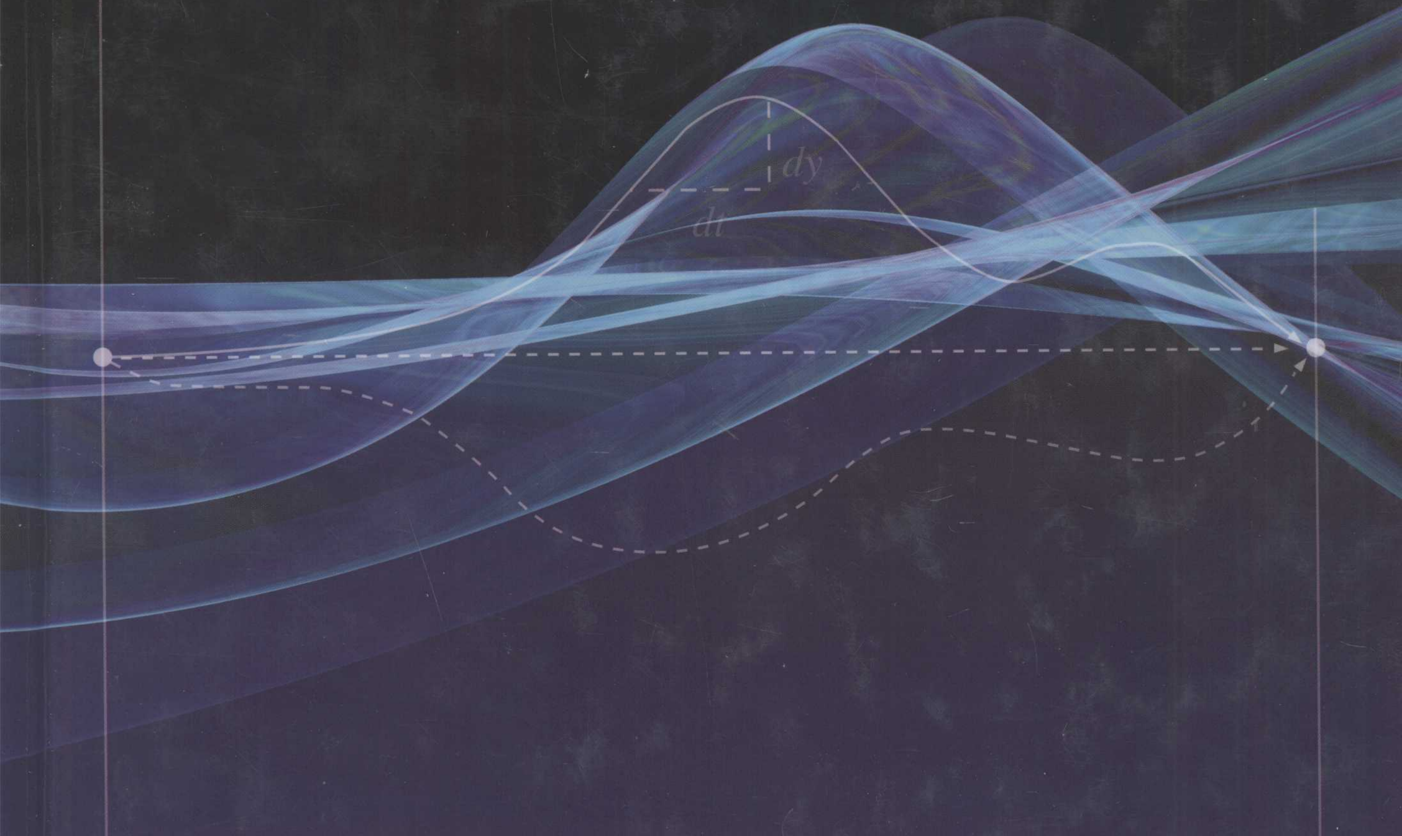
FUNDAMENTAL METHODS OF

Mathematical Economics

FOURTH EDITION



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Fundamental Methods of Mathematical Economics

Fourth Edition

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About the cover: The graph in Figure 20.1 on page 635 illustrates that the shortest distance between two points is a straight line. We chose it as the basis for the cover design because such a simple truth requires one of the most advanced techniques found in this book.

