DICTIONARY of MATHEMATICS

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by

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First published 1961

Printed in Great Britain by
J. W. Arrowsmith Ltd., Bristol 3

PREFACE

To SOME, mathematics appears difficult and complicated; to others, a subject of endless fascination and interest. This book should prove useful to readers in both these categories; as well as to that intermediate group—the students—in transition from first to second of these classes.

For the reader with little previous knowledge of mathematics, this dictionary will provide immediate assistance whenever specific problems are encountered; casual reading will also reveal something of the immense scope and power of the mathematical tools.

The student and teacher will find in this book ready reference to hundreds of the most commonly required definitions, processes, formulae and tables, as well as brief notes on the classic mathematicians. The level is suitable for use up to degree standard, though equally valuable for less advanced study.

In such a vast subject, the problem has been in deciding what to exclude as much as what to include; the aim has been to provide the maximum possible usefulness in a single compact book. The choice has thus invariably fallen on the inclusion of words and terms of most frequent occurrence, rather than to devote undue space to the more abstract philosophical conceptions.

In the modern world our lives are increasingly dominated by mathematics: I trust that this dictionary will help to guide many readers to an easier—and perhaps a little happier—relationship with this fundamental science.

C. C. T. BAKER

DICTIONARY OF MATHEMATICS

A

Abacus. A framework of wires carrying beads, and used for arithmetical calculations.

Abel. (1802–1829.) Abel proved the impossibility of solving the general quintic equation, $ax^5+bx^4+cx^3+dx^2+ex+f=0$, by means of radicals. He wrote several papers on the convergence of series, "Abelian" integrals, and elliptic functions. One of the first to insist on rigor in Mathematics.

Abridged Notation, the Method of. The method of using a single letter, equated to zero, to represent a given locus. Thus, $l=0$ may represent the straight line $Ax+By+C=0$, and $S=0$ may represent the point-conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0.$$

This makes expressions more manageable. For example, let the line $x\cos\alpha+y\sin\alpha-p_1=0$ be represented by $l_1=0$, and the line $x\cos\beta+y\sin\beta-p_2=0$ be represented by $l_2=0$. Then any straight line through the point of intersection of the two lines is represented by $l_1-kl_2=0$, where k is an arbitrary constant. Similarly, the equation of the conic passing through the points of intersection of the conics $S_1=0$ and $S_2=0$ is $S_1=kS_2$.

Abscissa. Using rectilinear coordinates, it is the perpendicular distance of a point from the y-axis, with the proper sign attached. Using inclined axes, it is the distance from the point to the y-axis along a line parallel to the x-axis. In Fig. A-1, PN is the abscissa of P.

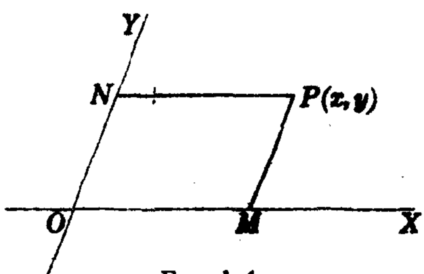

FIG. A-1

Absolute Convergence. A series is absolutely convergent if the sum of the absolute values of the terms is convergent. Thus, if the series $\Sigma|u_n|$ is convergent, then the series Σu_n is absolutely convergent, where u_n may be real or complex.

Absolute Number. A number like 2, or 5, or $\sqrt{3}$, as distinct from a quantity represented by a symbol.

Absolute Term. In an algebraic expression it is the term which does not contain the variable. In the equation

$$lx+my+n = 0,$$

n is the absolute term. It is also called the constant term.

Absolute Units. In the British system, the ft–lb–sec system, the unit of length is the foot, the unit of mass is the pound, and the unit of time is the second. In the internationally adopted metric system, the m.k.s. system, the unit of length is the metre, the unit of mass is the kilogram, and the unit of time is the second. These units are independent of the value of g, the acceleration due to gravity, which is not constant for all points on the Earth's surface. Since the weight of a pound, and the weight of a kilogram do depend upon g, they are called gravitational units.

Absolute Value of a Complex Number. The absolute value of the complex number $x+iy$, where x and y are real, is $+\sqrt{(x^2+y^2)}$. It is sometimes called the modulus of the number. The absolute value of $4+3i$ is $\sqrt{(16+9)}$, i.e. 5.

