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DSP FIRST:

A MULTIMEDIA APPROACH

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To Carolyn, who transformed my life. JHMc

To Dorothy, who will always be First. RWS

To Sarah, for her helping me to know better every day the World's Greatest Engineer, who is truly time-invariant and not at all linear.

MAY

Preface

Signal processing is concerned with processing signals. That obvious statement is often overlooked in digital signal processing (DSP) textbooks, where the concentration is on the mathematical techniques and methods needed to develop DSP algorithms. Such emphasis is probably correct in graduate courses, but this text and the accompanying CD-ROM were developed to introduce undergraduate students to DSP and real signals. The CD-ROM contains many actual signals so that students can implement and see the results of actual processing. In our effort to define an introductory course based on DSP, we have concentrated on examples such as sound and image signals.

We have used the term *multimedia* to describe this approach, because sound and images are integral to multimedia. More important, however, is the fact that sound and images are manipulated using the methods of DSP. So if one agrees that these signals are known as multimedia, then DSP is concerned with the infrastructure of multimedia because it provides the tools for developing and implementing that infrastructure. Perhaps the foremost example is signal compression. The movement of large audio and image files over networks demands data compression. The "signal" nature of audio and images can be exploited through filtering and transforms to obtain very compact representations such as the JPEG standard. Although these signal compression algorithms are often based on sophisticated mathematical descriptions and demand considerable expertise to design, we believe that it is possible in a first course to teach the fundamentals of DSP that underlie multimedia systems. Furthermore, we believe that working with the actual signals provides a high level of motivation to learn more about the theory and potential applications of DSP.

Many DSP researchers and engineers have designed lowpass filters, but how many have "seen" or "heard" the effects of a lowpass filter? By that we mean, how many have processed sound through a lowpass filter and listened to the result? Nowadays, it is not a difficult task to implement a lowpass filter and process audic, but it is rarely done in college courses. Why? One likely reason is that the facilities and materials to do so have not been readily available until now. The *DSP First CD-ROM* addresses the availability issue because it contains many sound files, both raw and processed, that can be viewed with a Web browser or imported into MATLAB. When used in conjunction with MATLAB, these sounds can be processed in many different ways. New filters can be designed and then applied, so that the concept

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of frequency response takes on a new meaning when the human auditory sense is involved. Likewise for visual stimulation where the effects of lowpass and highpass filtering of photographs can be seen. Questions such as how highpass filtered speech sounds or what a lowpass filter does to an image can be answered.

We provide a CD-ROM bundled with this textbook, and hope that the readers and their teachers value the CD-ROM material as more important than the written text. If so, then a first step in a new direction will have been taken to enhance learning. The CD-ROM contains four types of materials: demos, labs, exercises, and homework problems with solutions. Of these, the first two are the key items. Each chapter of the text has associated demos. These were developed for classroom use, but have now been preserved so they can be "browsed" outside of class. In this form, the demos have two uses: First, they define some key presentation that should be made during lecture time; second, they provide a resource for self-study. Over time, the database of good demos will grow, and we hope that the present CD-ROM is just a beginning for this sort of resource. At Georgia Tech, we will host a web site containing the material in its evolving form.¹

We have also written a text. It is a conventional book not unlike any other on the subject of DSP, although, as our title DSP First suggests, the distinguishing feature of the text (and the CD-ROM) is that it presents DSP at a level consistent with an introductory ECE course, i.e., the sophomore level in a typical U.S. university. The list of topics in the book is standard, but since we must combine DSP with some introductory ideas, the progression of topics may strike some teachers as unconventional. Part of the reason for this is that DSP has typically been treated as a senior or first-year graduate level course in electrical engineering, for which a traditional background of circuits and linear systems is assumed. We believe strongly that there are compelling reasons for turning this order around, since the early study of DSP affords a perfect opportunity to show electrical and computer engineering students that mathematics and digital computation can be the key to understanding familiar engineering applications. Furthermore, this approach makes the subject much more accessible to students in other majors such as computer science and other engineering fields. This point is increasingly important because non-specialists are beginning to use DSP techniques routinely in many areas of science and technology.

