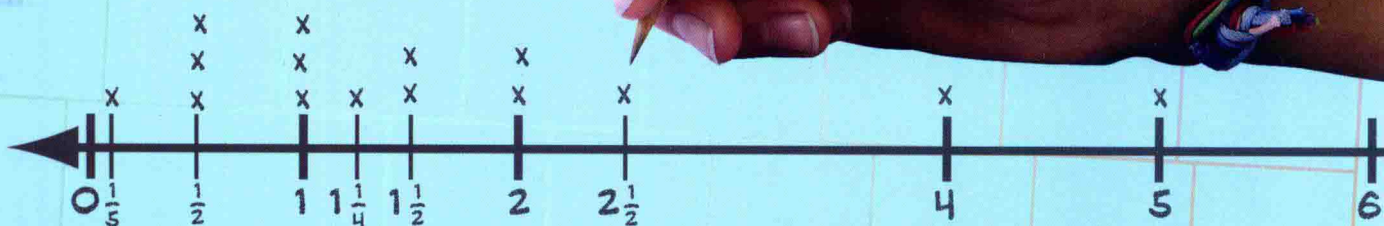


*Aligns to the
Common Core
State Standards*

Strategies for Teaching

FRACTIONS

Using Error Analysis for
Intervention and Assessment



David B. Spangler

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Common Core
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Strategies
for Teaching
FRACTIONS

Preface

S*trategies for Teaching Fractions* provides a practical intervention model with targeted hands-on materials for teachers to use when working with struggling learners. This strategy book is a comprehensive resource—providing the “what,” the “why,” and the “how”—to guide and support classroom work with fractions *at whatever grades students may need intervention* for these skills and concepts. As the second book in a series, *Strategies for Teaching Fractions* provides a natural continuation of the first book, *Strategies for Teaching Whole Number Computation*.

There is probably universal agreement that student proficiency with fractions is essential. According to *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (2008), “A major goal for K–8 mathematics education should be proficiency with fractions, for such proficiency is foundational for algebra” (p. xvii).

In 2010, the Council of Chief State School Officers and the National Governors Association released the *Common Core State Standards for Mathematics* (CCSS). These standards may be downloaded at www.corestandards.org/the-standards. The CCSS are organized into two related categories: (1) Standards for Mathematical Content (defining what students should understand and be able to do) and (2) Standards for Mathematical Practice (describing ways in which students should engage with mathematics based on processes and proficiencies). The CCSS place an emphasis on the teaching of fractions at Grades 3–6. The mathematical content and instructional activities in this book were written to closely align to these standards. Citations to the CCSS are frequently made where key standards are addressed. An overview of the Common Core State Standards for fractions is provided on the following page.

“The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected.”

—National Mathematics Advisory Panel
(2008, p. 18)

Common Core State Standards for Mathematics (CCSS)

Overview of Number and Operations—Fractions

Grade 3:

- Develop understanding of fractions as numbers.

Grade 4:

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Grade 5:

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Overview of The Number System

Grade 6:

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

—National Governors Association Center for Best Practices and
Council of Chief State School Officers (2010, pp. 22, 28, 34, and 41)

It should be noted that the topics described in the CCSS carry an expectation that they be mastered by students no later than at the grade levels cited. However, it is assumed that students engage in appropriate prerequisite work prior to those grade levels and meaningful review and extensions subsequent to those grade levels.

This book also addresses key concepts and skills related to fractions recommended by the National Council of Teachers of Mathematics (NCTM, 2000a, 2000b) in the *Number and Operations* Content Standard of their *Principles and Standards for School Mathematics*. At Grades 3–5, the NCTM expectations are for students to use models and benchmarks, to understand the various meanings of fractions, and to find equivalent fractions. At Grades 6–8, students are expected to compare and order fractions and to be fluent with all four operations with fractions to solve problems.