Absolute Value of a Real Number. The actual magnitude of a quantity regardless of its sign. The absolute value of x is written $|x|$. (Also known as modulus or "mod" x.) It is the numerical value. The absolute value of $+3$ and -3 is the cardinal number 3.

Absolute Velocity. The true, or real, velocity of a body as observed from the earth. The velocity of a body as observed from a point which is itself moving relative to the earth is called the relative velocity of the body.

Abstract. Not connected with anything material or concrete, such as the abstract number 2.

Abstract Geometry. There can be any number of abstract geometries, since they are creations of the mind built upon a collection of postulates.

Acceleration. The rate of change of velocity with time. If the velocity increases by equal amounts in successive equal times, the accelèration is said to be constant, or uniform. A negative acceleration is called a retardation. If the velocity of a point after time t is v, its acceleration is dv/dt. If its displacement after time t is x, its velocity is dx/dt, and its acceleration is

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

The acceleration of a point is also

$$\frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}.$$

In the fluxional notation, velocity is written $\dot{x}$ and acceleration $\ddot{x}$. With this notation, acceleration may also be written $\dot{v}$.

Acceleration of a Falling Body. The acceleration produced by the force of the Earth's gravitation. It is approximately 32·2 ft/sec/sec, or 981 cm/sec/sec. If a body falls from rest through a height h in time t under acceleration g, then $h = \frac{1}{2}gt^2$.

Acnode. An isolated point or conjugate point of a curve. A point in whose neighbourhood there is no other point of the curve. See Isolated Point.

Action and Reaction. See Newton's Laws of Motion.

Acute Angle. An angle less than a right angle.

Adam's Property of Tangent to a Conic. Fig. A-2 shows a conic. T is a point on the tangent at P. TU and TI are perpendiculars from T to SP and the directrix respectively. Then $SU = e\,.\,TI$.

Additive Function. If $f(x+y) = f(x)+f(y)$, then $f(x+y)$ is an additive function.

Adiabatic Expansion. If the expansion of a gas takes place isothermally, there is no alteration of temperature and, ideally, Boyle's Law, pv = constant, will hold. If the expansion is adiabatic, no heat is taken from, or supplied to, the gas, and the relation between p and v takes the form pv^γ = constant, where γ is a numerical constant, being the ratio of the two specific heats of the gas (measured respectively at constant volume and constant pressure).

Ad Infinitum. Usually refers to a series which continues without end. Indicated by dots ..., or by $\rightarrow \infty$.

Aggregation. The collecting together of terms to be treated as a single term. They are enclosed in brackets ( ), [ ], or { }, or placed below a line ——— called a vinculum.

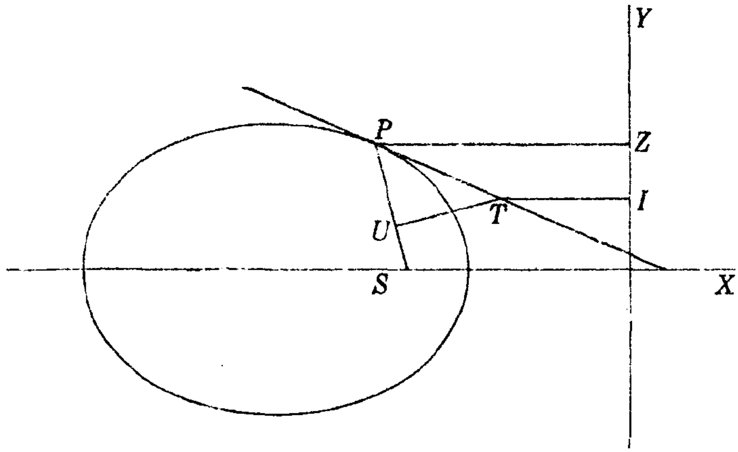

FIG. A-2.