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Fundamental Methods of Mathematical Economics

To Emily, Darryl, and Tracey

—Alpha C. Chiang

To Skippy and Myrtle

—Kevin Wainwright

About the Authors

Alpha C. Chiang received his Ph.D. from Columbia University in 1954, after earning a B.A. in 1946 from St. John's University (Shanghai, China) and an M.A. in 1948 from the University of Colorado. In 1954 he joined the faculty of Denison University in Ohio, where he assumed the chairmanship of the Department of Economics in 1961. From 1964 on, he taught at the University of Connecticut where, after 28 years, he became Professor Emeritus of Economics in 1992. He also held visiting professorships at New Asia College of the Chinese University of Hong Kong, Cornell University, Lingnan University in Hong Kong, and Helsinki School of Economics and Business Administration. His publications include another book on mathematical economics: *Elements of Dynamic Optimization,* Waveland Press, Inc., 1992. Among the honors he received are awards from the Ford Foundation and National Science Foundation fellowships, election to the presidency of the Ohio Association of Economists and Political Scientists, 1963–1964, and listing in *Who's Who in Economics: A Biographical Dictionary of Major Economists 1900–1994,* MIT Press.

Kevin Wainwright is a faculty member of the British Columbia Institute of Technology in Burnaby, B.C., Canada. Since 2001, he has served as president of the faculty association and program head in the Business Administration program. He did his graduate studies at Simon Fraser University in Burnaby, B.C., Canada, and continues to teach in the Department of Economics there. He specializes in microeconomic theory and mathematical economics.

Preface

This book is written for those students of economics intent on learning the basic mathematical methods that have become indispensable for a proper understanding of the current economic literature. Unfortunately, studying mathematics is, for many, something akin to taking bitter-tasting medicine—absolutely necessary, but extremely unpleasant. Such an attitude, referred to as "math anxiety," has its roots—we believe—largely in the inauspicious manner in which mathematics is often presented to students. In the belief that conciseness means elegance, explanations offered are frequently too brief for clarity, thus puzzling students and giving them an undeserved sense of intellectual inadequacy. An overly formal style of presentation, when not accompanied by any intuitive illustrations or demonstrations of "relevance," can impair motivation. An uneven progression in the level of material can make certain mathematical topics appear more difficult than they actually are. Finally, exercise problems that are excessively sophisticated may tend to shatter students' confidence, rather than stimulate thinking as intended.

With that in mind, we have made a serious effort to minimize anxiety-causing features. To the extent possible, patient rather than cryptic explanations are offered. The style is deliberately informal and "reader-friendly." As a matter of routine, we try to anticipate and answer questions that are likely to arise in the students' minds as they read. To underscore the relevance of mathematics to economics, we let the analytical needs of economists motivate the study of the related mathematical techniques and then illustrate the latter with appropriate economic models immediately afterward. Also, the mathematical tool kit is built up on a carefully graduated schedule, with the elementary tools serving as stepping stones to the more advanced tools discussed later. Wherever appropriate, graphic illustrations give visual reinforcement to the algebraic results. And we have designed the exercise problems as drills to help solidify grasp and bolster confidence, rather than exact challenges that might unwittingly frustrate and intimidate the novice.

In this book, the following major types of economic analysis are covered: statics (equilibrium analysis), comparative statics, optimization problems (as a special type of statics), dynamics, and dynamic optimization. To tackle these, the following mathematical methods are introduced in due course: matrix algebra, differential and integral calculus, differential equations, difference equations, and optimal control theory. Because of the substantial number of illustrative economic models—both macro and micro—appearing here, this book should be useful also to those who are already mathematically trained but still in need of a guide to usher them from the realm of mathematics to the land of economics. For the same reason, the book should not only serve as a text for a course on mathematical methods, but also as supplementary reading in such courses as microeconomic theory, macroeconomic theory, and economic growth and development.

