

Intermediate Algebra

MARK DUGOPOLSKI



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MARK DUGOPOLSKI

Southeastern Louisiana University

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Preface

This text is designed for a one term course in intermediate algebra, with an emphasis on the basics in the textual material as well as in the exercises. Although a complete development for each topic is provided, it is assumed that the student has had some prior training in algebra. (The text *Elementary Algebra* in this series is available for students with no prior experience in algebra.) The unifying theme of this text is first the development of the skills necessary for solving equations and inequalities and then application of those skills to solving applied problems.

My primary goal in writing this book was to write one that students can read, understand, and enjoy, while gaining confidence in their ability to use mathematics. Toward this end, I have endeavored to design a comprehensive, yet flexible, presentation of intermediate algebra that will satisfy the individual needs of instructors and a variety of students. I have taken what I refer to as a “common-ground” approach. Whenever possible, the text reminds students of ideas they are already familiar with and then builds on those ideas. This approach allows the instructor to meet the students on a common ground, before advancing to a new level.

Key Features

- Every chapter begins with a **Chapter Opener**, which features an application of an idea developed in that chapter. This gives the student a concrete idea of what will be accomplished in that chapter.

- ▶ Every section begins with a list of topics that tells what is **In This Section**. The sections are divided into subsections and the subsection titles correspond to the topics from the In-This-Section list.
- ▶ Important ideas are set apart in boxes for quick reference. These boxes are used for **definitions, rules, summaries, and strategies**.
- ▶ Simple **fractions and decimals** are used throughout the text. This feature helps to reinforce the basic arithmetic skills that are necessary for success in algebra.
- ▶ One of the main goals of the text is the development of the skills necessary for solving equations and inequalities, and then application of those skills to solving **applied problems**. For this reason, word problems occur wherever possible. This text contains over 400 word problems and they occur in over 50% of the exercise sets.
- ▶ **Inequalities** are used in applications as well as equations. This helps students to see the need for solving inequalities. There are word problems involving linear inequalities, absolute value inequalities, and quadratic inequalities.
- ▶ There are numerous **geometric word problems** throughout the text, which are designed to review basic geometric facts and figures. A summary of common geometric facts can be found inside the front cover.
- ▶ Each exercise set is preceded by **Warm-ups**, a set of 10 simple statements that are to be marked either true or false. These exercises are designed to bridge the gap between the lecture and the exercise sets and to stimulate discussion of the concepts presented in the section. Many of the false statements point out examples of common student errors. The answers to all of these exercises are given in the answer section.
- ▶ The exercise sets throughout the text contain **keyed and non-keyed exercises**. The exercises at the end of each section follow the same order as the textual material and they are keyed to the examples. This organization allows instructors the flexibility of being able to cover only part of a section and easily see which exercises are appropriate to assign. The keyed exercises always give the student a place to start and build confidence. Wherever appropriate, non-keyed exercises follow the keyed exercises and are designed to bring all of the ideas together. Answers to all odd-numbered exercises are in the answer section.
- ▶ **Calculator exercises** are included whenever appropriate. These exercises may be omitted by instructors who do not require calculators in their courses. However, to attain the maximum benefit from this course a calculator is essential. Topics such as quadratic equations, compound interest, and logarithms are more understandable when students can find approximate decimal answers as well as the exact answers.
- ▶ **Mental Exercises** occur at the end of many exercise sets and in the chapter review exercises. These exercises consist mainly of simple equations that can be solved in one or two steps, or simple expressions that can be simplified mentally. These

exercises are meant to be done (perhaps more than once) only after the written exercises. Mental exercises point out to all students what many of them would discover on their own.

- ▶ **Challenging Exercises** are included for instructors wishing to assign more difficult problems to the more able students. These optional exercises are indicated by an asterisk.
- ▶ **Logarithmic and exponential functions** are covered from a modern point of view. A scientific calculator should be required when discussing logarithmic and exponential functions. Calculating with logarithms is not discussed. However, a common logarithm table with instructions on how to use it is given in Appendix B for those instructors who use tables. The applications of logarithmic and exponential functions are stressed, as well as their properties and graphs.
- ▶ Each chapter contains a **Math at Work** feature that illustrates how an idea from that chapter occurs in a real-life situation. The applications are drawn from a variety of fields to maximize student interest. Each Math at Work feature contains an optional exercise pertaining to the application presented.
- ▶ Each chapter ends with a four part **Wrap-up**.

The **Chapter Summary** is a summary of the important concepts from the chapter along with brief illustrative examples.

The **Review Exercises** are intended to provide a review of each section of the chapter. The Review Exercises contain exercises that are keyed to the sections of the chapter, as well as miscellaneous exercises. Answers to the odd-numbered Review Exercises are in the answer section.

The **Chapter Test** is designed to help the student test his or her readiness for a chapter test. To help the student to be independent of the examples and sections, the Chapter Test has no keyed exercises. All answers for each Chapter Test are given in the answer section.

Tying It All Together exercises are designed to help students fit the current ideas in with ideas from the previous chapters. They may also contain exercises that will help in understanding the upcoming chapter. These exercises could be worked after a chapter test, but prior to starting the next chapter. All answers for these exercises are given in the answer section.

Ancillary Materials

- ▶ An **Annotated Instructor's Edition** is available to instructors. In addition to the material presented in the student edition, the AIE contains marginal notes to the instructor, in-text answers to all numerical exercises, and answers to all even-numbered graphing exercises in the instructor's answer section, which follows the To the Instructor introductory material.
- ▶ The **Instructor's Solution Manual** contains a detailed solution to each exercise in the text.

