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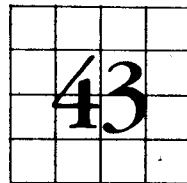
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Hydraulic Components

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43.1 INTRODUCTION

Hydraulic components find their widest application as the actuation (power-output) elements of power-control systems. The more important advantages of hydraulic systems are summarized as follows:

1. High-pressure hydraulic power can be generated efficiently, with pump efficiencies of 92 percent common.
2. Hydraulic components are comparatively light in weight compared with equivalent mechanical and electrical components because the highly stressed structures of the hydraulic system make very efficient use of structural material. Hydraulic pumps and motors, with a power density of less than 1 lb/hp are common. This light weight is made possible by the high pressures now available from commercially available pumps. Hydraulic systems operating at 3000 lb/in² are quite common and higher-pressure systems are available.
3. As seen by the load, the hydraulic actuator is extremely stiff compared with an equivalent pneumatic or electrical system. The hydraulic approach permits the maintenance of load position against significant and varying load forces, with lower loop gain and higher response speed.
4. Hydraulic actuation offers the highest torque (or force) to inertia ratio in comparison with most mechanical, pneumatic, and electrical systems. This property, coupled with the incompressible nature of the medium, results in exceptionally fast response and high power output.

Some of the disadvantages of hydraulic systems follow:

1. Most hydraulic systems use organic-base fluids which present serious fire and explosion hazards because of fluid spillage from leaks in piping or seals as well as breaks. The wide use of hydraulics in the die-casting and machine-tool field has resulted in the search for nonflammable hydraulic fluids, with good lubricity and no toxicity. Although good progress has been made, no fully satisfactory fluid is available at this time. The article on fluid properties will discuss some of the fluids and their relative fire-hazard problems.
2. Associated with the fire-hazard problem is the inherent difficulty of preventing leaks in normal usage and the subsequent "messiness" of the hydraulic system. This problem can be avoided at the expense of maintainability by all-welded plumbing.
3. High-speed-of-response hydraulic systems are sensitive to solid contaminants in the fluid which interfere with the smooth and proper function of the control valves and have been known to cause either poor operation or outright failure. Cleanliness in manufacture, assembly, normal operation, and service is an absolute necessity to assure good reliability. Adequate provision for filtering is only one of the necessities of good hydraulic design.

Even a cursory review of the design requirements for such systems as high-speed steel rolling-mill tension controls, jet-engine controls, or radar-antenna drives conveys the need for

the building of comprehensive and representative analytical models to assure satisfactory dynamic response and adequate stability throughout widely varying operating conditions and realistic tolerances.

The purpose of this section is to provide the designer with necessary basic building blocks to permit the preparation of an analytical model of the hydraulic system for experimentation and design specification. For illustrations of present design practice of commercially available hydraulic components, see Ref. 38.

43.2 PROPERTIES OF HYDRAULIC FLUIDS

43.2.1 Density and Related Properties

Density ρ is defined as the mass per unit of volume.

Specific weight W is defined as the weight per unit of volume.

Specific gravity σ is the ratio of the density of the substance in question to that of water at 60°F.

The petroleum industry uses a measure of relative density called "API gravity." API gravity in terms of specific gravity is given by Eq. (43.1).

$$\text{Degrees API} = 141.5/(\sigma 60^\circ\text{F}/60^\circ\text{F}) - 131.5 \quad (43.1)$$

$\sigma 60^\circ\text{F}/60^\circ\text{F}$ represents the specific gravity of the substance at 60°F relative to water at 60°F.

Specific gravity in terms of degrees API is given by

$$\sigma 60^\circ\text{F}/60^\circ\text{F} = 141.5/(\text{deg API} + 131.5) \quad (43.2)$$

The density of a liquid is a function of both pressure and temperature. At any one temperature a good approximation is given by Eq. (43.3)

$$\rho = \rho_0(1 + aP - bP^2) \quad (43.3)$$

where P is pressure and ρ_0 is the density at the reference condition. Empirical constants a and b are functions of temperature. Typical values for hydraulic oils at 60°F are¹

$$a = 4.38 \times 10^{-6} \text{ in}^2/\text{lb}$$

$$b = 5.65 \times 10^{-11} \text{ in}^4/\text{lb}^2$$

The decrease of density with increasing temperature (thermal expansion) is approximated by Eq. (43.4).

$$\rho = \rho_1[1 - \alpha(T - T_1)] \quad (43.4)$$

where α is called the cubical-expansion coefficient. This linear approximation is accurate within 0.5 percent for most hydraulic fluids over temperature ranges of 500°F.

