affairs.

In most engineering problems our primary interest lies not in the molecular behavior of a substance, but rather in how the substance behaves as a continuous medium. In our study of heat conduction, we will therefore neglect the molecular structure of the substance and consider it to be a continuous medium-*continuum*, which is fortunately a valid approach to many practical problems where only macroscopic information is of interest. Such a model may be used provided that the size and the mean-free-path of molecules are small compared with other dimensions existing in the medium, so that a statistical average is meaningful. This approach, which is also known as the *phenomenological* approach to heat conduction, is simpler than microscopic approaches and usually gives the answers required in engineering. On the other hand, the information is not so complete with the continuum approach and certain parameters such as thermodynamic state and transport properties have to be introduced empirically. Parallel to the study of heat transfer processes by continuum approach, molecular considerations can also be used to obtain information on transport and thermodynamic properties. In this book we shall study phenomenological heat conduction theory.

1.4 DEFINITIONS AND CONCEPTS IN THERMODYNAMICS

In thermodynamics, a *system* is defined as an arbitrary collection of matter of fixed identity bounded by a closed surface which can be real or imaginary. All other systems outside the surface that interact with the system under consideration are known as *surroundings*. In the absence of any mass-energy conversions, the mass of a system not only remains constant, but the system must be made up of exactly the same submolecular particles. The four general laws listed in Section 1.1. are always stated in the first instance in terms of a system. In fact, one cannot meaningfully apply the general laws until a definite system is identified.

A *control volume* is any defined region in space across the boundaries of which matter, energy, and momentum may flow, within which changes of matter and energy, and momentum storage may take place, and on which external forces may act. The control volume may change its position or size with time. However, most often we deal with control volumes which are fixed in space and are of fixed size and