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STOCHASTIC TRANSPORT IN COMPLEX SYSTEMS

FROM MOLECULES TO VEHICLES

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STOCHASTIC TRANSPORT IN COMPLEX SYSTEMS

Dedicated to
Martina, Indrani, and Yumiko

PREFACE

Historically, statistical mechanics was developed as the molecular kinetic theory of matter. Its formalism for systems in thermodynamic equilibrium has been established on a solid foundation. Twentieth century has witnessed many triumphs of this formalism in solving many mysteries of bulk matter in thermodynamic equilibrium. Encouraged by the overwhelming success of the concepts and techniques of statistical mechanics in understanding the physical properties of inanimate matter, some statistical physicists have taken bold steps to explore virgin territories beyond the traditional boundaries of physics using the same tool box. Some of these systems were subjects of earlier investigation in other traditional disciplines. Such unconventional applications of statistical mechanics have opened up new horizons of interdisciplinary research; one of the most successful among such joint ventures is the area of soft matter.

But, despite relentless efforts of statistical physicists over the last century, an equally strong foundation of nonequilibrium statistical mechanics remains elusive. One class of nonequilibrium systems that has received lot of attention in recent years consists of mutually interacting particles, which are driven by an external field. Such intrinsically nonequilibrium systems can settle down to a nonequilibrium steady state which, in principle, can be far from the equilibrium state that it could attain in the absence of the external field. Many physical systems have been successfully studied in the last two decades, using the techniques developed for models of such field-driven interacting particles.

In more recent times, nonequilibrium statistical mechanics has found even more unusual applications in research on traffic flow of various types of objects. A list of such systems would include not only vehicular traffic, but also traffic-like collective phenomena in living systems, as well as in colonies of organisms. The common modeling strategy is to represent the motile elements (i.e., vehicles or an organism) by self-propelled particles, which convert chemical energy (derived from fuel or food) into mechanical energy required for the forward movement in a dissipative environment. In such generic models, the mutual influences of the motile elements on the movement of each other are captured by postulating appropriate interparticle interactions.

Just as nonequilibrium statistical mechanics has helped in getting deep insight into the general physical principles governing the spatio-temporal organization and traffic flow in a wide variety of complex systems, which were earlier subjects of investigation in other branches of science and engineering, these disciplines have also enriched physics by proving wider testing grounds for the tool box of nonequilibrium statistical mechanics. The problems that we are studying here are classic examples of complex systems. Their collective behavior is not just a simple superposition of the individual behaviors, but the interactions among the particles/vehicles/agents... lead to new features of the dynamics with, sometimes, surprising consequences.

In Part I of this book, we give a pedagogical introduction to the conceptual framework and technical tools of nonequilibrium statistical mechanics. Although our main focus is on driven systems of interacting particles, we present the subject from a much broader perspective. In Part II, we illustrate the use of the formalisms developed in Part I by applying these to vehicular and pedestrian traffic, as well as to traffic-like collective phenomena in biology.

Modern life is crucially dependent on vehicular traffic. Our travel to our work places, to supermarkets, to hospitals, to schools and universities all depend on vehicular traffic. The supply of our daily needs also mostly come by trucks. We are all too familiar with the irritating traffic jams. But, traffic jam is not a modern phenomenon; it was familiar to inhabitants of cities even before the invention of motorized vehicles. Even in ancient cities like Rome, carts and chariots were often stuck in traffic jam and elaborate plans for traffic control were made even by Leonardo da Vinci.

However, study of traffic as a branch of science and engineering started growing only in the second half of the twentieth century. Interestingly, new approaches of investigations in traffic science have been opened up mostly by physicists, perhaps, because of the close analogy between vehicular traffic and systems of interacting particles driven far from equilibrium. The notable among the early contributors include, for example, Montroll, Potts, Prigogine, and others. The conceptual framework and technical tool box of statistical physics has turned out to be extremely useful in modeling traffic phenomena and analyzing the properties of the models.

Nature has dealt with traffic-like phenomena for billions of years. In the recent years, it has become clear that the environment inside a cell is, in many respects, similar to urban traffic system where molecular motors carry cargo over long distances by moving along filamentary tracks. Molecular traffic jam can lead to diseases. Therefore, fundamental understanding of these traffic phenomena will not only help in diagnosis but also in the control of those diseases that arise from malfunctioning of the molecular motor traffic in living systems.

