Fundamentals of Fluid Mechanics

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FLUID MECHANICS

Preface

This book is intended as an introductory textbook for undergraduate students in engineering and the applied sciences. A complete course in calculus including differential equations is assumed. A firm grasp of calculus is necessary for a thorough understanding of a basic mechanics course such as fluid mechanics. The text builds upon this mathematical foundation and every effort has been made to explain the significance of the mathematical equations. However, it is equally important that the student have a clear physical picture of the various types of flow problems under consideration. To this end detailed explanatory material has been included to prevent the mathematics from overshadowing the physical significance of a given flow.

Most undergraduates have had an introduction to vectors by the time they start their third year (and in fact have often used vector approaches in their previous mechanics courses). Thus, vectors have been introduced and used wherever appropriate. However, in most such situations multiple approaches are included and a complete undergraduate course can be structured around the book without any measurable introduction of vectors. An appendix which contains the basic vector definitions and operations is included.

In keeping with the trend toward a complete changeover to S.I. units in the future, the S.I. system is used throughout the book along with the conventional English units. The mix of S.I. and English units is roughly 50/50 in both the illustrative examples and in the problem sets at the end of each chapter. The emphasis in the book is strongly toward developing the ability of the student to work comfortably in either system rather than convert back and forth between the two systems. An appendix has been included to help familiarize the student with the general characteristics of the S.I. system.

The various basic equations formulated for a system which the students have become familiar with in their dynamics courses are transformed to equations appropriate for the control volume approach through the Reynolds transport theorem. These equations—continuity, energy and momentum—are ultimately compared with the similar equations which result from Newtonian Physics. The basic relationships are all followed by a variety of applications illustrated by numerous worked examples.

Approximately two-thirds to three-fourths of the text can be covered during the normal three-unit course. This would include most or all of the first nine chapters plus selected material from the remaining chapters as the instructor sees fit or as is appropriate to the type of engineering students taking the course. It is also possible to alter the order of the material introducing such subjects as compressible flow (Chapter 11) or ideal flow (Chapter 12) earlier in the course. As another alternative, much of the theoretical portion of the first nine chapters can be deleted so that more of the remaining material can be covered leading to a much broader, more practically oriented course.

Generally, problem solving aids, such as the tables to calculate normal depth in an open channel or the compressible flow tables, are not included. It was felt that in the brief encounter the student has with these specialized subjects during an introductory course, the slight amount of extra time required to obtain the solutions is rewarded by a better understanding of what is being calculated.

Fluid mechanics is one of the more difficult undergraduate courses for most students. This thought has been constantly kept in mind during the development of this book. Every attempt has been made to include lucid explanations so that the learning process will be at least fruitful if not altogether pleasant. I would like to take this opportunity to thank the typists who waded through several, nor always easy to follow, drafts of the manuscript. I also would like to express my appreciation to my family for their patience and understanding during the writing of the book.

Finally, I would like to acknowledge the immeasurable help of the many engineering students both in California and South Dakota who suffered through some of the early drafts of the book and whose comments and criticisms play a significant role in the final form of the book.

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1

Introduction

1–1 INTRODUCTION, DEFINITIONS, AND GENERAL ASSUMPTIONS

The air we breathe, the water we drink, and the blood and other liquids that flow in our bodies demonstrate the close dependence of our lives on various fluids. Not only must these fluids, as well as many others, be present when we need them, but they must be present where we need them with not only satisfactory quality but also sufficient quantity.

Although our many rivers can provide water for agriculture, industry, personal use, navigation, recreation, and power, management and control are required to reap these benefits. Even when managed, rivers can go on a rampage and destroy whole communities built near their banks. It is vital, then, that their flow be understood, controlled where possible, and the damaging effects minimized. Our atmosphere, which supports life on the earth in another sense, supports aircraft in flight and conceivably will support land craft separated from the earth by a cushion of air.

