MACHINERY'S HANDBOOK GUIDE

23rd Edition

CONTENTS

SECTION I

DIMENSIONS AND AREAS OF CIRCLES

HANDBOOK Pages 54 and 60

Circumferences of circles are used in calculating speeds of rotating machine parts, including: drills, reamers, cutters, grinding wheels, gears and pulleys. These speeds are variously referred to as: surface speed, circumferential speed, and peripheral speed; meaning in each case the distance a point on the surface or circumference would travel per minute. This distance is usually expressed as feet per minute. Circumferences are also required in calculating the circular pitch of gears, laying out involute curves, finding the lengths of arcs, and in solving many geometrical problems. Letters from the Greek alphabet are frequently used to designate angles, and the Greek letter π (pi) is always used to indicate the ratio between the circumference and the diameter of a circle:

$$\pi = 3.14159265 \ldots = \frac{\text{circumference of circle}}{\text{diameter of circle}}$$

For most practical purposes the value of $\pi = 3.1416$ may be used.

Example 1:– Find the circumference and area of a circle whose diameter is 8 inches.

On Handbook page 54 the circumference C of a circle is given as $3.1416d$. Therefore, $3.1416 \times 8 = 25.1328$ inches.

On the same page, the area is given as $0.7854d^2$. Therefore, A (area) $= 0.7854 \times 8^2 = 0.7854 \times 64 = 50.2656$ square inches.

Example 2:– From page 60 of the Handbook the area of a cylindrical surface equals $S = 3.1416 \times d \times h$. For a diameter of 8 inches and a height of 10 inches, the area is $3.1416 \times 8 \times 10 = 251.328$ square inches.

Example 3:– For the cylinder in Example 2 but with the area of both ends included, the total area is the sum of the area found in Example 2 plus two times the area found in Example 1. Thus, $251.328 + 2 \times 50.2656 = 351.8592$ square inches. The same result could have been obtained by using the formula for total area given on Handbook page 60: $A = 3.1416 \times d \times (\frac{1}{2}d + h) = 3.1416 \times 8 \times (\frac{1}{2} \times 8 + 10) = 351.8592$ square inches.

Example 4.—If the circumference of a tree is 96 inches, what is its diameter? Since the circumference of a circle $C = 3.1416 \times d$, $96 = 3.1416 \times d$ so that $d = 96 \div 3.1416 = 30.558$ inches.

Example 5.—The table on page 1005 of the Handbook provides values of revolutions per minute required to produce various cutting speeds for workpieces of selected diameters. How are these speeds calculated? Cutting speed in feet per minute is calculated by multiplying the circumference in feet of a workpiece by the rpm of the spindle: cutting speed in fpm = circumference in feet × rpm. Transposing this formula as explained in Section 3,

$$\text{rpm} = \frac{\text{cutting speed, fpm}}{\text{circumference in feet}}$$

For a 3-inch diameter workpiece (¼-foot diameter) and for a cutting speed of 40 fpm, rpm $= 40 \div (3.1416 \times \frac{1}{4}) = 50.92 = 51$ rpm, approximately, which is the same as the value given on page 1005 of the Handbook.

PRACTICE EXERCISES FOR SECTION 1

For answers to all practice exercise problems or questions see Section 22

1. Find the area of a circle 10 mm in diameter. Its circumference.

2. On Handbook page 1007, for a 5 mm diameter rotating at 318 rpm the corresponding cutting speed is given as 5 meters per minute. Check this value.

3. For a cylinder 100 mm in diameter and 10 mm high, what is the surface area not including the top or bottom?

4. A steel column carrying a load of 10,000 pounds has a diameter of 10 inches. What is the pressure on the floor in pounds per square inch?

5. What is the ratio of the area of a square of any size to the area of a circle having the same diameter as one side of the square?

6. What is the ratio of the area of a circle to the area of a square inscribed in that circle?

7. The drilling speed for cast iron is assumed to be 70 feet per minute. Find the time required to drill two holes in each of 500 castings if each hole has a diameter of ¾ inch and is 1 inch deep. Use 0.010 inch feed and allow one-fourth minute per hole for set-up.

8. Find the weight of a cast-iron column 10 inches in diameter and 10 feet high. Cast iron weighs 0.26 pound per cubic inch.