Traffic-like phenomena occur in biological systems at almost all levels of organization- from individual molecules and molecular-self assemblies to cells and organisms. The collective movements of ants toward the food source and their return to the nest appears very similar to vehicular traffic. Surprisingly, quantitative studies of this traffic phenomenon started only a couple of years ago despite the possible applications of the results in various ant-based routing algorithms in communication networks, in swarm intelligence, and even in decentralized management.

In the past, capturing large number of empirical facts and phenomena with a few mathematical equations have always led to great progress in scientific theories. Maxwell unified all the empirical phenomena in electricity and magnetism in terms of the four equations named after him. Similarly, our attempt has been to highlight the "conceptual unity" among the apparent diversity of systems covering a long range of length scales and time scales. A common conceptual thread runs through all these traffic-like phenomena. We hope this monograph will stimulate further exchange of ideas across disciplinary boundaries of physics, chemistry, biology, as well as technology and engineering enriching all.

HOW TO USE THIS BOOK

In Part I, we present a systematic pedagogical treatment of the theoretical formalisms which we, then, use in the Part II to develop and analyze models of transport and traffic phenomena. This book is self-contained in the sense that all the formal theoretical methods required in Part II are available in Part I.

The formalisms discussed in Part I are essentially based on the concepts and techniques of nonequilibrium statistical mechanics. For the convenience of nonexperts, we minimize subtle technical details in the main chapters of Part I. Instead, for the benefit of readers interested in such details, lengthy mathematical calculations have been given in the appendices.

In Chapter 1, while familiarizing the reader with the inventory of the toolbox of nonequilibrium statistical mechanics, we also mention their potential use in the subsequent chapters in the context of traffic science. In Chapter 2, we train the beginner in using some of the most powerful tools. In particular, we introduce several analytical, numerical, as well as phenomenological approaches for studying (quasi-) one-dimensional driven diffusive systems. In Chapter 3, we focus more specifically on a technique that, in recent years, has been used successfully in the mathematical treatment of many particle-hopping models. In Chapter 4, we give a comprehensive overview of our current understanding of the asymmetric simple exclusion process (ASEP), which is the most important particle-hopping model relevant for theoretical modeling of traffic. In fact, most of the particle-hopping models of vehicular traffic, which are described in Part II, are extensions of ASEP appropriate for capturing some specific traffic phenomena.

We classify the traffic models in Chapter 5 so that the reader does not lose sight of the forest for the trees. We review the well-known empirical facts about traffic phenomena in Chapter 6. In Chapter 7, we discuss all the theoretical treatments of the Nagel–Schreckenberg model, the minimal and most basic model of vehicular traffic on highways, in detail. All the other CA models of vehicular traffic, many of which are related to the Nagel–Schreckenberg model, are considered in Chapter 8. For the sake of completeness of the overview of theoretical approaches to vehicular traffic, we also present the non-CA models in Chapter 9. The models of highway traffic have been modified and extended appropriately to capture some of the key features of vehicular traffic on networks of roads and highways in cities and greater urban areas. We present these models in Chapter 10 along with similar models of internet traffic. Theories of pedestrian traffic and traffic-like collective phenomena in biological systems are now making rapid progress. We provide a glimpse of this fast emerging frontier area in Chapters 11 and 12.

There are at least two different routes how to use this book.

ROUTE 1: INTRODUCTION TO STOCHASTIC SYSTEMS

Part I of this book can be used as a textbook for a specialized course on stochastic transport in systems driven far from equilibrium. In such a course, depending on the academic background and research interest of the students, a few topics from Part II can be selected to show interesting applications of the methodology.

ROUTE 2: INTRODUCTION TO THE MODELING OF TRAFFIC PHENOMENA

Part II of this book can serve as a comprehensive introduction to the models of traffic and traffic-like collective phenomena. A reader, who is mainly interested in applied research in traffic, may need to refer to Part I if and only if (s)he wants to delve into any particular formalism.

A long list of references is provided as a guide to the literature. Therefore, this compendium will also serve as a valuable reference for experts actively engaged in research on traffic models and related phenomena in complex systems.

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However, we, the authors, are solely responsible for any conceptual error that might have crept in.

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