Nikhil R. Pal Michio Sugeno (Eds.)

# Advances in Soft Computing-AFSS 2002

2002 AFSS International Conference on Fuzzy Systems Calcutta, India, February 2002 Proceedings



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## Advances in Soft Computing – AFSS 2002

2002 AFSS International Conference on Fuzzy Systems Calcutta, India, February 3-6, 2002 Proceedings



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#### **Preface**

It is our great pleasure to welcome you all to the 2002 AFSS International Conference on Fuzzy Systems (AFSS 2002) to be held in Calcutta, the great City of Joy. AFSS 2002 is the fifth conference in the series initiated by the Asian Fuzzy Systems Society (AFSS). AFSS 2002 is jointly being organized by the Indian Statistical Institute (ISI) and Jadavpur University (JU). Like previous conferences in this series, we are sure, AFSS 2002 will provide a forum for fruitful interaction and exchange of ideas between the participants from all over the globe. The present conference covers all major facets of soft computing such as fuzzy logic, neural networks, genetic algorithms including both theories and applications. We hope this meeting will be enjoyable academically and otherwise.

We are thankful to the members of the International Program Committee and the Area Chairs for extending their support in various forms to make a strong technical program. Each submitted paper was reviewed by at least three referees, and in some cases the revised versions were again checked by the referees. As a result of this tough screening process we could select only about 50% of the submitted papers. We again express our sincere thanks to all referees for doing a great job. We are happy to note that 19 different countries from all over the globe are represented by the authors, thereby making it a truly international conference. We are proud to have a list of distinguished speakers including Profs. Z. Pawlak, J. Bezdek, D. Dubois, and T. Yamakawa.

We are thankful to the Asian Fuzzy Systems Society and its members, and in particular, to Prof. M. Mukaidono and Prof. Z. Bien, who have extended their cooperation in many forms in spite of their busy schedule. We are grateful to the co-sponsoring societies including IFSA, ISFUMIP (India), SOFT (Japan), CFSAT (Taiwan), FMFSAC (China), the World Federation of Soft Computing, and other international affiliates of AFSS.

We are grateful to Prof. S.B. Rao, former Director, ISI and Prof. S.C. Som, former Vice-Chancellor, JU, for their active help in initiating this conference. Thanks are also due to Prof. A.N. Basu, Vice-Chancellor, JU and Prof. K.B. Sinha, Director, ISI, who have taken special interest on many occasions to help the organizer in many ways and continuously supported us in making this conference a reality. Thanks are due to the Finance Chair, Prof. R. Bandyopadhyay, and the Tutorial Chair, Prof. M.K. Chakraborty for organizing an excellent tutorial program.

We would like to express our sincere thanks to the members of the Organizing Committee for their whole hearted support. Special mention must be made of the organizing Co-chairs, Prof. D. Patranabis and Prof. J. Das, and the organizing coordinators, Dr. R.K. Mudi and Dr. S. Raha and our colleague Dr. Srimanta Pal for their initiative, cooperation, and leading roles in organizing the conference. We would also like to express our special thanks to Prof. Bimal Roy for his help and support. We gratefully acknowledge the help of Prof. S. Sanyal, Prof. K. Ray,

Dr. D.P. Mandal, and Ms. T. Pal. The staff members of the Electronics and Communication Sciences Unit of ISI have done a great job and we express our thanks to them. We are also grateful to the Computer and Statistical Services Center, ISI, for its continuous support. Things will remain incomplete unless we mention two names, Mr. D. Chakraborty and Mr. P. Mohanta without whose help, it would have been impossible for us to make this conference a successful one. We must have missed many other colleagues and friends who have helped us in many ways. We express our thanks to them also.

We gratefully acknowledge the financial support provided by different organizations as listed below. Without their support it would have been impossible to hold this conference on this scale.

Last, but surely not the least, we express our sincere thanks to Mr. Alfred Hofmann of Springer-Verlag for his excellent support in bringing out these proceedings on time.

December 2001

Michio Sugeno Nikhil R. Pal

#### **Funding Organizations**

- All India Council of Technical Education
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- Indian Space Research Organization
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- Indian Statistical Institute
- Jadavpur University

#### Message from the Organizing Co-chairs

With great pleasure we extend a very warm welcome to all delegates and participants at the International Conference on Fuzzy Systems, AFSS 2002, and wish an excellent and fruitful program for everyone. This conference is the first of its kind to be held in Calcutta, the heart of the culture and economy of eastern India. We hope this conference provides an excellent opportunity for the academic fraternity and industry personnel in the related field to interact on the state of the art in fuzzy logic and other soft computing technologies. We are pleased to bring out the proceedings of the AFSS 2002 containing 73 papers including 5 plenary talks and a few invited ones. We are thankful to the Program Chair, Prof. M. Sugeno, for doing an excellent job in selecting quality papers.

Soft Computing has acquired a huge dimension in recent years percolating through to almost all strata of life. Starting from navigational systems to health care, identification problem to control of domestic appliances, process control to load despatch – in fact, all known areas of social and techno-economic growth have been aptly supported by the technique in which fuzzy logic has been the predominant tool / factor. It is, therefore, quite appropriate that such an international symposium on soft computing be held in this part of the continent and be repeated as frequently as possible.

