Theory of thermoelasticity with applications

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Preface

Although the theory of thermoelasticity has a long history, its foundations having been laid in the first half of the nineteenth century by Duhamel and Neumann, wide-spread interest in this field did not develop until the years subsequent to World War Two. There are good reasons for this sudden and continuing revival of interest. First, in the field of aeronautics, the high velocities of modern aircraft have been found to give rise to aerodynamic heating; in turn, this produces intense thermal stresses and, by lowering the elastic limit, reduces the strength of the aircraft structure. Secondly, in the nuclear field, the extremely high temperatures and temperature gradients originating in nuclear reactors influence their design and operation. Likewise, in the technology of modern propulsive systems, such as jet and rocket engines, the high temperatures associated with combustion processes are the origin of unwelcome thermal stresses. Similar phenomena are encountered in the technologies of space vehicles and missiles, in the mechanics of large steam turbines, and even in shipbuilding, where, strangely enough, ship fractures are often attributed to thermal stresses of moderate intensities.

The investigations of these, and similar, problems have brought forth a remarkable number of research papers, both theoretical and experimental, in which various aspects of thermal stresses in engineering structures are described. It is noteworthy, however, that those of but a mere handful of writers have decided to compile and order the available material, for the purpose of casting it into the form of a comprehensive exposition. Probably the first of these authors was H. Parkus, who, in cooperation with E. Melan, published, in 1953, a volume on stationary thermal stresses;[1] this was later com-

[1]E. Melan and H. Parkus, *Thermal Stresses Due to Stationary Temperature Fields* (in German), Springer, Vienna, 1953. A book by N. N. Lebedev (1937), reported by Russian authors, was never available outside the U.S.S.R.

plemented by two additional, no less valuable, volumes, one of which examined transient states.[2] In the interim, there appeared the publication of B. E. Gatewood,[3] which, like the booklet of D. J. Johns[4] (published eight years later), was well adapted to the needs of the designer.

In 1960, B. A. Boley and J. H. Weiner gave a profound and comprehensive exposition of various facets of the theory of thermoeleasticity, now considered classic.[5] In the same year, W. Nowacki published, in Polish, the results of his distinguished research of many years, later translated into English.[6] A companion volume on dynamic problems, by the same author, appeared in print a few years later.[7]

The list of previous writers apparently ends with the name of A. D. Kovalenko,[8] whose book, published in Russian in 1970, includes a number of interesting examples.[9]

The present book has its origins in a course of lectures on thermoelasticity given by the author during a number of years, in the Department of Mechanical and Aerospace Engineering of the University of Delaware and elsewhere.

It is well known that teaching in the classroom, by the spoken word, imposes more stringent obligations upon the instructor than teaching by the written word, that is, by means of a textbook. First, everything spoken should remain clear to everyone present, including the least bright and least prepared listeners. Secondly, the reasoning in classroom communication should proceed in a much more connected manner than that which is required in written communication; for statements lacking full motivation or logical gaps in the argument emerge vividly, raising doubts concerning the soundness of the development.

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[2]H. Parkus, *Transient Thermal Stresses* (in German), Springer, Vienna, 1959; H. Parkus, *Thermoelasticity*, Blaisdell, Waltham, Mass., 1968.

[3]B. E. Gatewood, *Thermal Stresses with Applications to Airplanes, Missiles, Turbines and Nuclear Reactors*, McGraw-Hill, New York, 1957.

[4]D. J. Johns, *Thermal Stress Analysis*, Pergamon, Oxford, 1965.

[5]B. A. Boley and J. H. Weiner, *Theory of Thermal Stresses*, Wiley, New York, 1960.

[6]W. Nowacki, *Thermoelasticity*, Pergamon Press, Oxford, 1962.

[7]W. Nowacki, *Dynamic Problems of Thermoelasticity* (in Polish), P.W.N., Warsaw, 1966.

