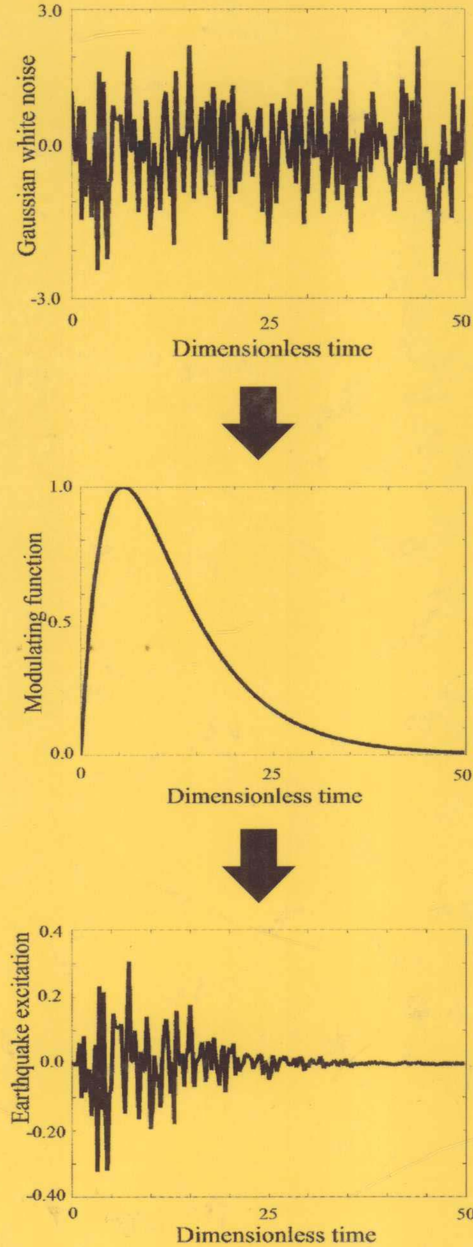
Nonlinear Random Vibration

Analytical Techniques and Applications

Second Edition



Cho W.S. To

CRC Press
Taylor & Francis Gr
A BALKEMA BOOK

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CRC Press
Taylor & Francis Group
Boca Raton London New York Leiden

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A BALKEMA BOOK

CRC Press/Balkema is an imprint of the Taylor & Francis Group, an informa business

Printed and bound in Great Britain by Antony Rowe (a CPI Group Company), Chippenham, Wiltshire

Published by: CRC Press/Balkema
P.O. Box 447, 2300 AK Leiden, The Netherlands
e-mail: Pub.NL@taylorandfrancis.com
www.crcpress.com – www.taylorandfrancis.co.uk – www.balkema.nl

Library of Congress Cataloging-in-Publication Data
Applied for

ISBN: 978-0-415-89897-3 (Hbk)
ISBN: 978-0-203-14462-6 (eBook)

To My Parents

Preface to the first edition

The framework of this book was first conceptualized in the late nineteen eighties. However, the writing of this book began while the author was on sabbatical, July 1991 through June 1992, at the University of California, Berkeley, from the University of Western Ontario, London, Ontario, Canada. Over half of the book was completed before the author returned to Canada after his sabbatical. With full-time teaching, research, the arrival of a younger daughter, and the moving in 1996 from Canada to the University of Nebraska, Lincoln the author has only completed the project of writing this book very recently. Owing to the long span of time for the writing, there is no doubt that many relevant publications may have been omitted by the author.

The latter has to admit that a book of this nature is influenced, without exception, by many authors and examples in the field of random vibration. The original purpose of writing this book was to provide an advanced graduate level textbook dealing, in a more systematical way, with analytical techniques of nonlinear random vibration. It was also aimed at providing a textbook for a second course in the analytical techniques of random vibration for graduate students and researchers.

In the introduction chapter reviews in the general areas of nonlinear random vibration appeared in the literature are quoted. Books exclusively dealing with and related to are listed in this chapter. Chapter 2 begins with a brief introduction to Markovian and non-Markovian solutions of stochastic nonlinear differential equations. Chapter 3 is concerned with the exact solution of the Fokker-Planck-Kolmogorov (FPK) equation. Chapter 4 presents the methods of statistical linearization (SL). Uniqueness and accuracy of solutions by the SL techniques are summarized. An introduction to and discussion on the statistical nonlinearization (SNL) techniques are provided in Chapter 5. Accuracy of the SNL techniques is addressed. The methods of stochastic averaging are introduced in Chapter 6. Various stochastic averaging techniques are presented in details and their accuracies are discussed. Chapter 7 provides briefly the truncated hierarchy, perturbation, and functional series techniques.