9. If machine steel has a tensile strength of 55,000 pounds per square inch, what should be the diameter of a rod to support 36,000 pounds if the safe working stress is assumed to be one-fifth of the tensile strength?

10. Moving the circumference of a 16-inch automobile flywheel two inches, moves the camshaft through how many degrees? (The camshaft rotates at one-half the flywheel speed.)

11. The tables beginning on page 962 give lengths of chords for spacing off circumferences of circles into equal parts. Is another method available?

SECTION 2

CHORDAL DIMENSIONS, SEGMENTS, AND SPHERES

HANDBOOK Pages 62 and 962–964

A chord of a circle is the distance along a straight line from one point on the circumference to any other point. A segment of a circle is that part or area between a chord and the arc it intercepts. The lengths of chords and the dimensions and areas of segments are often required in mechanical work.

Lengths of Chords.—The table of chords, Handbook page 962, can be applied to a circle of any diameter as explained and illustrated by examples on that page. This table is given to six decimal places so that it can be used in connection with precision tool work.

Example 1:—A circle has 56 equal divisions and the chordal distance from one division to the next is 2.156 inches. What is the diameter of the circle?

The chordal length in the table for 56 divisions and a diameter of 1 equals 0.05607; therefore, in this case

$$2.156 = 0.05607 \times \text{diameter}$$

$$\text{Diameter} = \frac{2.156}{0.05607} = 38.452 \text{ inches}$$

Example 2:—A drill jig is to have 8 holes equally spaced around a circle 6 inches in diameter. How can the chordal distance between adjacent holes be determined when the table, Handbook page 962, is not available?

One-half the angle between the radial center-lines of adjacent holes = 180 ÷ number of holes. If the sine of this angle is multiplied by the diameter of the circle, the product equals the chordal distance. In this example we have 180 ÷ 8 = 22.5 degrees. The sine of 22.5 degrees (see page 104) is 0.38268; hence, the chordal distance = 0.38268 × 6 = 2.296 inches. The result is the same as would be obtained with the table on Handbook page 962 because the figures in the column "Length of Chord" represent the sines of angles equivalent to 180 divided by the different numbers of spaces.

Use of the Table of Segments of Circles—Handbook pages 70 and 71.—This table is of the unit type in that the values all apply to a radius of 1. As explained above the table, the value for any other radius can be obtained by multiplying the figures in the table by the given radius, except in the case of areas when the *square* of the given radius is used. Thus, the unit type of table is universal in its application.

Example 3:—Find the area of a segment of a circle, the center angle of which is 57 degrees and the radius 2½ inches.

First locate 57 degrees in the center angle column; opposite this figure in the area column will be found 0.07808. Since the area is required, this number is multiplied by the square of 2½. Thus,

$$0.07808 \times (2\tfrac{1}{2})^2 = 0.488 \text{ square inch}$$

Example 4:—A cylindrical oil tank is 4½ feet in diameter, 10 feet long, and is in a horizontal position. When the depth of the oil is 3 feet 8 inches, what is the number of gallons of oil?

The total capacity of the tank equals $0.7854 \times 4\tfrac{1}{2}^2 \times 10 = 159$ cubic feet.

One U. S. gallon equals 0.1337 cubic foot (see Handbook page 2404); hence, the total capacity of the tank equals $159 \div 0.1337 = 1190$ gallons.

The unfilled area at the top of the tank is a segment having a height of 10 inches or $^{10}/_{27}$ (0.37037) of the tank radius. The nearest decimal equivalent to $^{10}/_{27}$ in Column *h* of the table on pages 70 and 71 is 0.3707; hence, the number of cubic feet in the segment-shaped space $= (27^2 \times 0.401 \times 120) \div 1728 = 20.3$ cubic feet and $20.3 \div 0.1337 = 152$ gallons. Therefore, when the depth of oil is 3 feet 8 inches, there are $1190 - 152 = 1038$ gallons. (See also Handbook page 48 for additional information on the capacity of cylindrical tanks.)

Spheres—Handbook page 62.—Handbook page 62 gives formulas for calculating spherical volumes.

Example 5.—If the diameter of a sphere is 24⅝ inches what is the volume, given the formula:

$$\text{Volume} = 0.5236\, d^3$$

The cube of 24⅝ = 14932.369; hence, the volume of this sphere = 0.5236 × 14932.369 = 7818.5 cubic inches.