Compressibility K is a most important property to the designer of fast-response hydraulic systems. It is defined as the unit rate of change of density with change in pressure.

For small pressure changes

$$K \triangleq (1/\rho)(d\rho/dP) \quad (43.5)$$

The *bulk modulus* β of a fluid is the reciprocal of compressibility,

$$\beta \triangleq \rho(dP/d\rho) \quad (43.6)$$

From Eq. (43.3), compressibility and bulk modulus are respectively

$$K = (a - 2bP)/(1 + aP - bP^2) \quad (43.7)$$

$$\beta = (1 + aP - bP^2)/(a - 2bP) \quad (43.8)$$

Typical values of β are given in the table of fluid properties (Table 43.1).

The computed values of bulk moduli for a given fluid should be used with great caution in the analysis of hydraulic control systems. The effective compressibility as seen by the flow source must be considered. Entrained air and transmission conduit compliance can drastically reduce the bulk modulus of the pure liquid.

Table 43.1 Properties of Hydraulic Fluids

Type	Light turbine	Phosphate ester	Chlorinated hydrocarbon	SAE 30	Petroleum base MIL-H -5606 A	Phosphate ester
Use.....	Petroleum industry DTE light	Industrial fire resistant Pydraul F9	Industrial fire resistant Aroclor 1242	Mobile petroleum	Aircraft missile	Aircraft missile
Typical commercial.....						
Specific gravity 60°F/80°F.....	0.865	1.20	1.32	0.887	0.848	Skydrol 500A
Expansion coefficient α , 1/°F.....	4.21×10^{-4}	4×10^{-4}		4.21×10^{-4}	0.5×10^{-4}	1.086
Viscosity, cs:						
-40°F.....						
0°F.....		18,000/20°F	2,000/30°F		500	240
100°F.....	33	47	18	120	1	11.5
210°F.....	5.2	6.3	2.0	12	14.3	3.92
400°F.....		2.5/300°F			5.1	
550°F.....					1.9	
700°F.....						
Viscosity-temperature coefficient, λ , 1/°F.....	1.85×10^{-1}				1.04×10^{-1}	
Pour point, °F.....	0	-5	2	0	-90	-85
Fire point, °F.....	445	675	None	495	235	425
Flash point, °F.....	400	430	345	435	225	360
Autogenous ignition temperature, °F.....		1100			700	>1100
Bulk modulus $\times 10^{-4}$ psi.....	≈ 250	387		≈ 250	270	387
Thermal conductivity, Btu/(hr)(ft ²)(°F/ft).....		0.065/70°F	0.053	0.082	0.0615	0.0365
Specific heat, Btu/(lb)(°F).....		0.33	0.285/60°F		0.5/100°F	0.42/68°F

Table 43.1 Properties of Hydraulic Fluids (Continued)

Type	Halogenated silicone	Silicone ester	Diester MIL-L 7808 C	Poly-phenyl ether	NAK 77	Mercury	JP 6
Use.....	Aircraft missile G.E. F50	Aircraft missile Oronite 8200	Jet-engine lubrication	Aircraft nuclear	Aircraft nuclear	Jet fuel
Typical commercial.....	1.03	0.93	0.93	1.18	0.87	12.76/602°F, 13.55/68°F	0.75 to 0.84
Specific gravity, 60°F/60°F.....							
Expansion coefficient α , 1/°F.....		0.445×10^{-4} 2400/-65°F	0.44×10^{-4}				
Viscosity, cs:							
-40°F.....	934	630	1920	5.22
0°F.....	287	195	230	0.1305	2.6
100°F.....	60	32.5	14.2	609	6.7	0.1087	0.97
210°F.....	21.2	11.3	3.6	6.0	0.5	0.0935	0.53
400°F.....	6.5	3.8	1.3	1.3	0.4	0.0797	
550°F.....	3.5	2.2	0.8	0.9	0.3	0.0745	
700°F.....	2.2	0.5	0.2	0.0710	
Viscosity-temperature coefficient, λ , 1/°F.....							
Four point, °F.....	-100	< -100	< -75	+5	+10		
Fire point, °F.....	>650	450	485	NA		
Flash point, °F.....	>825	390	430	NA		
Autogenous ignition temperature, °F.....	900	760	485	465	NA		
Bulk modulus $\times 10^{-4}$ psi.....	150/75°F, 30/700°F	218	290/0°F, 60/440°F	2,600/90°F, 1,400/600°F	449 170/100°F, 80/200°F, 30/400°F, 0.087/0°F, 0.082/210°F, 0.43/0°F, 0.53/210°F
Thermal conductivity, Btu/(hr)(ft²)(°F/ft).....		0.070	0.084/200°F	4.7/50°F, 8.1/600°F	
Specific heat, Btu/(lb)(°F).....	0.37/75°F, 0.51/500°F	0.47/68°F	0.35/65°F, 0.60/400°F	0.033 2240°F(Tc)* -38°F(Tc)† 674.4°F(BP)‡	