When you think of the basic ideas needed to understand signals and systems, and also DSP, a few key ones must be included: Frequency response is one; sampling is another, filtering is a third. Beyond that, most other topics in our text are needed because they support those key concepts. For example, to explain frequency response, one must know about sinusoids because the frequency response is really concerned with the sinusoidal response. Then it is convenient to represent sinusoids in a complex number representation, so we treat phasors. In a conventional (circuits-based) curriculum, these basic notions are covered under the umbrella of AC circuits, so in effect we have developed an alternative to that path. This idea is not ours, but

¹ http://www.ece.gatech.edu/~dspfirst

one that we have adapted from Prof. Ken Steiglitz (Princeton University), whose books on introductory DSP have blazed this trail. Indeed much of our work has been inspired by two of Steiglitz's books.^{2, 3}

This project owes a considerable debt to the many students who have spent countless hours developing MATLAB code and HTML code, as well as running the laboratories at Georgia Tech and coaching the sophomores who have been exposed to this new format for signal processing education. In many cases, they have developed complex demonstrations and have found great challenge and reward in outperforming their professors. In other cases, they have brought skills and insights to the project that would have eluded us. We would like to recognize each of the following major contributors: Dr. Jeff Schodorf, who served as a TA for the EE-2200 course at Georgia Tech for over two years and who did most of the reconstruction demos in Chapter 4, as well as overseeing the archiving of much of the lab material for the CD-ROM. David Anderson, who served as the primary TA for 2200 and has developed new labs and refined old ones for the CD-ROM, as well as providing much needed quality testing of the final CD-ROM. Craig Ulmer, who developed PeZ, a full-featured pole-zero editor, to demonstrate the interrelationships among the time, frequency, and transform domains; and Brad North, who made improvements in PeZ. Emily Loadholt, who provided the music recordings for the labs and the music spectrogram viewer. Amer Abufadel, who created the FIR image-filtering demos, and Joseph Stanley, who provided a real boost by creating the artwork that transformed the CD-ROM from its bland beginnings. Scott McClellan, who scanned most of the homework problems and solutions late at night. Robbie Griffin, who was inspired by the music synthesis labs, and immediately after taking the course became a developer of new labs for these topics.

We want to acknowledge the contributions of our editor, Tom Robbins at Prentice-Hall. Very early on, he bought into the concept of *DSP First* and supported and encouraged us at every step in the project. He also arranged for reviews of the book and the CD-ROM by some very thoughtful and careful reviewers including Filson Glanz, S. Hossein Mousavinezhad, Geoffrey Orsak, Rogelio Palomera-Garcia, Stan Reeves, and Mitch Wilkes. An extraordinary thorough review by Robert Strum had a significant impact on the final result.

We also want to thank our colleagues and key members of our respective Institutes for their support and for providing resources such as: MAY's sabbatical from Rose-Hulman, which led to this team of authors; the John and Mary Franklin Foundation, which has provided continuous support of RWS; and support from the Georgia Tech Foundation and EduTech that financed additional course development by MAY and the graduate students. JHMc and RWS would like to acknowledge the freedom and opportunities afforded them by other faculty in the Georgia Tech School of ECE to initiate this new course as a part of on-going curriculum revisions. Likewise, MAY

² K. Steiglitz, An Introduction to Discrete Systems, John Wiley & Sons, 1972.

³ K. Steiglitz, A Digital Signal Processing Primer: with Applications to Computer Music, Addison-Wesley Publishing Company, 1996.

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would like to thank his colleagues at Rose-Hulman for their support and encouragement while developing this new approach and for designing a curriculum that uses this new foundation.

Finally, we want to recognize the understanding and support of our wives (Carolyn McClellan, Dorothy Schafer, and Sarah Yoder) who watched in amazement as this project consumed most of our energy and time during the final few fanatical fortnights. Also, MAY would like to thank his eight children for understanding "Why Daddy has to go to work when it is Saturday." In closing, we make the observation that this project is not really finished. At the outset, we decided to create material for this course on a rapid time schedule and to deliver that material to others as soon as possible. As a result, many ideas are left unfulfilled. But that is the appeal of the Web-based approach. It can easily grow to incorporate the innovative visualizations and experiments that others will provide.

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