[8]A. D. Kovalenko, *Thermoelasticity*, Wolters-Noordhoff, Groningen, 1969.

[9]It is perhaps not out of place to mention the notes from the course held in 1972 by I. N. Sneddon: *The Linear Theory of Thermoelasticity*, Intern. Centre of Mech. Sci., publ. 110, Udine, 1974.

The satisfaction of both these demands is usually not a simple matter; so it was in the author's case when, as it usually happens in the extension courses, the members of his audience ranged from young students with bachelor's degrees to mature doctors of philosophy coming from industry in order to broaden or refresh their formal knowledge, acquired perhaps many years earlier. In such a situation, it is never clear who knows or who remembers what, and (usually to the noticeable relief of the audience) it is often advisable not to anticipate too much erudition on the part of the listeners. Whether ultimately beneficial or harmful, such an approach was taken by the author. Naturally, this reflected strongly on the content of the lectures and, eventually, has greatly influenced the content of this book, making it rather self-contained and suitable for independent study. Although, in this sense, the character of the book is elementary, the contents of the book are not; and the author would like to think that it acquaints the reader quite thoroughly with many aspects of thermoelasticity, affording him a solid grounding upon which he may proceed to construct his own investigations. Of course, there is no doubt that experts in the field will find some portions of the book too discursive.

As a rule, each chapter starts with a relatively detailed exposition of the concepts essential for the comprehension of the subsequent text. The theoretical introductions are illustrated by one or more examples, solved in full. References to the pertinent literature are given apropos of each formula or statement that (due to limitations on space) cannot be derived or sufficiently motivated in the text. These references constitute an integral part of the text and may be of service to those readers who wish to supplement their study. It is believed that the emphasis on the fundamentals, rather than on a (necessarily limited) number of ready-to-use recipes, better prepares the student for independently attempting the solution of problems which he may encounter in future.

To facilitate the reading, Chapter 2 gives some rudiments of the indicial notation, used extensively throughout the book, and includes a list of formulas either referred to later in the text or hard to find in the literature. Those elements of the tensor calculus which are provided in Section 2.1 should satisfy the needs of the readers who would like to study all advanced topics of linear thermoelasticity.

Chapters 3 through 6 give a compact description of the main ideas of thermodynamics, heat transfer, and elasticity. Chapter 7 discusses the inadequacy of the classical, parabolic, equation of heat conduction; Chapter 9 contains a rather detailed analysis of the temperature field. Certain methods of solution of the differential equation of heat conduction are given here with an eye toward their application in the solution of general thermoelastic problems, not necessarily those of heat conduction alone.

To the investigation of the stress fields and the problem of solidification are devoted Chapters 8 and 10 through 13. These chapters complete the description of the theoretical foundations of thermoelasticity and close the first part of the book.

The second part includes a number of applications of the theory; it examines thermoelastic equilibrium and stability of bars, plates, and shells, as well as the influence of cracks. Dynamic problems discussed include certain of those associated with moving and periodic temperature fields, as well as with thermoelastic vibrations and waves. The effects of coupling of the mechanical and thermal fields are described in Chapter 21, while applications of variational methods are taken up in Chapter 26. Chapters 22 to 25 touch upon a few subjects which, at the present time, probably find no direct applications in engineering but (at least in the author's opinion) may be of importance in the not-too-distant future. Because of limited space, a number of originally projected and prepared chapters on the variational methods in heat conduction, function-space methods, bodies with temperature-dependent properties, nonlinear thermoelasticity, and thermoelasticity of micropolar materials could not be included in the text. The same can be said for numerical methods, although the book features the solution of a quantity of numerical examples.

Certainly it is true that, in a work of this sort, touching upon a variety of disciplines within the engineering sciences, there is always a chance for a slip of the pen or an insidious error to creep in; the most one can hope is that they are few and venial: the author will be thankful if they are reported.

The material has been selected from the literature according to the author's preference; however, many outstanding contributions had to be left out. Whenever possible credit has been given to the original sources.