In 2006, NCTM expanded upon those standards with its publication *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*. That document describes a new approach to curriculum development that focuses on areas of emphasis within each grade—recommending the most significant mathematical concepts and skills that should be taught at each grade level. *Strategies for Teaching Fractions* addresses the depth of coverage envisioned by the focal points with respect to number and operations for fractions for Grades 3–6. The focal points include the following: At Grade 3, students

understand the meanings of fractions as parts of a whole, parts of a set, or points or distances on a number line. They also compare and order fractions using various strategies, including benchmarks. At Grade 4, students connect fractions and decimals. At Grade 5, students add and subtract fractions with like and unlike denominators. At Grade 6, students multiply and divide fractions. At all grades, students make estimates and solve problems.

The pedagogy employed in this book is aligned with all five Process Standards outlined in the *Principles and Standards for School Mathematics* (NCTM, 2000a) as described below.

- *Problem solving*: Problem-solving opportunities are embedded throughout the text.
- *Communication*: Suggested questions that may be used directly with students are provided throughout the Intervention Activities; suggestions for student writing opportunities are also provided.
- *Reasoning and proof*: Many of the suggested questions, along with the writing opportunities, require students to *explain their reasoning and thinking*.
- *Connections*: This book makes connections to the study of *probability, measurement, and geometry*—thus providing a strong context for fractions. Because probability is often addressed in the *final* chapter of a textbook (and is often barely touched on in class), it is especially important to integrate work with it when studying fractions.
- *Representation*: The concept development in the Intervention Activities utilizes manipulatives, number lines, fraction strips and circles, and other models and diagrams.

Three types of knowledge crucial for teaching mathematics at any level are described at right. O'Donnell (2009) citing Hill et al. (2004) notes that teachers illustrate *pedagogical content knowledge* by “generating representations, interpreting student work, and analyzing student mistakes.” *Strategies for Teaching Fractions* addresses all three types of knowledge in a concise, user-friendly way. With a focus on *how* to apply specific pedagogy to specific content, this resource is especially strong in the area of pedagogical content knowledge. Because this book develops *mainstream mathematics concepts in a way that is truly meaningful for students*, teachers should find this book to be an effective supplement to *any* textbook program—from those on the traditional end of the continuum to those that are based on reform.

NCTM Process Standards

Problem Solving

“Problem solving is an integral part of all mathematics learning.”

—NCTM (2000a, p. 52)

Communication

“Reflection and communication are intertwined processes in mathematics learning.”

—NCTM (2000a, p. 61)

Reasoning and Proof

“Teachers can understand students’ thinking when they listen carefully to students’ explanations.”

—NCTM (2000a, p. 126)

Connections

“The opportunity for students to experience mathematics in context is important.”

—NCTM (2000a, p. 66)

Representation

“Seeing similarities in the ways to represent different situations is an important step toward abstraction.”

—NCTM (2000a, p. 138)

Content Knowledge

Knowledge of the subject

Pedagogical Knowledge

Knowledge of instructional practice

Pedagogical Content Knowledge

Knowledge of mathematical content for its use in the classroom

—Schulman (1987)

RESEARCH BASE FOR THE BOOK

Strategies for Teaching Fractions is informed by academic research conducted and analyzed during the past 35 or more years. In developing this book, the author provides a mix of this extensive research base with his personal experience of about 40 years in mathematics education. The author's experience includes teaching at various levels, including classroom (Grades 5–8), community college (in a remedial teaching lab), and university (methods courses).

Although some errors that occur in students' work with fraction concepts and computation are due to carelessness or incorrect recall of number facts, many are due to misconceptions and the use of incorrect strategies. According to Pincus et al. (1975), "Too often, when teachers find errors in a child's work, they mark the example wrong, assume that the child did not master the basic facts, and prescribe further drill. *Careful analysis of errors through observation and interviews with individual children is essential*" (p. 581).

A key premise of this book is that if teachers (1) analyze student work for *error patterns* (revealed through diagnostic tests, practice, activities, and student discourse/oral interviews) and (2) then provide timely, targeted, and meaningful intervention, student errors will decrease in frequency—while at the same time student understanding of concepts will increase. *In particular, error analysis should enable a teacher to build on what students are doing correctly, while focusing on those areas for which the student needs additional work.* By taking into account academic research on how students learn, the intervention strategies illustrated in this book should result in improved student performance and more positive student dispositions toward learning mathematics.