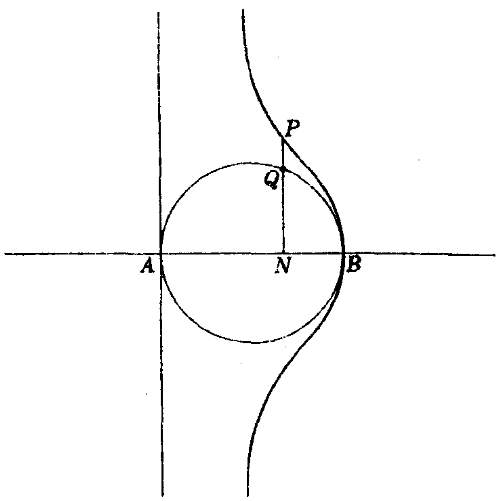

FIG. A-3

Agnesi, the Witch of. Sometimes called the versiera. It is a plane cubic curve. Draw a circle with AB as diameter, Fig. A-3. Draw PQN perpendicular to AB such that

$$QN : PN = AN : BN.$$

Then the locus of P is a curve called the witch of Agnesi, after Donna Maria Gaetana Agnesi. If A is the origin and AB is along the x-axis, its equation is $xy^2 = 4a^2(2a-x)$, where a is the radius of the circle.

Ahmes Papyrus. Sometimes called the Rhind papyrus, this treatise was written by an Egyptian scribe about 1600 B.C. It is probably the oldest book on Mathematics, and contains about 85 mathematical problems. It was bought by an Egyptologist named Henry Rhind in the 19th century, and is now in the British Museum. It deals with simple algebraic equations like $x-\frac{1}{8}x = 17$, giving the answers correctly, or in unit fractions, although the subject Algebra did not exist in those days. It also contains five problems on the mensuration of pyramids, and the earliest known mention of an approximate value for π.

Algebra. Generalised arithmetic.

Algebra, Fundamental Theorem of. Every polynomial of degree n ($n \geqslant 1$) with complex coefficients has at least one real root which is a complex number, either real or imaginary. It was first proved by Gauss.

Algebraic Expression. One containing only algebraic symbols and operations. Examples: $3x+5$; $2x^2+3x+4$; $\sqrt{x}$. Expressions which are not algebraic are called transcendental, of which $\log x$, $\sin^{-1}x$, a^x, $\tan x$ and e^x are examples.

Algebraic Function. A function which contains only algebraic terms and symbols. An expression which can be put in the form $Ax^n+Bx^{n-1}+Cx^{n-2}+ \ldots$, where A, B, C, etc. are rational integral functions of x.

Algorithm, Euclid's. A method of finding the highest common factor of two numbers. It can also be used for polynomials in algebra.

Alternate Angles. The angles on opposite sides of a transversal which cuts two lines. In Fig. A-4, a and d are alternate angles; so are b and c. If AB and CD are parallel and are cut by the transversal PQ, the alternate angles a and d are equal, and the alternate angles b and c are equal.

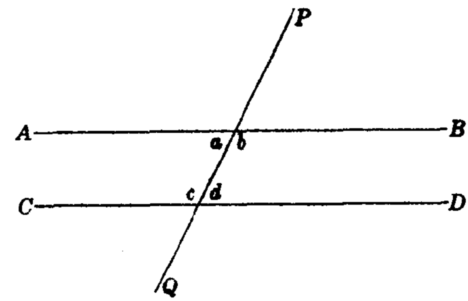

FIG. A-4

Alternating Series. A series of terms which are alternately positive and negative.

Altitude of a Triangle. The perpendicular distance from a vertex to the opposite side, then called the base.

Ambiguous. Doubtful, uncertain, or not having a unique solution.

Ambiguous Case in the Solution of a Plane Triangle. This occurs when two sides and an angle other than the included angle are given. If AB and BC and angle A are given, Fig. A-5, C may be at C_1 or at C_2.

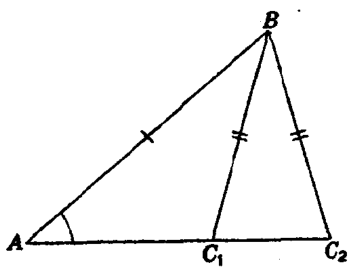

FIG. A-5

Amplitude. The maximum displacement from the mean position in vibratory motion. In the periodic function $y = a \sin \omega t$, the amplitude is a. If a complex number is represented in polar coordinates, it takes the form $r(\cos\theta + i\sin\theta)$, where θ is the amplitude, or argument (phase number or phase angle). It is the angle which the vector representing a complex number makes with the positive horizontal axis. See Argand Diagram.