PRACTICE EXERCISES FOR SECTION 2

For answers to all practice exercise problems or questions see Section 22

1. Find the lengths of chords when the number of divisions of a circumference and the radii are as follows: 30 and 4; 14 and 2½; 18 and 3½.

2. Find the chordal distance between the graduations for thousandths on the following dial indicators: (*a*) Starrett has 100 divisions and 1⅜-inch dial. (*b*) Brown & Sharpe has 100 divisions and 1¾-inch dial. (*c*) Ames has 50 divisions and 1⅝-inch dial.

3. The teeth of gears are evenly spaced on the pitch circumference. In making a drawing of a gear, how wide should the dividers be set to space 28 teeth on a 3-inch diameter pitch circle?

4. In a drill jig, 8 holes, each ½ inch diameter, were spaced evenly on a 6-inch diameter circle. To test the accuracy of the jig, plugs were placed in adjacent holes and the distance over the plugs was measured with a micrometer. What should be the micrometer reading?

5. In the preceding problem, what should be the distance over plugs placed in alternate holes?

6. What is the length of the arc of contact of a belt over a pulley 2 feet 3 inches in diameter if the arc of contact is 215 degrees?

7. Find the areas, length of chords and heights, of the following segments: (*a*) radius 2 inches, angle 45 degrees; (*b*) radius 6 inches, angle 27 degrees.

8. Find the number of gallons of oil in a tank 6 feet in diameter and 12 feet long if the tank is in a horizontal position and the oil measures 2 feet deep.

9. Find the surface area of the following spheres, the diameters of which are 1½; 3⅜; 65; 20¼

10. Find the volume of each sphere in the above exercise.

11. The volume of a sphere is 1,802.725 cubic inches. What is its surface area and diameter?

SECTION 3

FORMULAS AND THEIR REARRANGEMENT

HANDBOOK Page 24

A formula may be defined as a mathematical rule expressed by signs and symbols instead of in actual words. In formulas, letters are used to represent numbers or *quantities*, the term "quantity" being used to designate any number involved in a mathematical process. The use of letters in formulas, in place of the actual numbers, simplifies the solution of problems, and makes it possible to condense into small space the information that otherwise would be imparted by long and cumbersome rules. The figures or values for a given problem are inserted in the formula according to the requirements in each specific case. When the values are thus inserted, in place of the letters, the result or answer is obtained by ordinary arithmetical methods. There are two reasons why a formula is preferable to a rule expressed in words. 1. The formula is more concise, it occupies less space, and it is possible to see at a glance, the whole meaning of the rule laid down. 2. It is easier to remember a brief formula than a long rule, and it is, therefore, of greater value and convenience.

Example 1:—In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the diametral pitch. Then the formula would be:

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter (D) of the gear equals 2 added to the number of teeth (N), and this sum is divided by the pitch (P).

If the number of teeth in a gear is 16 and the diametral pitch 6, then simply put these figures in the place of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{16 + 2}{6} = \frac{18}{6} = 3 \text{ inches}$$

Example 2:—The formula for the horsepower of a steam engine is as follows:

$$H = \frac{P \times L \times A \times N}{33{,}000}$$

in which H = indicated horsepower of engine;
P = mean effective pressure on piston in pounds per square inch;
L = length of piston stroke in feet;
A = area of piston in square inches;
N = number of strokes of piston per minute.

Assume that $P = 90$, $L = 2$, $A = 320$, and $N = 110$; what would be the horsepower?

If we insert the given values in the formula, we have:

$$H = \frac{90 \times 2 \times 320 \times 110}{33{,}000} = 192$$

From the examples given, we may formulate the following general rule: ***In formulas, each letter stands for a certain dimension or quantity; when using a formula for solving a problem, replace the letters in the formula by the figures given for a certain problem, and find the required answer as in ordinary arithmetic.***

Omitting Multiplication Signs in Formulas.—In formulas, the sign for multiplication (×) is often left out between letters the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$H = \frac{P \times L \times A \times N}{33{,}000} \text{ can also be written } H = \frac{PLAN}{33{,}000}$$

If $A = 3$, and $B = 5$, then: $AB = A \times B = 3 \times 5 = 15$.