It is these types of considerations involving such a wide range of applications—from the field of medicine to the design of a submarine, from the blowing of wheat through a pipeline to the pouring of molten iron ore—that makes the study of fluid mechanics so important today. Some of the aforementioned subjects are relatively new, others have been around since the dawn of man. In particular, problems involving the maintenance of a water supply have always beset mankind. As man started settling in concentrated areas, the problems started taking on an engineering nature. Historical records are full of early hydraulic works dating back thousands of years in China, Egypt, Greece, and Rome, to mention a few examples. The ruins of many ancient dams, irrigation canals, and aqueducts are still in existence; and a few are still in use!

The development of engineering through the centuries has more often than not been on a trial-and-error basis, and the design relationships that resulted were largely empirical in nature. It has only been during the last century that fluid mechanics developed as an engineering science with a theoretical formulation that could be applied to practical problems. For a thorough presentation of the history leading up to modern fluid mechanics, the reader is referred to the references by Rouse listed at the end of the chapter.

The paragraphs above were designed to give you an idea of the breadth and depth of the subject of fluid mechanics. However, a more rigorous definition is now in order. *Fluid mechanics* is the study of the motion and stresses that result when a fluid has a given set of forces and boundary conditions imposed upon it. A *fluid* is often described as a substance that takes the shape of its container, a gas completely filling the container while a liquid fills only that lower portion which is consistent with its nearly constant volume. As a somewhat more sophisticated definition, we will also take a fluid to be a substance that will begin to deform when a shear force is applied to it and continue to deform as long as force is applied. By way of contrast, a solid will deform a fixed amount under an applied stress and rebound part or all of the way when the force is removed, depending on whether or not the elastic limit was exceeded. A few general assumptions and restrictions need to be made at the beginning. Forces imposed on a fluid may be either *surface forces* (i.e., applied at a particular point, or distributed over a particular surface) or *body forces* (considered to act throughout the body of fluid). Of the latter type only gravity will be considered in this text.

Flow relationships developed in this book will be obtained using the assumption that the fluid is a continuum. Although certain fluid properties will be explained on the basis of molecular considerations, the formulation of the basic relationships is based on a hypothetical continuous fluid, a fluid that can be continually subdivided without thought of a molecular structure. This approach avoids the difficulty of dealing with the complexity of molecular motion itself. Provided that the boundary dimensions of interest in a problem are large relative to the mean free path of the molecules, as is usually the case, the assumption is amply justified.[1]

[1]For air at atmospheric pressure, this requires that all open spaces be in excess of approximately 10^{-3} mm.

A final assumption that requires brief mention is that no slip can occur either between the fluid and a boundary or internally in the fluid itself. In other words, the velocity of a fluid at a boundary must be identical to the velocity of the boundary and there may be no velocity discontinuities within the fluid. This is a companion assumption to the continuum model and like the foregoing is amply justified by experience.

1–2 FUNDAMENTAL DIMENSIONS AND BASIC UNITS

The fundamental dimension of length (L) is perhaps the best understood of the various dimensions. The distance between two points in space, whether it is the shortest distance, via a straight line, or is one of a large number of curvilinear or irregular routes, is measured in units of length. Commonly, in English units we will use inches, feet, or miles, while SI units of length will include millimeters, centimeters, and meters.[2] The concepts of area and volume involving units of square inches, square feet, or square miles, and cubic inches, cubic feet, or even cubic miles, respectively, in English units, are simply the same fundamental dimension of length to the second and third powers. Although physically hard to visualize, we will involve the length dimension to noninteger powers as in Fig. 1-1, where the liquid velocity discharging from an orifice a distance H below the water surface is proportional to $H^{1/2}$ (or the length dimension of depth to the one-half power).

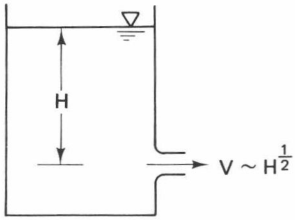

Fig. 1-1. Discharge from an orifice.

As equations are developed throughout the text, they will usually be general and therefore applicable to more than one fluid. This requires that the equations be dimensionally homogeneous; that is, all terms in a given equation must have identical dimensions. In the problem illustrated by Fig. 1-1, we will find that the appropriate equation is

$$H = \frac{V^2}{2g}$$

[2]A discussion of the SI (Système International d'Unités) system is given in Appendix B, as are conversion factors between English and SI units. See also Example 1-2.