C.W.S. To
Lincoln, Nebraska 2000

Preface to the second edition

Various theoretical developments in the field of nonlinear random vibration have been made since the publication of the first edition. Consequently, the latter has been expanded somewhat in the present edition in which a number of errors and misprints has been corrected.

The organization of the present edition remains essentially the same as that of the first edition. Chapter 1 is an updated introduction to the reviews in the general areas of nonlinear random vibration. Books exclusively dealing with and related to analytical techniques and applications are cited. Chapter 2 is concerned with a brief introduction to Markovian and non-Markovian solutions to stochastic nonlinear differential equations. Exact solutions to the Fokker-Planck-Kolmogorov (FPK) equations are included in Chapter 3. Methods of statistical linearization (SL) with uniqueness and accuracy of solutions are presented in Chapter 4. Some captions and labels of figures in this chapter have been changed to commonly used terminology. Chapter 5 deals with the statistical nonlinearization (SNL) techniques. Section 5.5 is a new addition introducing an improved SNL technique for approximating multi-degree-of-freedom nonlinear systems. Methods of stochastic averaging are presented in Chapter 6. In the present edition, more detailed steps are added and some reorganization of steps are made. Chapter 7 includes truncated hierarchy, perturbation, and functional series techniques. In the present edition, more steps have been incorporated in the Volterra series expansion techniques. An appendix presenting a brief introduction to the basic concepts and theory of probability, random variables, and random processes has been added to the present edition. This new and brief addition is aimed at those readers who need a rapid review of the prerequisite materials.

C.W.S. To

Lincoln, Nebraska 2011

Acknowledgements

ACKNOWLEDGEMENTS FOR THE FIRST EDITION

The author began his studies in random vibration during his final year of undergraduate program, between 1972 and 1973, at the University of Southampton, United Kingdom. The six lectures given by Professor B.L. Clarkson served as a stimulating beginning. After two years of master degree studies at the University of Calgary, Canada, in October 1975 the author returned to the University of Southampton to work as a research fellow in the Institute of Sound and Vibration Research for his doctoral degree under the supervision of Professor Clarkson. The fellowship was sponsored by the Admiralty Surface Weapons Establishment, Ministry of Defence, United Kingdom. During this period of studies, the author was fortunate enough to have attended lectures on random vibration presented by Professor Y.K. Lin who was visiting Professor Clarkson and the Institute in 1976. The year 1976 saw the gathering of many experts and teachers in the field of random vibration at the *International Union of Theoretical and Applied Mechanics, Symposium on Stochastic Problems in Dynamics*. The author was, thus, influenced and inspired by these experts and teachers.

The conducive atmosphere and the availability of many publications in the libraries at the University of California, Berkeley and the hospitality of emeritus Professors J.L. Sackman, J.M. Kelly, Leo Kanowitz and other friends at Berkeley had made the writing enjoyable, and life of the author and his loved ones memorable.

Case (ii) in page 131 and the excitation processes in almost all the examples in Chapter 6 have been re-written and changed as a result of comments from one of the reviewers. Section 7.4 has been expanded in response to the suggestion of another reviewer. The author is grateful to them for their interest in reviewing this book.

Thanks are due to the author's two present graduate students, Ms. Guang Chen and Mr. Wei Liu who prepared all the drawings in this book.

Finally, the author would like to express his gratitude to his friend, Professor Fai Ma for his encouragement, and wishes to thank the Publisher, Mr. Martin Scrivener and his staff for their publishing support.

ACKNOWLEDGEMENTS FOR THE SECOND EDITION

Since the publication of the first edition in 2000 various theoretical developments in the field of nonlinear random vibration have been made. It is therefore appropriate to publish the present edition at this time. The author has taken the opportunity to make a number of corrections.

The appendix on Probability, Random Variables and Random Processes is the result of the suggestion of a reviewer of the proposal for the present edition. The reviewer's suggestion and comments are highly appreciated.

Finally, the author wishes to thank Mr. Janjaap Blom, Senior Publisher, Ms. Madeline Alper, Customer Service Supervisor, and their staff for their publishing support.