It is only the multiplication sign (×) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic. The multiplication sign can never be left out between two figures: 35 always means thirty-five, and "three times five" must be written 3×5; but "three times A" may be written $3A$. As a general rule the figure in an expression such as "$3A$" is written first, and is known as the *coefficient* of A. If the letter is written first, the multiplication sign is not left out, but the expression is written "$A \times 3$."

Rearrangement of Formulas.—A formula can be rearranged or "transposed" to determine the values represented by different letters of the formula. To illustrate by a simple example, the formula for determining the speed (s) of a driven pulley when its diameter (d), and the diameter (D) and speed (S) of the driving pulley are known, is as follows: $s = \frac{S \times D}{d}$. If the speed of the driven pulley is known and the problem is to find its diameter or the value of d instead of s, this formula can be rearranged or changed. Thus: $d = \frac{S \times D}{s}$.

Rearranging a formula in this way is governed by four general rules.

Rule 1. An independent term preceded by a plus sign (+) may be transposed to the other side of the equals sign (=) if the plus sign is changed to a minus sign (−).

Rule 2. An independent term preceded by a minus sign may be transposed to the other side of the equals sign if the minus sign is changed to a plus sign.

As an illustration of these rules, if $A = B - C$, then $C = B - A$, and if $A = C + D - B$, then $B = C + D - A$. That the foregoing is correct may be proved by substituting numerical values for the different letters and then transposing them as shown.

Rule 3. A term which multiplies all the other terms on one side of the equals sign may be moved to the other side, if it is made to divide all the terms on that side.

As an illustration of this rule, if $A = BCD$, then $\frac{A}{BC} = D$ or according to the common arrangement $D = \frac{A}{BC}$. Suppose, in the preceding formula, that $B = 10$, $C = 5$, and $D = 3$; then $A = 10 \times 5 \times 3 = 150$, and $\frac{150}{10 \times 5} = 3$.

Rule 3. A term which divides all the other terms on one side of the equals sign may be moved to the other side, if it is made to multiply all the terms on that side.

To illustrate, if $s = \frac{SD}{d}$, then $sd = SD$, and, according to Rule 3, $d = \frac{SD}{s}$. This formula may also be rearranged for determining the values of S and D: thus $\frac{ds}{D} = S$, and $\frac{ds}{S} = D$.

If, in the rearrangement of formulas, minus signs precede quantities, the signs may be changed to obtain positive rather than minus quantities. All the signs on both sides of the equals sign or on both sides of the equation may be changed. For example, if $-2A = -B + C$, then $2A = B - C$. The same result would be obtained by placing all the terms on the opposite side of the equals sign which involves changing signs. For instance, if $-2A = -B + C$, then $B - C = 2A$.

Fundamental Laws Governing Rearrangement.—After a few fundamental laws which govern any formula or equation are understood, its solution usually is very simple. An equation states that one quantity equals another quantity. So long as both parts of the equation are treated exactly alike the values remain equal. Thus, in the equation $A = \frac{1}{2}ab$, which states that the area A of a triangle equals one half the product of the base a times the altitude b, each side of the equation would remain equal if we added the same amount. $A + 6 = \frac{1}{2}ab + 6$; or we could subtract an equal amount from both sides: $A - 8 = \frac{1}{2}ab - 8$; or multiply both parts by the same number: $7A = 7(\frac{1}{2}ab)$; or we could divide both parts by the same number and we would still have a true equation.

One formula for the total area T of a cylinder is: $T = 2\pi r^2 + 2\pi rh$, where r = the radius and h = the height of the cylinder. Suppose we want to solve this equation for h. $2\pi rh + 2\pi r^2 = T$. Transposing the part which does not contain h to the other side by changing its sign, we get: $2\pi rh = T - 2\pi r^2$. In order to obtain h, we can divide both sides of the equation by any quantity which will leave h on the left-hand side thus:

$$\frac{2\pi rh}{2\pi r} = \frac{T - 2\pi r^2}{2\pi r}$$

It is clear that in the left-hand member, the $2\pi r$ will cancel out, leaving: $h = \frac{T - 2\pi r^2}{2\pi r}$. The expression $2\pi r$ in the right hand member cannot be cancelled because it is not an independent factor since the numerator equals the difference between T and $2\pi r^2$.