- ▶ The **Student's Solution Manual** contains detailed solutions to all of the odd-numbered exercises as well as solutions to all exercises from the Warm-ups, Chapter Tests, and Tying It All Together sections.
- ▶ The **Student's Study Guide** includes diagnostic tests for each chapter as well as detailed solutions and explanations for each test question.
- ▶ There are several options available for **Computerized Testing**.

AWTest (Apple) is an algorithm-based testing system keyed to the text. AW-Test will generate multiple choice, open-ended, and true-false questions.

Omni Test (IBM) is a powerful new testing system developed exclusively for Addison-Wesley. It is an algorithm-driven system that allows the user to create up to 99 perfectly parallel forms of any test effortlessly. It also allows the user to add his or her own test items and edit existing items with an easy to use, on screen "What-You-See-Is-What-You-Get" text editor.

Mac Test (Macintosh) is a test item bank containing ten questions for each objective drawn from the text. Questions are in a multiple choice format.

- ▶ The **Printed Test Bank** contains three alternate forms of tests for each chapter, as well as cumulative review tests and final exams.
- ▶ **Tutorial Software** is available in several forms.

IMPACT: An Interactive Mathematics Tutorial (IBM & Mac) is a tutorial package correlated to each section of the text. This package will generate exercises similar to those in the text exercise sets. If the student gets an exercise wrong, he or she can see either an example or a solution. The program will lead students interactively through step-by-step solutions to the exercises so that they can easily pinpoint specific trouble areas if they make an error.

Students using **The Math Lab** (IBM & Apple II) pick the topic area, level of difficulty, and number of problems. If they get a wrong answer, the program will prompt them with the first step of the solution. This program keeps detailed records of students' results, which can be stored on the disk.

In **Professor Weissman's Software** (IBM), students pick the topic area and level of difficulty. The program generates problems and gives step-by-step solutions if they get a wrong answer. Problems increase in difficulty as students are successful. This program also keeps records of students' scores.

For the **Algebra Problem Solver** (IBM), students choose a topic and can either request an exercise from the computer or enter their own homework exercise. The computer will show them a step-by-step solution to the problem.

- ▶ An extensive set of **video tapes** is available free upon adoption. These videos cover every topic presented in the text. Particular attention has been paid to working examples and giving complete explanations of the steps involved.
- ▶ **The Math Hotline** is a new idea in teaching and learning packages created especially for this text. Suppose a student is at home studying and can't get a problem

started. Now, simply by calling the Math Hotline from a touch-tone phone and responding to a few questions with the keypad, a student can receive a hint for any odd-numbered exercise. This means thousands of hints are as close as the phone. The Math Hotline will provide hints 24-hours a day, 365 days a year.

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Hammond, Louisiana

M.D.

To the Student

This text was written to help you learn algebra. I have made the explanations as clear as possible and I have provided plenty of examples to illustrate the ideas. This text was written with you in mind. You should be aware of the many features that were designed into this text to make the text easy to use.

The exercises follow the same order as the textual material and they are keyed to the examples in the text. This feature is to help you get started. Don't try to understand an entire section before beginning the exercises. Get started by doing exercises. If you don't understand how to do an exercise, then refer to the example that corresponds to that exercise. If you miss class and you need to work on your own, read the section in pieces. Read until you get through an example, then work the corresponding exercises.

To learn algebra you must do algebra and doing algebra means working problems. I have written an abundance of problems for this text. I have tried to avoid overly complicated exercises to give you plenty of practice with the basics. The answers to all odd-numbered exercises are given in the back of the book. These answers should be consulted only after you have worked a problem and checked your work.

To help you bring each chapter together, a four-part Wrap-up is included at the end of each chapter. In each Wrap-up you will find a Chapter Summary that lists the

key ideas in each chapter along with illustrative examples. You will also find Review Exercises that are keyed to the sections of the chapter, along with miscellaneous exercises from the chapter. A Chapter Test is included for each chapter to let you check your progress. Complete answers for chapter tests are given in the back of the book. The exercises in the Chapter Tests are not keyed to the section from which they come. After each Chapter Test (starting with Chapter 2) I have included some exercises called Tying It All Together. These exercises are designed to help you fit current ideas in with ideas from previous chapters. Sometimes these exercises include problems that will be helpful to review before starting the next chapter. The Tying It All Together exercises could be worked after an in-class test but prior to the next class meeting.

If you need additional help with the exercises, there is a Student's Solution Manual available that includes complete solutions to those problems that have answers in the book. There is also a Student's Study Guide available to help you to prepare for tests. I have also prepared a computerized help line that you can call from a touch tone phone and get a brief message on how to work a problem. Help is available 24 hours a day for those exercises that have answers in the answer section of the book. Relax and do your homework faithfully, and I am sure that you will be successful in algebra.

Mark Dugopolski

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Front Endpaper

Some formulas

CHAPTER 1

Basics



Why can you put your groceries on the checkout counter in any order and still get the same total?

Do we get the same amount of tax if the sales tax is computed on each item and then totaled, or if the items are totaled first and then the tax computed?

Why is \$8 divided among 4 people not the same as \$4 divided among 8 people?

What do these questions have to do with algebra? The answers to these questions are found in the fundamental properties of arithmetic. Success in algebra strongly depends on a solid foundation of arithmetic. The computational skills developed in arithmetic are essential in algebra. However, in algebra we must understand the underlying concepts of arithmetic as well. In this chapter we will review some of the concepts of arithmetic, learn the basic terminology of algebra, and discover the answers to the above questions (and many others).