Table of contents

1 Introduction

For safety, reliability and economic reasons, the nonlinearities of many dynamic engineering systems under environmental and other forces that are treated as random disturbances must be taken into account in the design procedures. This and the demand for precision have motivated the research and development in nonlinear random vibration. Loosely speaking, the field of nonlinear random vibration can be subdivided into four categories. The latter include analytical techniques, computational methods, Monte Carlo simulation (MCS), and system identification with experimental techniques. This book is mainly concerned with the first category and therefore the publications quoted henceforth focus on this category. The subject of computational nonlinear random vibration is dealt with in a companion book that is published recently [1.1].

It is believed that the first comprehensive review on nonlinear random vibration was performed by Caughey [1.2]. Subsequently, other reviews appeared in the literature [1.3-1.15], for example. There are books exclusively concerned with and related to nonlinear random vibration [1.16-1.24]. Many books [1.25-1.39] also contain chapter(s) on nonlinear random vibration.

While it is agreed that there are many techniques available in the literature for the analysis of nonlinear systems under random excitations, the focus of the present book is, however, on those frequently employed by engineers and applied scientists. It also reflects the current interests in the analytical techniques of nonlinear random vibration.

Chapter 2 begins with a brief introduction to Markovian and non-Markovian solutions of stochastic nonlinear differential equations. This serves as a foundation to subsequent chapters in this book.

Chapter 3 presents the exact solutions of the Fokker-Planck-Kolmogorov (FPK) equations. Solution of a general single degree-of-freedom (dof) system and applications to engineering systems are included. Solution of a multi-degrees-of-freedom (mdof) system and stochastically excited Hamiltonian systems are also considered.

Chapter 4 deals with the methods of statistical linearization (SL). Solutions to single dof and mdof nonlinear systems with examples of engineering applications are given. Uniqueness and accuracy of solutions by the SL techniques are summarized.

Chapter 5 provides an introduction to and discussion on the statistical nonlinearization (SNL) techniques. Single dof and mdof nonlinear systems are considered. Accuracy of the SNL techniques is addressed.

Chapter 6 treats the methods of stochastic averaging. The classical stochastic averaging (CSA) method, stochastic averaging method of energy envelope (SAMEE), and various other stochastic averaging techniques are introduced and examples given. Accuracy of the stochastic averaging techniques is discussed.

Chapter 7 introduces briefly several other techniques. The lattter include truncated hierarchy, perturbation, and functional series techniques. The truncated hierarchy techniques include Gaussian closure schemes and non-Gaussian closure schemes, while the functional series techniques encompass the Volterra series expansion techniques, and Wiener-Hermite series expansion techniques.

It is assumed that the readers have a first course in random vibration or similar subject. Materials in Chapters 2 and 3 are essential and serve as a foundation to a better understanding of the techniques and applications in subsequent chapters.

An outline of the basic concepts and theory of probability, random variables and random processes is included in the appendix for those who need a rapid review of the essential background materials.

2 Markovian and Non-Markovian Solutions of Stochastic Nonlinear Differential Equations

2.1 Introduction

Within the field of nonlinear random vibration of structural and mechanical systems the statistical complexity of a stochastic process (s.p.) is determined by the properties of its distribution functions. Two types of classifications are important in the analysis. These are classification based on the statistical regularity of a process and classification based on its memory.

In this section the above two types of classifications are introduced in Sub-sections 2.1.1 and 2.1.2. Then in Sub-section 2.1.3 the kinetic equation associated with the s.p. is derived. This provides the basis for distribution and density functions that are important to subsequent analysis. Section 2.2 contains the basic material for Markovian solution of stochastic nonlinear differential equations. Essential features and relevant information for non-Markovian solution of stochastic nonlinear differential equations are included in Section 2.3.

2.1.1 Classification based on regularity

In this type of classification, s.p. are divided into two categories. They are the stationary stochastic processes (s.s.p.) and nonstationary stochastic processes (n.s.p.). Assuming time t is the parameter of the strict sense or strong s.s.p. $X(t)$, its statistical properties are all independent of time t or are all independent of the absolute time origin. On the other hand, for a n.s.p. all statistical properties of that process are dependent of time t.

When the absolute value of the expectation of the s.s.p. $X(t)$ is a constant and less than infinity, the expectation of the square of $X(t)$ is less than infinity, and the covariance of $X(t)$ is equal to the correlation function of $X(t)$, the s.p. is called a wide-sense or weak s.s.p. Of course, when such a s.s.p. is Gaussian it is completely specified by its means and covariance functions.