We have attempted to retain the principal objectives and style of the previous editions. However, the present edition contains several significant changes. The material on mathematical programming is now presented earlier in a new Chap. 13 entitled "Further Topics in Optimization." This chapter has two major themes: optimization with inequality constraints and the envelope theorem. Under the first theme, the Kuhn-Tucker conditions are developed in much the same manner as in the previous edition. However, the topic has been enhanced with several new economic applications, including peak-load pricing and consumer rationing. The second theme is related to the development of the envelope theorem, the maximum-value function, and the notion of duality. By applying the envelope theorem to various economic models, we derive important results such as Roy's identity, Shephard's lemma, and Hotelling's lemma.

The second major addition to this edition is a new Chap. 20 on optimal control theory. The purpose of this chapter is to introduce the reader to the basics of optimal control and demonstrate how it may be applied in economics, including examples from natural resource economics and optimal growth theory. The material in this chapter is drawn in great part from the discussion of optimal control theory in *Elements of Dynamic Optimization* by Alpha C. Chiang (McGraw-Hill 1992, now published by Waveland Press, Inc.), which presents a thorough treatment of both optimal control and its precursor, calculus of variations.

Aside from the two new chapters, there are several significant additions and refinements to this edition. In Chap. 3 we have expanded the discussion of solving higher-order polynomial equations by factoring (Sec. 3.3). In Chap. 4, a new section on Markov chains has been added (Sec. 4.7). And, in Chap. 5, we have introduced the checking of the rank of a matrix via an echelon matrix (Sec. 5.1), and the Hawkins-Simon condition in connection with the Leontief input-output model (Sec. 5.7). With respect to economic applications, many new examples have been added and some of the existing applications have been enhanced. A linear version of the IS-LM model has been included in Sec. 5.6, and a more general form of the model in Sec. 8.6 has been expanded to encompass both a closed and open economy, thereby demonstrating a much richer application of comparative statics to general-function models. Other additions include a discussion of expected utility and risk preferences (Sec. 9.3), a profit-maximization model that incorporates the Cobb-Douglas production function (Sec. 11.6), and a two-period intertemporal choice problem (Sec. 12.3). Finally, the exercise problems have been revised and augmented, giving students a greater opportunity to hone their skills.

Acknowledgments

We are indebted to many people in the writing of this book. First of all, we owe a great deal to all the mathematicians and economists whose original ideas underlie this volume. Second, there are many students whose efforts and questions over the years have helped shape the philosophy and approach of this book.

The previous three editions of this book have benefited from the comments and suggestions of (in alphabetical order): Nancy S. Barrett, Thomas Birnberg, E. J. R. Booth, Charles E. Butler, Roberta Grower Carey, Emily Chiang, Lloyd R. Cohen, Gary Cornell, Harald Dickson, John C. H. Fei, Warren L. Fisher, Roger N. Folsom, Dennis R. Heffley, Jack Hirshleifer, James C. Hsiao, Ki-Jun Jeong, George Kondor, William F. Lott, Paul B. Manchester, Peter Morgan, Mark Nerlove, J. Frank Sharp, Alan G. Sleeman, Dennis Starleaf, Henry Y. Wan, Jr., and Chiou-Nan Yeh.

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Our deep appreciation goes to Sarah Dunn, who served so ably and givingly as typist, proofreader, and research assistant. Special thanks are also due to Denise Potten for her efforts and logistic skills in the production stage. Finally, we extend our sincere appreciation to Lucille Sutton, Bruce Gin, and Lucy Mullins at McGraw-Hill, for their patience and efforts in the production of this manuscript. The final product and any errors that remain are our sole responsibility.

Suggestions for the Use of This Book

Because of the gradual buildup of the mathematical tool kit in the organization of this book, the ideal way of study is to closely follow its specific sequence of presentation. However, some alterations in the sequence of reading is possible: After completing first-order differential equations (Chap. 15) you can proceed directly to optimal control theory (Chap. 20). If going directly from Chap. 15 to Chap. 20, however, the reader may wish to review Sec. 19.5, which deals with two-variable phase diagrams.

If comparative statics is not an area of primary concern, you may skip the comparative-static analysis of general-function models (Chap. 8) and jump from Chap. 7 to Chap. 9. In that case, however, it would become necessary also to omit Sec. 11.7, the comparative-static portion of Sec. 12.5, as well as the discussion of duality in Chap. 13.

Alpha C. Chiang
Kevin Wainwright

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