It is a pleasure to acknowledge the help the author has received from his friends and former students: Dr. Allan Dallas, who read over the entire manuscript and suggested many improvements of the text and style; Dr. Hisaichi Ohnabe, who checked the calculations in the first portion of the book. My colleagues, Professors M. A. Beatty, A. C. Eringen, M. D. Greenberg, E. H. Kerner, R. J. Libera, and C. Y. Yang were kind enough to read and criticize particular chapters of the book (some not included). Of course, the author alone is responsible for the text as it stands.

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Grateful acknowledgment is made to several authors and publishers for permission to reproduce certain illustrations: Springer-Verlag, New York (Figs. 1 and 2 from *Acta Mechanica*, vol. 16, 1973, pp. 45–64; now Fig. 22.2); Dun-Donnelley Publ. Corp., New York, and Prof. F. Kreith (Figs. 6.1, 6.4, 10.18 from *Principles of Heat Transfer*, 1967; now Figs. 5.2 and 5.3); Prof. R. Muki and Jap. Soc. of Mech. Eng. (Figs. 1 to 3 and 5 from *Bulletin of JSME*, vol. 4, 1964, pp. 506–514; now Figs. 19.6 and 19.7); Prof. W. Fiszdon, Editor, *Arch. Mech. Stos.*, and P.W.N. (large portions and figures from author's papers in vol. 5, pp. 221–235, vol. 7, pp. 247–265, vol. 9, pp. 359–368; now Fig. 12.1); North-Holland Publ. Co., New York (Table 1 from *Progress in Sol. Mech.*, vol. 1, 1960, p. 279; now Table 20.2); Dr. J. J. Jaklitch, Editor, *ASME* (Fig. 4 from *Journ. Appl. Mech.*, 1969, pp. 763–767; now Fig. 13.2); Editor *ZAMP* (large portions of author's paper in vol. 12, 1961, pp. 132–148); McGraw-Hill Book Co., New York (Fig. 5.8 from E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*, 1972; Fig. 20 from I. S. Sokolnikoff and R. M. Redheffer, *Mathematics of Physics and Modern Engineering*, 1958; Fig. 14.8 from G. E. Dieter, Jr., *Mechanical Metallurgy*, 1961; now Figs. 9.18, 10.5, 17.4c); BSB B. G. Teubner Verlagsges., Leipzig, Fig. 88 on p. 71 (now Fig. 24.1) of Grimsehl, *Lehrbuch d. Phys.*, vol. 2, Elektromagn. Feld, 16 edition, 1963.

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Introduction

As clearly indicated by the name itself, thermoelasticity is a branch of applied mechanics concerned with the effects of heat on the deformation and stresses in solid bodies which are considered to be elastic. It is, thus, an extension of the conventional theory of isothermal elasticity to processes in which deformation and stresses are produced not by mechanical forces alone, but by temperature variation as well. A typical example of thermal deformation is the buckling of rails, which may result if the narrow gaps between the rails are not sufficient to allow for their free thermal expansion.

Thermoelastic processes are not totally reversible. For while the elastic part may be reversed, inasmuch as the deformations caused by heat are recoverable (at least theoretically) through cooling, the thermal part may not. This phenomenon owes its existence to the dissipation of energy taking place during heat transfer, in particular during heat conduction. That is, heat that spontaneously flows from hot to cold may not be transported back to its source without an external intervention, and the original thermal conditions may not be restored.

The effect of the temperature field on the deformation field is not a one-way phenomenon. It is an experimental fact that a deformation of the body produces changes in its temperature. In other words, a deformation acts as a source or sink of heat. As a familiar example, recall that in the standard tensile test the metallic sample at the moment of rupture may even be too hot to be touched.

The mechanical and thermal aspects are coupled, and inseparable. In fact, this coupling complicates considerably the computational aspect of solving actual thermoelastic problems. Practically speaking, however, it is generally possible to discount the coupling and to evaluate the temperature and deformation fields, in this order, separately. We shall have abundant opportunity to examine these questions later.