2.1.2 Classification based on memory

If s.p. are grouped in accordance with the manner in which the present state of a s.p. depends on its past history, then such a classification is called classification based on memory. This classification is centered around the Markov processes.

In accordance with the memory properties, the simplest s.p. is one without memory or is purely stochastic. This is usually called a zeroth order Markov process. Clearly, a continuous-parameter purely s.p. is physically not realizable since it implies absolute independence between the past and the present regardless of their temporal closeness. The white noise process is a purely s.p. The Markov process to be defined in Sub-section 2.2.1 is usually called a simple Markov process. There are higher order Markov processes that are not applied in this book and therefore are not defined here.

It may be appropriate to note that the memory of a s.p. is not to be confused with the memory of a nonlinear transformation. The latter is said to have memory if it involves with inertia.

2.1.3 Kinetic equation of stochastic processes

A technique that can give explicit results of joint distributions of the solution process is introduced in this sub-section. The foundation of the following derivation was presented by Bartlett [2.1] and Pawula [2.2], and subsequently by Soong [2.3].

A s.p. $X(t)$ with its first probability density function being denoted by $p(x,t)$ satisfies the relation that

$$p(x,t+\Delta t) = \int_{-\infty}^{\infty} p(x,t+\Delta t \mid y,t)\, p(y,t)\, dy\ , \tag{2.1}$$

where $p(x,t+\Delta t \mid y,t)$ is the conditional probability density function of $X(t+\Delta t)$ given that $X(t) = y$.

Let $\psi(u,t+\Delta t \mid y,t)$ be the conditional characteristic function of $\Delta X = X(t+\Delta t) - X(t)$ given that $X(t) = y$,

$$\begin{aligned}\psi(u,t+\Delta t \mid y,t) &= \left\langle e^{iu\Delta X} \mid y,t \right\rangle \\ &= \int_{-\infty}^{\infty} e^{iu\Delta x}\, p(x,t+\Delta t \mid y,t)\, dx\ , \quad \Delta x = x - y\ ,\end{aligned} \tag{2.2}$$

where the angular brackets denote the mathematical expectation.

By taking the inverse Fourier transformation, one has

$$p(x,t+\Delta t \mid y,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iu\Delta x}\,\psi(u,t+\Delta t \mid y,t)\,du \,. \qquad (2.3)$$

By expanding the conditional characteristic function $\psi(u,t+\Delta t \mid y,t)$ in a Taylor series about $u = 0$, Eq. (2.3) becomes

$$\begin{aligned} p(x,t+\Delta t \mid y,t) &= \sum_{k=0}^{\infty} \frac{c_k(y,t)}{2\pi k!}\int_{-\infty}^{\infty} (iu)^k e^{-iu\Delta x}\,du \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k c_k(y,t)}{k!}\,\frac{\partial^k[\delta(\Delta x)]}{\partial x^k}\,, \end{aligned} \qquad (2.4)$$

where

$$c_k(y,t) = \left\langle (\Delta X)^k \mid y,t \right\rangle = \left\langle [X(t+\Delta t) - X(t)]^k \mid X(t) = y \right\rangle .$$

These expectations are known as the incremental moments of $X(t)$.

Substituting Eq. (2.4) into (2.1) and after integration, one obtains

$$p(x,t+\Delta t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}\,\frac{\partial^k[c_k(x,t)\,p(x,t)]}{\partial x^k} \,.$$

This equation can be expressed as

$$p(x,t+\Delta t) - p(x,t) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k!}\,\frac{\partial^k[c_k(x,t)\,p(x,t)]}{\partial x^k} \,.$$

Upon dividing this equation by Δt and in the limit as $\Delta t \to 0$, it leads to

$$\frac{\partial p(x,t)}{\partial t} = \sum_{k=1}^{\infty} \frac{(-1)^k}{k!}\,\frac{\partial^k[\alpha_k(x,t)\,p(x,t)]}{\partial x^k}\,, \qquad (2.5)$$

where

$$\alpha_k(x,t) = \lim_{\Delta t \to 0}\left(\frac{1}{\Delta t}\right)\left\langle [X(t+\Delta t) - X(t)]^k \mid X(t) = x \right\rangle .$$

Equation (2.5) is known as the *kinetic equation* of the s.p. $X(t)$ and $\alpha_k(x,t)$ are the *derivate moments*. It is a deterministic parabolic partial differential equation and has important use in the solution of stochastic differential equations.