Analogy. Likeness, or resemblance. Methods for solving problems are sometimes suggested by methods which work for similar problems.

Analysis. Many of the principles of Mathematics are often illustrated graphically, or by drawings, or by models. Rigorous proofs of mathematical propositions are based on logic, and the study of such is called Mathematical Analysis. It uses mainly the methods of Algebra and the Calculus, rather than of Geometry.

Analytic Function of a Complex Variable. A function $f(z)$ of the complex variable $z = x+iy$ is analytic at a point in the z-plane if the function and its first derivative are finite and single valued there.

Analytic Function of a Real Variable. A function $f(x)$ is analytic for $x = h$ if it can be represented by a Taylor's series in powers of $(x-h)$ which is equal to the function for a value of x in the neighbourhood of h.

Analytical Geometry. Cartesian, or coordinate geometry. Position is represented by coordinates, and algebraic methods of reasoning are used.

Analytical Methods. Solutions using algebraic methods.

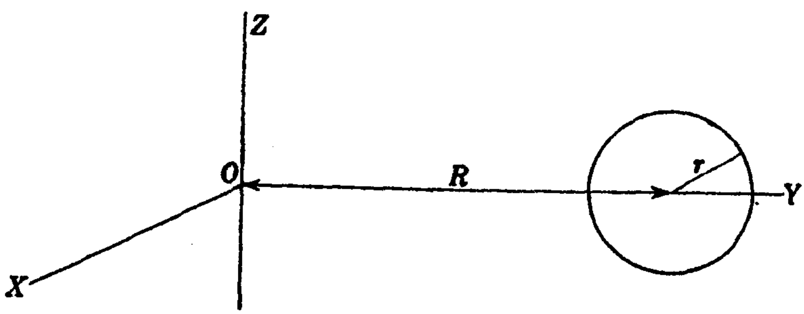

Fig. A-6

Anchor Ring. A torus, or tore, Fig. A-6. The surface generated by rotating a circle about a line in its plane and external to it.

If r is the radius of the rotating circle, R the distance of its centre from the axis of rotation, say the z-axis, then the equation of the generating circle is $(y-R)^2+z^2 = r^2$, and the equation of the anchor ring is $[\sqrt{(x^2+y^2)}-R]^2+z^2 = r^2$. The volume generated is $2\pi^2Rr^2$, and the surface generated is $4\pi^2Rr$.

Angle. A measure of the inclination of two straight lines to each other. In Fig. A-7, the inclination of the lines OA and OB is the angle AOB, written $\angle AOB$, or $A\hat{O}B$. In the sexagesimal system, the geometric unit is the right angle, which is subdivided into 90 equal parts called degrees. One complete revolution of the radius vector describes an angle of 360 degrees. Each degree is subdivided into 60 equal parts called minutes. Each minute is subdivided into 60 equal parts called seconds. An angle of 72 degrees 34 minutes 18 seconds is written as $72° 34' 18''$. If no symbol is placed with the size of the angle, it is understood to represent radians. The radian is a unit of circular measure. The angle 0·75 means an angle of 0·75 radians, and an angle π is an angle of π radians, which is 180 degrees. See Radian.

Angle Between a Line and a Plane. The acute angle between the line and its projection on the plane.

Angle Between Two Intersecting Curves. The angle between the tangents at the point of intersection of the curves.

Angle Between Two Lines in Space. The angle between two intersecting lines which are parallel to the two given lines.

Angle Between Two Planes. Called a dihedral angle. The angle between two lines, one in each plane, drawn from a point in the line of intersection of the two planes and perpendicular to it.

Angle Between Two Straight Lines in a Plane. If two straight lines are represented by the equations $y = m_1x+c_1$ and $y = m_2x+c_2$, then they intersect at an angle θ which is given by

$$\tan\theta = \frac{m_1-m_2}{1+m_1m_2}.$$

Angle of Contingence. If P and Q, Fig. A-8, are two points on a circle, with centre O, such that angle POQ is $\delta\psi$, the angle between the tangents at P and Q is also $\delta\psi$, and this is called the angle of contingence.