In spite of coupling between the temperature and deformation, heating is not always accompanied by stress. In this respect thermoelasticity differs from elasticity, in which there is no deformation without stress.

To illustrate, let us imagine that a body that has complete freedom to expand or contract under the influence of heating is exposed to a uniform temperature increase (Fig. 1.1). Since all elements of the body undergo the same deformation there is no reason why the freedom of an element to deform, such as the element *ABCD* in Fig. 1.1a, should be in any way impeded by the neighboring elements. Due to the absence of internal constraints, the deformation of elements, for example of the element *ABCD* into the element *A′B′C′D′* in Fig. 1.1b, proceeds without generating internal stresses. The body deforms but remains free of stress. In this process the internal coherence of the body remains intact since the deformations are compatible. Indeed, if the strain at each point of the body is the same, the compatibility equations, which involve second spatial derivatives of strain components,[1] become automatically and identically satisfied. It follows that the stress free deformation does not produce such unwelcome singularities as gaps, slips or penetrations of matter into matter.

However, there is little need to emphasize that, in general, the

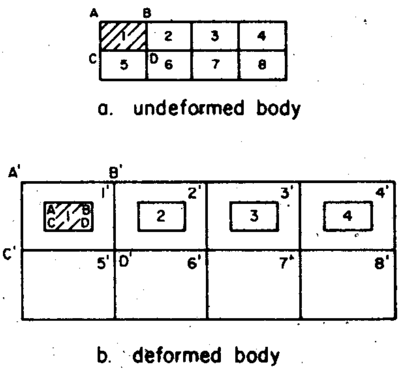

Fig. 1.1. A free thermal deformation.

[1]Compare, e.g., I. S. Sokolnikoff, *Mathematical Theory of Elasticity*, McGraw-Hill, New York, 1956, equations (10.10).

thermal deformation is *not* uniform, and the body cannot deform freely. In such situations the compatibility of deformations may be preserved only through the development of an adequate system of stress within the body.

Two particular cases in which a uniform temperature field produces stresses are worthy of mention:

(a) Existence of external constraints, as occurs, for example, if the ends of a beam, or the edges of a plate are sealed within a wall.

(b) Nonhomogeneity of the body, especially of a discontinuous type. An example of the latter is sketched in Fig. 1.2. Suppose the rectangular cross-section of a column heated evenly consists of two materials I and II of different coefficients of thermal linear expansion. Under the heating, if the elements I and II were disconnected along the interface AB, the latter would tend to occupy two positions $A'B'$ and $A''B''$, say. For the body to remain intact, stress must arise along AB, which induces the segments $A'B'$ and $A''B''$ to remain together.

Another extreme case involves a nonuniform temperature field, which does *not* produce stress. This occurs when the field is stationary, and the temperature is a linear function of position, and is independent of whether the body is simply or multiply connected.[2] We shall return to this question later on.

The preceding example constitutes, of course, an exception to a general rule that temperature changes produce stresses in solid bodies. Because the source of stress is heat, the stress generated by the temperature field is called a *thermal stress*. If the material of the body in which the latter originates is elastic, the stress is said to be *thermoelastic*. Thermal stresses arising in materials which are in-

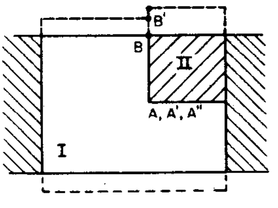

Fig. 1.2. A discontinuous nonhomogeneity.

[2]In simple terms, a body is multiply connected if it contains cavities or holes.

elastic are the subject of such disciplines as thermoplasticity and thermoviscoelasticity.

In the present book we confine ourselves to the study of thermoelastic stresses alone. The stress response of bodies to heating depends on mechanical and thermal properties which, in fact, vary with the temperature. It is, therefore, of interest to examine the elastic characteristics, at least for an isotropic body, recorded in the standard tensile test.