The symposium has received support from the experts in the area from all over the world and participation of them actively and in person has been of great significance considering the worldwide disturbance presently prevailing.

The organizing committee extends its thanks to all those who have been instrumental in making the symposium a success. Our special gratitude is due to the 'chief patrons' Prof. K. B. Sinha, Director, Indian Statistical Institute and Prof. A.N. Basu, Vice-chancellor, Jadavpur University. The committee is also grateful to the sponsors who have supported the conference financially and otherwise.

We hope the proceedings will make good reading for those interested in the relevant field of research and remain a valuable asset.

December 2001

D. Patranabis and J. Das

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### A New Perspective on Reasoning with Fuzzy Rules

D. Dubois<sup>1</sup>, H. Prade<sup>1</sup>, and L. Ughetto<sup>2</sup>

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Abstract. Fuzzy rules are conditional pieces of knowledge which can either express constraints on the set of values which are left possible for a variable, given the values of other variables, or accumulate tuples of feasible values. The first type are implicative rules, while the second are based on conjunctions. Consequences of this view on inference and interpolation between sparse rules are presented.

#### 1 Introduction

Fuzzy rule-based systems have been either used as a convenient tool for synthesizing control laws from data, or in a knowledge representation and reasoning perspective in Artificial Intelligence (AI) [3]. This paper focuses the second use. Indeed, fuzzy rules of the form "if X is A, then Y is B" (more generally, the condition part of a rule can be compound), where A and/or B are fuzzy sets, are often considered as a basic concept in fuzzy logic [20]. They have been used for knowledge representation, where implicative rules play an important place (e.g., [1]), as well as for data modeling, where conjunctive rules are preferred [16].

In AI, logic-based knowledge representation aims at delimiting a set of possible states of the world (the models of the propositions which constitute the knowledge base). Each interpretation satisfying the whole set of propositions in the base (representing the available knowledge) is then considered as possible, since it is not forbidden. Thus, the addition of new pieces of information will just reduce the set of possible states, since the set of models is reduced. The information is said to be complete if only one possible state for the represented world remains. A statement is true (resp. false) for sure if the set of its models contains (resp. rules out) the possible states of the world; in case of incomplete information, one can ignore if a statement is true or false.

With a different prospect, and a different tradition, databases generally use the closed world assumption. Only what is known as true is represented, and then this assumption allows for the deduction of what is regarded as false, by default: a statement is either true (present in the database) or considered as false, since it is not known as true. When new pieces of data are available, they are just added to the database, and the corresponding information, if considered as false until now, is then considered as true. There is no representation of ignorance, only the storage of accepted statements.

This basic distinction between positive and negative information is important, since (expert) knowledge is both made of restrictions or constraints on the possible values of tuples of variables (as in the AI view), often induced by general laws, and examples of values known for sure as being possible, generally obtained in the form of observations, or as reported facts (as in database practice). This distinction plays a central role for the representation as well as the handling of fuzzy information. The next section opposes implicative fuzzy rules which are of the constraint type, and conjunction-based rules which are of the data accumulation type. Section 3 discusses inference for the two types of rules, while section 4 deals with interpolation between sparse rules of the two types.

#### 2 Different Types of Fuzzy Rules with Different Semantics

#### 2.1 Implicative Rules

In possibility theory, the available information is represented by means of possibility distributions which rank-order the possible states of affairs in a given referential set or attribute domain. In fact, the main role of possibility distributions is to discard states of affairs inconsistent with the available knowledge. A piece of information "X is  $A_i$ ", where X is a variable ranging on a domain U, and  $A_i$  is a subset of U (maybe fuzzy), means here "the (ill-known) value for X is for sure in  $A_i$ ". It is represented by the constraint:

$$\forall u \in U, \ \pi_X(u) \le \mu_{A_i}(u) \ , \tag{1}$$

where  $\pi_X$  is a possibility distribution restricting the values of X [19]. Several such pieces of information are naturally aggregated conjunctively into:

$$\forall u \in U, \ \pi_X(u) \le \min_i \ \mu_{A_i}(u) \ . \tag{2}$$

Then, once all the constraints are taken into account, a minimal specificity principle is applied, which allocates to each value (or state of the world) the greatest possibility degree in agreement with the constraints. It leads to enforce the equality in (2).

Considering a knowledge base  $\mathcal{K} = \{A_i \to B_i, i = 1, \dots, n\}$ , made of n parallel fuzzy rules (i.e., rules with the same input space U and the same output space V), each rule "if X is  $A_i$ , then Y is  $B_i$ " (denoted  $A_i \to B_i$ ) is represented by a conditional possibility distribution  $\pi^i_{Y|X} = \mu_{A_i \to B_i}$  (the membership function of  $A_i \to B_i$ ), which is determined according to the semantics of the rule. X is the tuple of input variables (on which information can be obtained) and Y the tuple of output variables (about which we try to deduce information). According to (2), the possibility distribution  $\pi^{\mathcal{K}}$  representing the base  $\mathcal{K}$  is obtained as the (min-based) conjunction of the  $\pi^i_{Y|X}$ 's:

$$\pi^{\mathcal{K}} = \min_{i=1,\dots,n} \ \pi^i_{Y|X} \ . \tag{3}$$