*Tc—critical temperature. †FP—freezing point. ‡BP—boiling point.

43.2.2 Viscosity

A true fluid is commonly defined as matter which is unable to store energy in shear; it is not elastic in shear. Therefore, any shear force applied to a fluid film will result in a finite shear rate. A Newtonian fluid is one for which shear rate is proportional to the shear stress. A constant of proportionality μ , the absolute viscosity, is defined by the following expression:

$$\mu = \tau / (du/dx) \quad (43.9)$$

where τ is the shear stress, du the incremental change in velocity resulting from the shear stress, and x the direction of the shear stress.

Most hydraulic fluids behave like Newtonian liquids up to shear rates of about 1,000,000 lb/in²/s. However, some of the highly compounded high-temperature hydraulic fluids may lose as much as 40 percent viscosity as the long-chain molecules are oriented or sheared.² This loss is temporary in some cases, with the original viscosity restored after an appreciable lapse of time. In other cases the loss is permanent.

Since several units of viscosity are in use, they should be carefully defined:

Reyn. A very large inconvenient unit in the English system.

$$1 \text{ Reyn} \equiv 1 \text{ lb s/in}^2$$

Centipoise (cP) (metric system). One centipoise is the viscosity of a fluid such that a force of 1 dyne will give two parallel surfaces 1 cm² area, 1 cm apart, a velocity of 0.01 cm/s. The centipoise is thus 0.01 dyne-s/cm².

Centistoke (cSt). Absolute viscosity divided by density is defined as kinematic viscosity. The common units are the stoke and the centistoke corresponding to the poise and the centipoise, respectively, divided by the density in consistent units. The centistoke is thus 0.01 cm²/s.

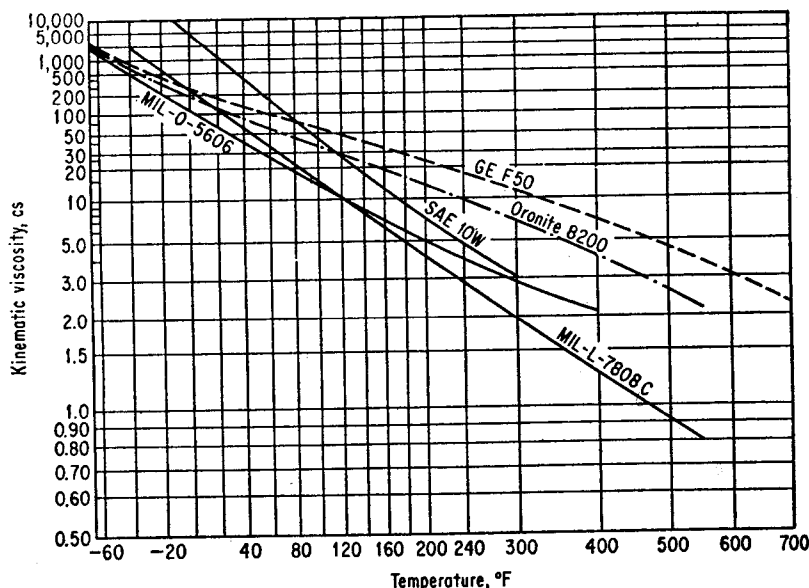


Fig. 43.1 Viscosity-temperature curves for various hydraulic fluids.

Saybolt Universal Seconds. The Saybolt viscosimeter is commonly used to determine the viscosity of petroleum products. The time required for 60 mL of the sample to flow through a 0.176-cm-diameter and 1.225-cm-long tube is measured and designated SSU.