Example 3:—Rearrange the formula for a trapezoid (Handbook page 52) to obtain h.

$$A = \frac{(a + b)h}{2}$$

$2A = (a + b)h$ (multiply both members by 2)

$(a + b)h = 2A$ (transpose both members so as to get the multiple of h on the left-hand side)

$\frac{(a + b)h}{a + b} = \frac{2A}{a + b}$ (divide both members by $a + b$)

$h = \frac{2A}{a + b}$ (cancel $a + b$ from the left-hand member)

Example 4:—The formula for determining the radius of a sphere (Handbook page 62) is as follows:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Rearrange to obtain a formula for finding the volume V.

$r^3 = \frac{3V}{4\pi}$ (cube each side)

$4\pi r^3 = 3V$ (multiply each side by 4π)

$3V = 4\pi r^3$ (transpose both members)

$\frac{3V}{3} = \frac{4\pi r^3}{3}$ (divide each side by 3)

$V = \frac{4\pi r^3}{3}$ (cancel 3 from left-hand member)

The procedure has been shown in detail to indicate the underlying principles involved. The rearrangement could be simplified somewhat by direct application of the rules previously given. To illustrate:

$$r^3 = \frac{3V}{4\pi} \quad \text{(cube each side)}$$

$$4\pi r^3 = 3V \quad \text{(applying } \textit{Rule 4} \text{ move } 4\pi \text{ to left-hand side)}$$

$$\frac{4\pi r^3}{3} = V \quad \text{(move 3 to left-hand side—} \textit{Rule 3}\text{)}$$

This final equation would, of course, be reversed to locate V at the left of the equals sign as this is the usual position for whatever letter represents the quantity or value to be determined.

Example 5.—It is required to determine the diameter of cylinder and length of stroke of a steam engine to deliver 150 horsepower. The mean effective steam pressure is 75 pounds; the number of strokes per minute is 120. The length of the stroke is to be 1.4 times the diameter of the cylinder.

First insert in the horsepower formula (Example 2) the known values:

$$150 = \frac{75 \times L \times A \times 120}{33{,}000} = \frac{3 \times L \times A}{11}$$

The last expression is found by cancellation.

Assume now that the diameter of the cylinder in inches equals D. Then $L = \frac{1.4D}{12} = 0.117D$, according to the requirements in the problem; the divisor 12 is introduced to change the inches to feet, L being in feet in the horsepower formula. The area $A = D^2 \times 0.7854$. If we insert these values in the last expression in our formula, we have:

$$150 = \frac{3 \times 0.117D \times 0.7854D^2}{11} = \frac{0.2757D^3}{11}$$

$$0.2757D^3 = 150 \times 11 = 1650$$

$$D^3 = \frac{1650}{0.2757} \qquad D = \sqrt[3]{\frac{1650}{0.2757}} = \sqrt[3]{5984.8} = 18.15$$

Hence, diameter of the cylinder should be about 18 inches, and the length of the [illegible] 75.4 [illegible] 75½ inches.

Solving Equations or Formulas by Trial.—One of the equations used for spiral gear calculations, when the shafts are at right angles, the ratios are unequal, and the center distance must be exact, is as follows:

$$R \sec \alpha + \operatorname{cosec} \alpha = \frac{2CP_n}{n}$$

In this equation

R = ratio of number of teeth in large gear to number in small gear;

C = exact center distance;

P_n = normal diametral pitch;

n = number of teeth in small gear.

The exact spiral angle α of the large gear is found by trial using the equation just given.

Equations of this form are solved by trial by selecting an angle assumed to be approximately correct, and inserting the secant and cosecant of this angle in the equation, adding the values thus obtained and comparing the sum with the known value to the right of the equals sign in the equation. An example will show this more clearly. Using the problem given in MACHINERY'S HANDBOOK (page [illegible]) as an example, $R = 3$; $C = 10$; $P_n = 8$; $n = 28$.

Hence, the whole expression

$$\frac{2CP_n}{n} = \frac{2 \times 10 \times 8}{28} = 5.714$$

from which it follows that

$$R \sec \alpha + \operatorname{cosec} \alpha = 5.714$$

In the problem given, the approximate spiral angle required is 45 degrees. The spiral gears, however, would not meet all the conditions given in the problem, if the angle could not be slightly modified. In order to determine whether the angle should be greater or smaller than 45 degrees, insert the values of the secant and cosecant of 45 degrees in the formula. The secant of 45 degrees is 1.4142 and the cosecant is 1.4142. Then