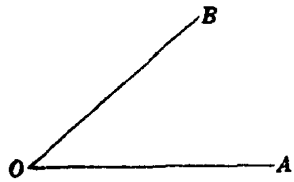

FIG. A–7

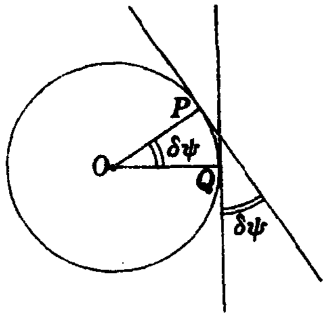

FIG. A–8

Angle of Depression. The angle *AOB*, Fig. A-9, *OA* being horizontal.

Angle of Elevation. The angle *POQ*, Fig. A-10, *OP* being horizontal.

Angle of Friction. See Friction.

Angle of Polygon. The sum of the angles of a convex polygon of n sides is $(2n-4)$ right angles. Each angle of a regular convex polygon of n sides is $(2n-4)/n$ right angles.

Angle, Phase. See Amplitude.

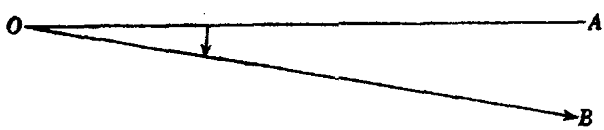

FIG. A–9

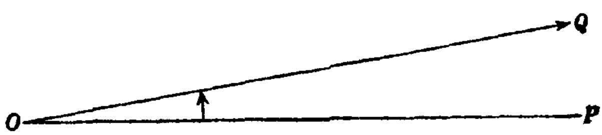

FIG. A–10

Angle, Solid. With a given point as centre, describe a sphere of unit radius. If radii are drawn from the given point through all points of the boundary of a given area, then these radii enclose a solid angle whose magnitude is that of the given area. The unit solid angle is called a steradian. It intersects unit surface area on a sphere of unit radius. The sum of all the solid angles which can be drawn at a given point without overlapping is equal to 4π, this being the area of a sphere of unit radius. Two solid angles are considered equal if they have the same magnitude, irrespective of their shape.

Angular Acceleration. The rate of change of angular velocity, i.e. $d\omega/dt$, or $d^2\theta/dt^2$, written $\dot{\omega}$ or $\ddot{\theta}$.

Angular Momentum. The same as moment of momentum. Fig. A-11 shows a particle of mass m rotating in a circle of radius r. Its linear velocity is v and its angular velocity is ω. Its linear momentum is mv. Its angular momentum is the moment of its linear momentum $= mvr = mr^2\omega$ (since $v = r\omega$) $= I\omega$ (since $I = mr^2$). Thus the angular momentum is the product of the moment of inertia and the angular velocity. It is a vector quantity. The effect of a torque on a body is to give it angular acceleration.

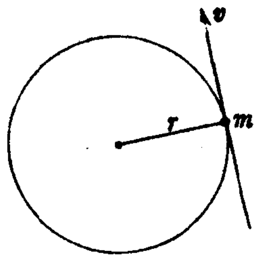

FIG. A-11

Angular Motion. Rotary motion. The formulae for angular motion are analogous to those for linear motion. Let ω_1 be the initial velocity, ω_2 the final velocity, α the angular acceleration and θ the angle turned through in time t, then the following formulae hold:

(1) $\omega_2 = \omega_1 + \alpha t.$ (2) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2.$

(3) $\theta = \dfrac{\omega_1 + \omega_2}{2} t.$ (4) $\omega_2^2 = \omega_1^2 + 2\alpha\theta.$

Angular Speed. The number of degrees or radians through which the radius vector passes in unit time. It is measured in degrees per second or in radians per second.

Angular Velocity. The rate of change of the angle, θ, rotated through, i.e. $d\theta/dt$, or $\dot{\theta}$, usually denoted by ω.

Anharmonic Ratio. The same as cross ratio.

Annulus. The area between two plane concentric circles. Its area is $\pi R^2 - \pi r^2 = \pi(R+r)(R-r)$, R being the radius of the larger, and r the radius of the smaller circle.