The following lists various conversion factors: $1 \text{ lb}\cdot\text{s}/\text{in}^2 = 68,747.2 \text{ poises}$, $1 \text{ lb}\cdot\text{s}/\text{ft}^2 = 478.8 \text{ poises}$, $1 \text{ in}^2/\text{s} = 6.4516 \text{ stokes}$, $1 \text{ ft}^2/\text{s} = 229.03 \text{ stokes}$, $1 \text{ dyne}\cdot\text{s}/\text{cm}^2 = 1 \text{ poise}$, and $1 \text{ cm}^2/\text{s} = 1 \text{ stoke}$.

Effect of Pressure. The viscosity of liquids increases with pressure in a manner approximated as follows:

$$\log_{10} \mu/\mu_0 = cP \quad (43.10)$$

where c is a constant which for petroleum products at room temperature is approximately³

$$c = 7 \times 10^{-4} \text{ in}^2/\text{lb}$$

Effect of Temperature. The viscosity of hydraulic fluids decreases markedly with increasing temperature. Of the many empirical formulas which have been proposed to describe the variation, Eq. (43.11) has found broadest acceptance.

$$\mu_r = \mu_0 e^{-\lambda(T-T_0)} \quad (43.11)$$

where T is the temperature °F, T_0 is the reference temperature, and λ is the temperature coefficient of viscosity.

For most petroleum-base oils an empirical formula called the *Walther formula* is a better approximation.⁴ It is valid for relatively pure oils in the temperature range -20 to $+160^\circ\text{F}$. The ASTM charts⁵ are based on this approximation. Figure 43.1 gives the viscosity-temperature characteristics of several typical hydraulic fluids on the ASTM chart. The temperature coefficients of viscosity of several typical hydraulic fluids are given in Table 43.1.

Pour Point.⁶ The pour point is the temperature at which a fluid will no longer pour from a standard container when tested according to a standard ASTM procedure. It is considerably below the temperature which can be considered a practical lower limit for use in hydraulic systems. A viscosity in excess of 2500 cSt is not generally practical for hydraulic-system use.

43.2.3 Chemical Properties

Thermal stability relates to the fact that some hydraulic fluids when heated to high temperatures either decompose to form gaseous, liquid, or solid products or polymerize to form gels, varnishes, or even cokes.

Oxidative stability relates to the reaction of the hydraulic fluid with the oxygen of either the atmospheric air, the dissolved air, or other oxidizing agents. Since the products of oxidation are often acidic in nature, corrosion problems may result. Sludges and varnishes which result can clog filters, freeze sleeve valves, and foul orifices. Oxidation problems arise primarily in high-temperature applications.

Hydrolytic stability relates to the reaction of the hydraulic fluid with free water.

Fire Safety. Standard tests have been devised to rate the relative "fire safety" of various fluids.⁷

The *flash point* is the temperature at which sufficient vapors are evolved from the fluid in a heated cup to cause a transient flame when a pilot flame is brought into the test area.

The *fire point* is the temperature at which the flame above a test cup will be self-sustaining.

The *autogenous ignition* temperature is considerably above the fire point and is the temperature at which a liquid droplet will ignite upon contact with heated air.

Compatibility of hydraulic fluid relates to the property of the fluid to either be affected or affect surrounding metallic and nonmetallic materials.

Toxicity. Several of the special hydraulic fluids for use in high-temperature applications contain special additives which may be injurious when inhaled as vapor.

43.2.4 Thermal Properties

The specific heat is the amount of heat which must be supplied to a unit mass to raise its temperature one degree.

For most fluids the specific heat increases with temperature. The specific heats of pure petroleum-base oils are well approximated by a Bureau of Standards equation

$$C = (1/\sqrt{\sigma})(0.388 + 0.000457T) \quad (43.12)$$

where C = specific heat, Btu/lb·°F

σ = specific gravity at 60°F/60°F

T = temperature, °F

The thermal conductivity for petroleum-base oils

$$k = (0.813/\sigma)[1 - 0.0003(T - 32)] \quad \text{Btu/h-ft}^2\text{°F-in} \quad (43.13)$$

43.2.5 Surface Properties

Two areas involving surface energies which are of interest to the designer of hydraulic equipment are foaming and boundary lubrication. A foam is an emulsion of gas bubbles in a liquid. Since foam is many times more compressible as the gas-free liquid, the presence of foam in a servosystem will drastically decrease the bulk modulus of the mixture depending on the volumetric proportions of foam and liquid in the chamber. Developers of hydraulic fluids frequently add antifoaming agents.⁶

Friction and wear between metal surfaces in sliding are strongly affected by the molecular structure of the fluid-metal interfaces. Boundary lubrication relates to physicochemical relations which occur in very thin films. "Oiliness" is sometimes defined as a property of a fluid which will give low coefficients of friction to two sliding surfaces (see Sec. 28).