First, it should be noted that the form of the stress-strain diagram changes markedly with temperature. The lower the temperature the steeper and narrower the diagram; this means that with a decreasing temperature materials acquire more and more properties attributed to brittleness. Hence, Young's modulus increases, and so does the yield point; on the other hand, relative elongation diminishes, and tends to zero as the temperature approaches absolute zero. Not surprisingly, heating has an opposite effect, and above certain temperatures materials exhibit so much plasticity or viscosity that their resistance to external forces may no longer be evaluated by the theory of elasticity.

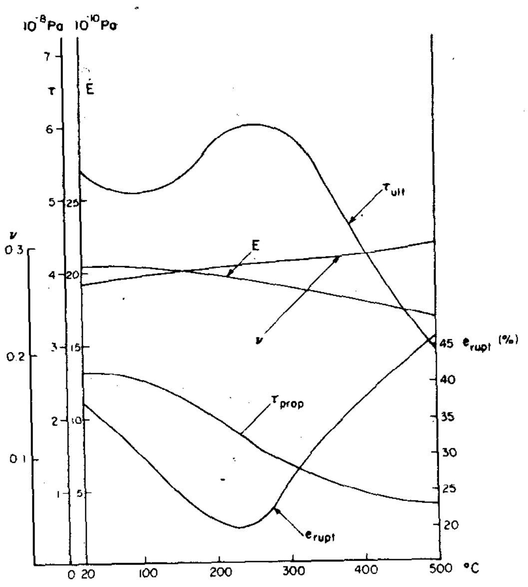

Fig. 1.3. Mechanical properties versus temperature.

Figure 1.3 displays the variation of some characteristic properties of low carbon steel with temperature.[3] It is seen that with an increase in temperature, Young's modulus (E) and stress at the proportional limit (τ_{prop}) decrease, while the relative elongation at rupture (e_{rupt}) initially decreases (up to about 250°C), and then increases rapidly. Actually, with the temperature approaching the melting point, the stress-strain diagram sinks down and flattens, more and more approaching the mode of zero stress and very large strain. Poisson's ratio (ν) for steels increases from about 0.28 at room temperature to almost 0.5 at elevated temperatures, that is to the theoretical limit for isotropic materials remaining elastic.[4]

Examples of materials, which violate the common rule of decreasing Young's modulus with an increasing temperature, include certain types of graphite specially prepared.[5] For these materials, Young's modulus is equal to about 79×10^2 Pa at room temperature and increases by almost 40% at 2000°C. They may also be considered elastic, at least for small strains, up to the temperatures as high as about 2200°C (compared with a sublimation temperature of about 3700°C).

Generally speaking, the variation of Young's modulus or of the modulus of shear (denoted by G or μ) with temperature may be approximated by a parabolic law such as

$$\mu = \mu_0(1 - \beta\theta^2), \tag{1.1}$$

where β is an experimental (positive) coefficient, and θ the temperature increment above the ambient temperature.

It is clear that because of the sensitivity of the mechanical characteristics to the temperature, the properties of materials subject to nonuniform temperature fields seem to vary from point to point: materials behave as though they are inhomogeneous. This fact becomes a source of computational difficulties since the coefficients in thermoelastic governing equations become functions of position. It is recalled in this connection that the solution of partial differential equations with variable coefficients is an intricate chapter of the theory of differential equations.

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[3]Data given in pascals, where 1 kgf/cm^2 = 9.807×10^4 Pa and 1 psi = 6.895×10^3 Pa.

[4]F. L. Everett and J. Miklowitz, Poisson's ratio at high temperatures, *J. Appl. Phys.*, vol. 15, 1944, p. 592.

[5]F. Faris, L. Green, Jr., and G. Smith, The thermal dependence of the elastic moduli of polycrystalline graphite, *J. Appl. Phys.*, vol. 21, 1951, p. 89.