2.2 Markovian Solution of Stochastic Nonlinear Differential Equations

There are many physical quantities, such as the response of a nonlinear system under a random excitation that can be represented by a white noise process, can be described as Markov processes. Rigorous fundamental treatment on the subject was presented by Kolmogorov [2.4]. The solution by the analytical techniques considered in this monograph is generally based on the concepts of Markov processes. Thus, it is essential to introduce these concepts. To this end, Markov and diffusion processes are defined in Sub-section 2.2.1 while the Stratonovich and Itô's integrals are presented in Sub-section 2.2.2. Sub-section 2.2.3 is concerned with the one-dimensional Fokker-Planck forward or Fokker-Planck-Kolmogorov (FPK) equation. To further clarify the use of Stratonovich and Itô's integrals a single-degree-of-freedom (sdof) quasi-linear system is included in Sub-section 2.2.4.

2.2.1 Markov and diffusion processes

A stochastic process $X(t)$ on an interval $[0,T]$ is called a Markov process if it has the following property:

$$P\left[X(t_n) < x_n \mid X(t_{n-1}) = x_{n-1}, \ldots, X(t_0) = x_0\right] = \\ P\left[X(t_n) < x_n \mid X(t_{n-1}) = x_{n-1}\right], \qquad T \geq t_n > \ldots > t_1 > t_0 \geq 0 \ , \tag{2.6}$$

where $P[.]$ designates the probability of an event and the conditional probability of the Markov process $X(t)$, $P[X(t) < x \mid X(t_0) = x_0]$ is known as the transition probability distribution function. Equation (2.6) means that the process *forgets* the past if t_{n-1} is being regarded as the present.

Applying the Markov property (2.6), one can show that

$$p(x_3, t_3 \mid x_1, t_1) = \int_{-\infty}^{\infty} p(x_3, t_3 \mid x_2, t_2) p(x_2, t_2 \mid x_1, t_1) dx_2 \ , \tag{2.7}$$

where $p(x_i, t_i \mid x_{i-1}, t_{i-1})$, $i = 2,3$, are the transition probability densities. Equation (2.7) describes the flow or transition probability densities from instant t_1 to another instant t_3. It is known as the *Smoluchowski-Chapman-Kolmogorov* (SCK) equation.

A Markov process $X(t)$ is called a diffusion process if its transition probability density satisfies the following two conditions for $\Delta t = t - s$ and $\varepsilon > 0$,

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|>\varepsilon} p(y,t \mid x,s)\, dy = 0 , \tag{2.8a}$$

and

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|\le\varepsilon} (y-x)\, p(y,t \mid x,s)\, dy = f(x,s) , \tag{2.8b}$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|y-x|\le\varepsilon} (y-x)^2\, p(y,t \mid x,s)\, dy = G^2(x,s) , \tag{2.8c}$$

where the drift and diffusion coefficients, $f(x,s)$ and $G(x,s)$, respectively, are independent of the time t when $X(t)$ is stationary because in this case $p(y,t \mid x,s) = p(y,t-s \mid x)$ only depends on the time lag Δt.

2.2.2 Itô's and Stratonovich integrals

$$\phi_{[a,b]}(t) = \begin{cases} 1 & \textit{if} \quad a \le t \le b , \\ 0 & \textit{otherwise} . \end{cases} \tag{2.9}$$

Consider a characteristic function on the interval $[a,b]$ for $0 \le a < b \le T$,

For $0 \le a < b \le T$, one defines

$$\int_0^T \phi_{[a,b]}(t)\, dB(t) \equiv B(b) - B(a) , \tag{2.10}$$

where $B(t)$ is the Brownian motion process which is a *martingale* because

$$\langle\, | B(t) |\, \rangle < \infty , \tag{2.11a}$$

and for all $t_1 < t_2 < \ldots < t_n$ and $a_1, \ldots, a_n$,

$$\langle x(t) \mid x(t_1) = a_1 , \ldots , x(t_n) = a_n \rangle = a_n . \tag{2.11b}$$

If $f(t)$ is a step function on $[0,T]$ and $0 = t_0 < t_1 < \ldots < t_m = b$, then