43.2.6 Choice of Hydraulic Fluid

System performance, both steady and transient, is affected by fluid properties as follows^{3,4}:

Viscosity affects damping effects, pipe flow, lubrication, leakage, motor and pump efficiencies.

Density affects orifice flow, acoustic effects, pump and motor efficiency.

Specific heat and thermal conductivity combined with viscosity and density affect temperature rise and heat rejection.

Compressibility is all-important in determining transmission characteristics, stability and response of closed-loop control systems, and pressure pulsations from pumps.

Vapor pressure and gas solubility affect cavitation effects and the associated pump-filling problem as well as the potential effects on compressibility.

Hydraulic-system life and reliability are closely associated with such fluid properties as:

Lubricity. Boundary lubrication affects wear in pumps and motors, and friction in sliding valves.

Thermal stability, where poor performance results in high acidity, gas evolution, solid-particle formation, gum formation, varnish formation, depending upon fluid and temperature level.

Compatibility. Poor performance may result in seal deterioration and other side effects.

Not to be overlooked should be *hydrolytic stability*, *oxidative stability*, and *flammability* for consideration of *fire safety*.

Certain of the disadvantages of the types of fluids listed in Table 43.1* are as follows:

1. Petroleum-base oils have poor viscosity-temperature characteristics, are volatile and highly flammable.
2. Esters of dibasic organic acids have poor low-temperature viscosity, poor thermal stability, and poor compatibility with elastomers and other organic materials.
3. Phosphate esters are good in many respects for aircraft and industrial applications, but thermal stability is poor above 250°F.
4. Polyglycols have poor thermal stability above 400°F and their decomposition is catalyzed by some metals. They are incompatible with even a trace of hydrocarbon, this incompatibility resulting in the formation of gummy precipitates.
5. Fluorocarbons and chlorinated hydrocarbons have poor viscosity-temperature characteristics, do not accept additives, and are heavy.
6. Silicones have poor lubricity and are unresponsive to additives.
7. Polysiloxanes and orthosilicate esters have poor hydraulic stability and are as flammable as petroleum-base oils.
8. Phenyl ethers have poor low-temperature characteristics limiting their use to above 0°F.
9. Liquid metals such as NAK 77 (a eutectic of 77 percent Na, 23 percent K) and mercury have poor lubricity, present serious handling problems. Mercury is highly toxic, NAK 77 highly flammable in air.

43.3 FUNDAMENTAL RELATIONSHIPS IN HYDRAULIC FLOW

43.3.1 Introduction

In the design of such hydraulic components as pumps, motors, valves, and conduits the observation and proper application of certain basic relationships will assure sound fundamental design. These concepts are⁴

1. Conservation of mass (continuity)
2. Conservation of momentum
3. Conservation of energy

Conservation of mass requires that the rate of mass flow into a defined volume equal the rate of mass flow out plus the rate at which mass accumulates within the control volume. Thus (see Fig. 43.2)

$$\int_{A_s} \rho V_n dA_s + d/dt \int_v \rho dv = 0 \quad (43.14)$$

where V_n = velocity normal to control surface

A_s = control surface

v = control volume

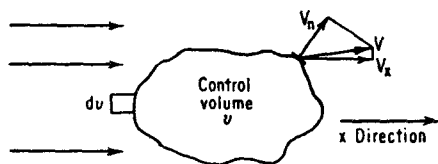


Fig. 43.2 Control volume in a flow field.

Conservation of momentum requires that the net rate of outflow of momentum in a specific direction x plus the rate at which x momentum accumulates within the control volume be equal to the force applied to the control volume in the x direction. Thus

$$\sum F_x = \frac{d}{dt} \int_V \rho V_x dv + \int_{A_s} \rho V_x V_n dA \quad (43.15)$$

EXAMPLE 1⁴: How is the pressure difference $P_1 - P_2$ related to the rate of change of flow of a frictionless incompressible fluid in a uniform tube of length L and area A ?

Conservation of mass requires that

$$\rho Q_1 = \rho Q_2 \quad \text{or} \quad Q_1 = Q_2 = Q \quad (\rho = \text{const}) \quad (43.16)$$

where Q = volume flow, in^3/s .