Antilogarithm. Inverse logarithm. The number whose logarithm is the given number.

Aperiodic Motion. Not periodic. The kind of motion obtained in the case of a pendulum which moves in a viscous fluid, in a "dead beat" galvanometer, and in some seismographs.

Apex. The highest point.

Aphelion. The point of the elliptic orbit of a planet which is farthest from the sun.

Apollonius. (260–170 B.C.) A Greek geometer who carried on the work of Euclid. Famous for his work on conics, about which he wrote 8 books, 7 of which survive. He introduced the parabola, ellipse, and hyperbola as sections of a circular cone. He proposed geometrical constructions with ruler and compasses only, and showed how to construct a circle to touch three given circles.

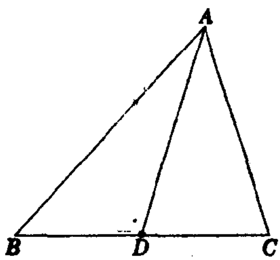

FIG. A–12

Apollonius's Circle. The locus of a point which moves in a plane so that the ratio of its distances from two fixed points in that plane is constant.

Apollonius's Theorem. In any triangle, the sum of the squares on two sides is equal to twice the square on half the third side together with twice the square on the median. In ΔABC, Fig. A-12, $AB^2+AC^2 = 2BD^2+2AD^2$.

Approximately Equal To. Written as $\doteqdot$ or $\simeq$.

Approximations. There are several methods of making approximations. (a) If the successive terms in the expansion $f(x) = a_0+a_1x+a_2x^2+ \ldots$ become rapidly smaller, a good approximation to the value of $f(x)$ is obtained by taking the first few terms. (b) A particular case of the binomial theorem states that:

$$(1+x)^n = 1+nx+\frac{n(n-1)}{1\,.\,2}x^2+\frac{n(n-1)(n-2)}{1\,.\,2\,.\,3}x^3 \ldots .$$

If the terms of the second and higher degrees are neglected, then $(1+x)^n = 1+nx$. This is called the first approximation. The second approximation is

$$(1+x)^n = 1+nx+\frac{n(n-1)}{1\,.\,2}x^2,$$

and so on for higher approximations. Other simple approximations are:

$$\frac{1}{1-x} = (1-x)^{-1} \simeq 1+x,$$

$$\frac{1}{1+x} = (1+x)^{-1} \simeq 1-x,$$

$$\frac{1}{(1-x)^2} = (1-x)^{-2} \simeq 1+2x,$$

$$\frac{1}{(1+x)^2} = (1+x)^{-2} \simeq 1-2x.$$

Similarly, if x and y are small compared with unity, their product and powers higher than the first may be neglected, and then

$$(1+x)(1+y) = 1+x+y.$$

Also, if h is small compared with x,

$$(x+h)^n \simeq x^n + nx^{n-1}h.$$

(c) Approximations may be made using differentials and differential coefficients.

Since

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx},$$

it follows that $\Delta y/\Delta x = dy/dx + E$, where $E \to 0$ as $\Delta x \to 0$.

Therefore

$$\Delta y = \frac{dy}{dx}\Delta x + E \,.\, \Delta x.$$

Thus if Δx is very small

$$\Delta y = \frac{dy}{dx}\Delta x,$$

approximately.

This is used to find approximate changes in the value of y for small changes in the value of x for, if x is increased or decreased by a small amount, y will increase or decrease by approximately dy/dx times that amount.

(d) Taylor's expansion gives better approximations than the differentials in (c), for it enables Δy to be found to any desired degree of approximation.

Let $y = f(x)$. Then $\Delta y = f(x+\Delta x) - f(x)$.

By Taylor's series,

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + f''(x)\frac{(\Delta x)^2}{2!} + f'''(x)\frac{(\Delta x)^3}{3!} + \dots .$$

Therefore

$$\Delta y = \frac{dy}{dx}\Delta x + \frac{1}{2!}\frac{d^2y}{dx^2}(\Delta x)^2 + \frac{1}{3!}\frac{d^3y}{dx^3}(\Delta x)^3 + \dots .$$

Approximations to Roots of Equations. See Newton's Method of Approximating to the Roots of an Equation.

A Priori. Starting from first principles.