The dimensions of the velocity V and the acceleration of gravity g must occur in a combination which results in the same dimensions for the term as that for the depth H. Upon solving for the velocity, its dependence on the square root of H should now be apparent.

The dimensions of time (T), generally expressed in seconds, may be combined with the length scale to give different kinematic[3] quantities. The various velocities and velocity components all have the dimension of length per unit time (L/T), or in usual English units, ft/s, while accelerations are expressed by length/time squared (L/T^2), or, for example, m/s^2 in SI units. The volume rate of flow, or discharge, is also kinematic and is expressed by volume per unit time (L^3/T). The most common English unit for discharge is the cubic foot per second (cfs). Other terms occurring in the literature equivalent to cfs are the second-foot and the cusec. On occasion other units, such as gallons per minute (gpm), will be employed. The most common SI units for discharge will be cubic meters per second (m^3/s), although for low flow rates, liters per unit time may be more convenient. In addition, kinematic mixing coefficients[4] on both the very small or molecular scale and the large or eddy scale will be introduced. These have the dimensions L^2/T, usually ft^2/s or m^2/s.

The remaining fundamental dimensions are force and mass. The units generally employed here will be the pound or newton as the unit of force, and the slug or kilogram as the unit of mass. The four dimensions of length, time, force, and mass are sufficient to describe the mechanics of fluid behavior. However, if thermodynamic effects are coupled with the fluid flow, then temperature would also have to be included as a fundamental dimension. The four dimensions do not exist independently but are related through Newton's second law; consequently, only three dimensions can be considered to be independent. This may be seen by writing Newton's second law,

$$F = Ma \tag{1-1}$$

in the fundamental dimensions

$$F \sim M\frac{L}{T^2}$$

If the proportional symbol is replaced by an equality sign and the mass, length, and time units specified, this becomes the defining equation for the force. Specifically, if the slug, foot, and second are selected, we have immediately

$$1 \text{ lb} = 1 \text{ slug-ft/s}^2$$

Alternatively, if the pound, foot, and second are specified, the companion

[3]*Kinematics* refers to the description of motion irrespective of force and mass.

[4]The kinematic viscosity ν (defined in Section 1-3) and the kinematic eddy viscosity ϵ (which is not defined until Section 7-3) are quantities that measure not only the intensity of mixing but also the resistance to flow.

equation,

$$M = \frac{FT^2}{L}$$

becomes the defining equation for mass:

$$1 \text{ slug} = 1\,\frac{\text{lb-s}^2}{\text{ft}}$$

Similarly, if as basic units, the kilogram, meter, and second are chosen,

$$1 \text{ newton} = 1 \text{ kilogram meter/second}^2$$

As a result of these considerations, the dynamic variables of mechanics may be expressed in alternative dimensions. Consider either the fluid pressure or a shear stress. Both are usually expressed as a force per unit area (F/L^2) or pounds per square foot in English units. They can also be obtained in mass, length, and time units through the equality of dimensions given above. The resulting dimensions are M/LT^2, or slugs/ft-s^2 in English units. Other quantities, such as the dynamic viscosity, to be introduced later, can be expressed equally well with either set of dimensions. In the F, L, and T system the English viscosity units are pound-seconds/square foot, while in the M, L, and T system the viscosity units become slugs/foot-second. The fundamental dimensions in both systems, the most common units, and the appropriate symbol for most variables used in the text are given in Table A-I of Appendix A.

EXAMPLE 1-1

A relatively famous relation which will be developed in a later chapter is the *Bernoulli equation*. The equation states that the sum of three terms must remain constant. This may be written as

$$y + \frac{p}{\gamma} + \frac{V^2}{2g} = C$$

where y is the elevation, p the pressure, γ the specific weight, V the velocity, g the acceleration of gravity, and C the Bernoulli constant. With reference to Table A-1, show that the equation is dimensionally homogeneous and determine the dimensions of the constant C.

SOLUTION:

Using *FLT* dimensions from Table A-1, we find that the symbols have the following dimensions:

y	L (length)
p	F/L^2 (force/length^2)
γ	F/L^3
V	L/T
g	L/T^2