Conservation of momentum requires that

$$P_1 A - P_2 A = \rho Q V - \rho Q V + (d/dt)(\rho V A L) \quad (43.17)$$

Thus

$$P_1 - P_2 = (\rho L/A)(dQ/dt) \quad (43.18)$$

EXAMPLE 2: One-dimensional, frictionless, incompressible, streamline flow of a fluid (Fig. 43.3)

Conservation of mass demands that

$$V dA + A dV = 0 \quad (43.19)$$

Conservation of momentum demands that

$$-g\rho dZ - dP = \rho V^2(dA/A) + \rho V dV \quad (43.20)$$

Combining Eqs. (43.19) and (43.20) yields Euler's equation

$$V dV + dP/\rho + g dZ = 0 \quad (43.21)$$

Since ρ is constant, Euler's equation can be integrated yielding Bernoulli's equation

$$V^2/2g + P/\rho g + Z = \text{const} \quad (43.22)$$

Conservation of energy requires that the increase in internal energy of a system of fixed identity be equal to the work done on the system plus the heat added to the system. Applied to a control volume fixed in space and not subject to acceleration, conservation of energy yields (see Fig. 43.4)

$$dQ_n/dt + dW_x/dt - dE/dt + \int_{A_s} (P/\rho + e)\rho V_n dA \quad (43.23)$$

where Q_n = heat flow to the control volume

W_x = shaft and shear work done on the system

E = total internal energy of fluid inside the control volume

e = total internal energy per unit of mass ($e = u + gZ + V^2/2$)

u = intrinsic internal energy per unit of mass of fluid

Z = height above the reference point, in (gZ is the potential energy per unit of mass)

P = pressure on an element of area at the surface of the control volume

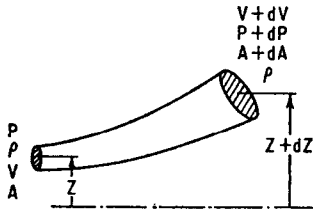


Fig. 43.3 One-dimensional, frictionless, steady, incompressible streamline flow with area change.

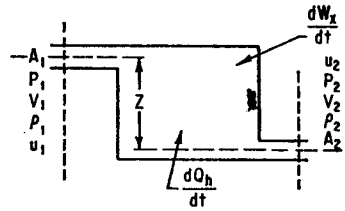


Fig. 43.4 Conservation of energy applied to a simple control volume.

43.3.2 Orifice Flow

The design of valves for control and regulation purposes and the design of pumps and motors require the analysis of flow through rounded and sharp-edged orifices.

Consider the case of *frictionless flow through a nozzle or orifice* as illustrated in Fig. 43.5. At condition 1 the velocity is negligible. From Bernoulli's equation and $\rho AV = \text{constant}$,

$$\rho Q = A_2 \sqrt{2p(P_1 - P_2)} \quad (43.24)$$

The discharge coefficient C_d is defined

$$C_d \triangleq A_2/A_0 \quad (43.25)$$

where A_0 is the nominal area of the constriction. The discharge coefficient varies from 0.6 to 1.0 depending upon the geometry of the upstream passage. With this definition the mass-flow equation becomes

$$\rho Q = C_d A_0 \sqrt{2p(P_1 - P_2)} \quad (43.26)$$

The fact that friction phenomena from viscous effects are neglected requires that the Reynolds number of the flow at section 2 must be in excess of a certain critical value, which varies from 260 for the sharp-edged annular orifice to 100,000 for certain circular orifices in pipes. The Reynolds number, a dimensionless quantity, is defined by

$$Re \triangleq \rho V D_e / \mu$$

where D_e is the equivalent hydraulic diameter of the constriction, μ is the absolute viscosity, V is the velocity, and ρ is the density. For Re larger than the critical value the discharge coefficient C_d is constant and thus independent of Re . For Re less than the critical value C_d is a function of both geometry and Re , as demonstrated in the following paragraphs.

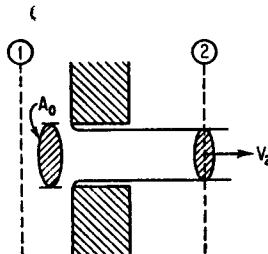


Fig. 43.5 Frictionless flow through a nozzle or orifice.

The Sharp-Edged Circular Orifice. Orifice meters for flow measurement commonly consist of sharp-edged circular orifices in pipes with pressure taps on each side of the orifice plate. Sharp-edged circular orifices are also commonly used in flow-control valves as fixed restrictions. The effect of upstream pipe diameter is commonly included as the ratio of orifice diameter to pipe diameter β . The mass-flow equation thus becomes

$$\rho Q = C_d A_o \sqrt{2\rho(P_1 - P_2)/(1 - \beta^4)} \quad (43.27)$$

For a circular orifice

$$Re = 4\rho Q/\pi\mu D \quad (43.28)$$

Figure 43.6 presents data for knife-edged orifices⁹ with 45° chamfers. As a properly constructed measuring device, the sharp-edged orifice will give a measurement accuracy of about ± 0.5 percent. As a control device, the sharp-edged circular orifice can be expected to give the most reproducible results as compared with other orifice configurations (see Fig. 43.7b).

The thick-plate circular orifice, particularly in the smaller sizes and for diameter-to-length ratio of unity, gives a 30 percent spread in discharge coefficient at any one Reynolds number for Reynolds numbers from 100 to 6000. A diameter length ratio below 0.2 is recommended in order to assure reasonable insensitivity to upstream conditions and manufacturing variations.

The quadrant-edge orifice is reported to give excellent results over large ranges of Reynolds number and diameter ratios¹⁰ (see Table 43.2 and Fig. 43.7c).

The Sharp-Edged Annular Orifice. For sliding-type control valves with sharp edges (see Fig. 43.7a) the discharge coefficient C_d is reasonably constant

$$C_d = 0.60 \text{ to } 0.65$$

above the critical value of the Reynolds number of 300. Below the critical value of Re , the discharge coefficient increases to about 0.95, then decreases almost linearly to $Re = 0$ (see Fig. 43.8). With decreasing pressure drop the peak value of C_d tends to decrease. This phenomenon becomes pronounced for ΔP below about 150 lb/in².

Poppet-type valves show a similar variation of discharge coefficient with Re . Experiments¹¹ show general agreement with Fig. 43.8 over pressure-drop ranges of 500 to 200 lb/in² and 45° poppet angles. The geometry of the valve and the valve chamber should be considered in applying any data, as the effect may be large for special cases.

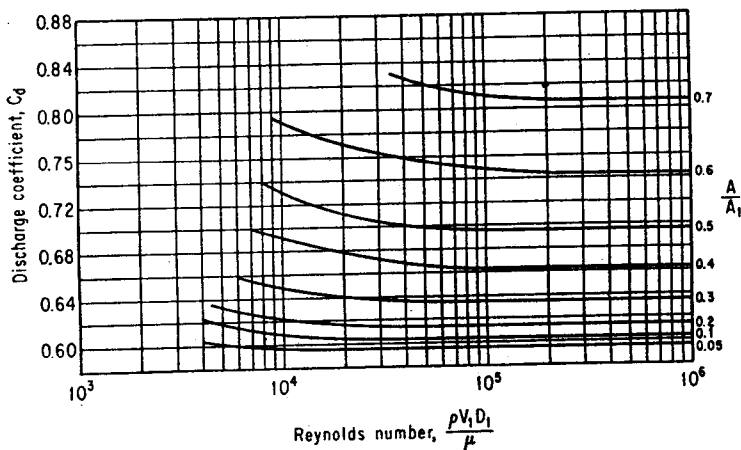


Fig. 43.6 Data for knife-edged orifices.

Table 43.2 Discharge Coefficients for Quadrant-Edged Orifices¹⁰

β	r/d	$C_d = K(\sqrt{1 - \beta^4})$	Limits of constant C_d within $\pm 0.5\%$	
			R_{Dmin}	R_{Dmax}
0.225	0.100	0.77	760	56,000
0.400	0.114	0.78	650	140,000
0.500	0.135	0.801	430	230,000
0.600	0.209	0.838	350	250,000
0.625	0.285	0.859	330	250,000
0.656	0.380	0.890	270	250,000
0.692	0.446	0.890	250	250,000

The "Flapper-Valve" Type of Annular Orifice. The "flapper valve" has been for many years and is today frequently employed in control valves. Figure 43.9a shows the sharp-edged orifice. Figure 43.9b is the more practical chamfered and flat design. Experiments for a two-dimensional glass-sided model gave discharge coefficients of 0.65 for Reynolds numbers in excess of 600 for the sharp-edged orifices as shown in Fig. 43.10. Figure 43.11 presents the results for several flats and chamfers as shown in Fig. 43.9b.¹² Outside chamfer angles α can vary from 30 to 60° without materially affecting the result. The results indicate that internally chamfered orifices of the flapper type can have discharge coefficients up to 0.90, which remain constant for Reynolds numbers in excess of 600, where Re is defined by

$$Re = \rho Vx/\mu \quad (43.29)$$

$$\text{where } V = \sqrt{2(\Delta P)/\rho} \quad (43.30)$$

x = flapper opening, in

ΔP = pressure drop, lb/in²

ρ = density of fluid, lb·s/in⁴

V = average velocity, in/s

since
and

$$Q = C_d A \sqrt{2(\Delta P)/\rho} \quad (43.31)$$

$$A = \pi D x \quad (43.32)$$

$$Re = Q/\pi D C_d \mu \quad (43.33)$$

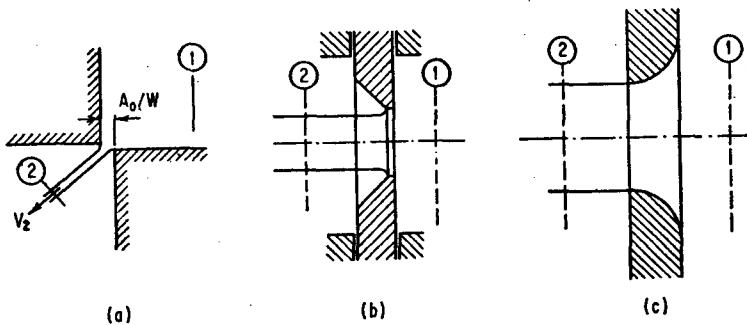


Fig. 43.7 Flow through nozzles and orifices. (a) Sharp-edged annular orifice. (b) Sharp-edged orifice. (c) Quadrant-edged orifice.

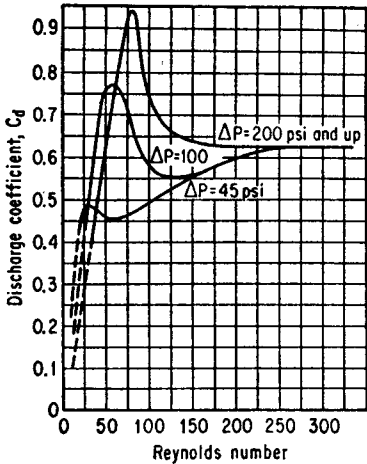


Fig. 43.8 Orifice coefficients for annular sharp-edged orifices as a function of orifice Reynolds number.

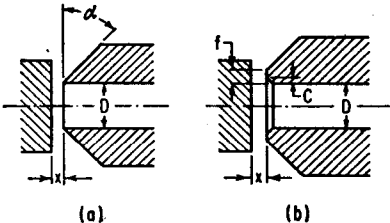


Fig. 43.9 Schematic diagrams of flapper orifice configurations.

43.3.3 Laminar Flow

In the design of such hydraulic components as certain closely fitted valves, the pistons and valve plates of positive-displacement pumps and motors, and certain seals, the equations of fully developed laminar flow are of interest. The following cases assume constant fluid viscosity and density.⁴

Steady Flow through Circular Pipes (see Fig. 43.12).

$$dP/dx = -128 \mu Q / \pi D^4 \tag{43.34}$$

$$dP/dx = -32 \mu V_{av} / D^2 \tag{43.35}$$

$$Q = (\pi D^4 / 128 \mu L) (P_1 - P_2) \tag{43.36}$$

$$u = 2V_{av} (1 - 4r^2/D^2) \tag{43.37}$$

$$V_{av} = 4Q / \pi D^2 \tag{43.38}$$

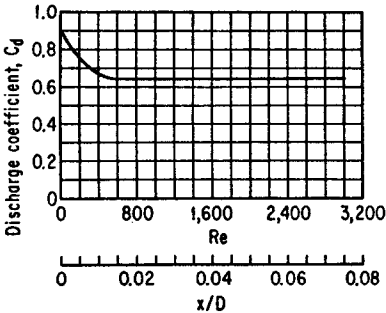


Fig. 43.10 Discharge coefficients as a function of flapper opening for sharp-edged flapper nozzle with 60° outside chamfer.

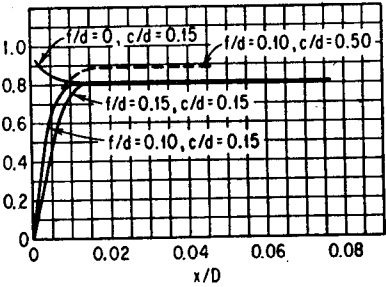


Fig. 43.11 Discharge coefficients as a function of flapper opening for several nozzles with flaps and inside chamfers at 45°.