HOW THE BOOK IS ORGANIZED

The book begins with a substantial section, "A Look at the Academic Research: Intervention in the Mathematics Classroom." This academic research supports the goals and premises of the book, the pedagogical practices utilized in the Intervention Activities, and other aspects of intervention. These practices

include accessing language, activating prior knowledge, scaffolding, using representations, using estimation and mental math, introducing alternative algorithms, differentiating instruction, participating in instructional games, using technology, and more.

The research section concludes with a survey of key research findings related to the teaching of fraction concepts and computation. Included is an extensive discussion on the various meanings, or models, of fractions (addressed by NCTM at left). A hallmark of this book is the ongoing development of fraction concepts and computation through the use of a variety of fraction models.

NCTM Content Standard

Number and Operations

In grades 3–5, all students should

- develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers.

—NCTM (2000a, p. 148)

Next is a two-part section titled “Big Ideas in Fractions and Problem Solving.” The first part provides an overview of key understandings related to fraction concepts and fraction computation. These ideas include an overview of terminology, properties, and overall fraction number sense. The second part of this section is titled “Actions and Operations: Problem Structures for Addition, Subtraction, Multiplication, and Division.” Here, illustrative models based on fractions are provided for teachers to use with children to help them decide which operation to use to solve a given problem by thinking of *actions* that can be done with objects (or other representations) that relate to mathematical operations. By focusing on real-world fraction contexts for the operations, key connections are made between *problem solving* and computation. For illustrative models for each operation based on whole numbers, see Spangler (2010, pp. 17–23).

According to Kilpatrick, Swafford, and Bradford (2001), “Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics” (p. 420). The author suggests that the problem structures described in this section be used with students as they study each operation. Students should then be asked to write their own word problems based on these structures to enhance their understanding of the operation. Because problem solving is not the major focus of this book, teachers are encouraged to integrate additional resources for problem solving when they teach computation.

Three main units then follow—one for fraction concepts, one for addition/subtraction of fractions and mixed numbers, and one for multiplication/division of fractions and mixed numbers. The units follow a predictable format. Each begins with a diagnostic test (in multiple-choice format), followed by an Item Analysis Table that keys student incorrect test responses to specific error patterns. (Each distractor on the tests is based on a specific error pattern.) A detailed section of error patterns with *step-by-step* Intervention Activities (the heart of the book) then follows. The Intervention Activities often include an alternative algorithm to go along with the traditional algorithm. It should be noted that the Intervention Activities (both for the traditional and for the alternative algorithms) may be used with students as part of the *initial instruction*—and are not just intended for use with students *after* they may have struggled with a concept.

A hallmark of this book is its strong focus on *teaching for understanding* in developing the Intervention Activities for fraction concepts and for each operation. Guided questions (with suggested student responses) and hands-on experiences (including a guided discovery lesson to introduce each operation) are used to achieve student understanding. Following the Intervention Activities for each unit is a short section of practice (keyed to the Item Analysis Table). Supplemental material (blacklines) for estimation, instructional games, and follow-up activities is also included for each unit.

“Basic skills with numbers continue to be vitally important for a variety of everyday uses. They also provide crucial foundation for the higher-level mathematics essential for success in the workplace which must now also be part of a basic education.”

—Ball et al. (2005, p. 1056)

“A curriculum built on focal points . . . has the potential to offer opportunities for the diagnosis of difficulties and immediate intervention, thus helping students who are struggling with important mathematics content.”

—NCTM (2006, p. 5)

Each main unit, along with the sections on academic research and “Big Ideas,” concludes with a set of *Questions for Teacher Reflection*. These open-ended questions are intended to provide springboards for discussion among teachers or preservice teachers who may be using this book in a professional development setting, workshop, or methods course.

INTENDED USES AND AUDIENCE FOR THE BOOK

Strategies for Teaching Fractions is intended to serve a wide audience of educators, and there are a variety of ways to use the book.

- **Intervention program for fraction concepts and computation in pullout or full-classroom situations:** The book may be used as a full-fledged intervention program for work with fractions. The book provides a comprehensive tool for quickly diagnosing and pinpointing trouble spots students are encountering—with specific Intervention Activities tailored to address the weaknesses identified.
- **Resource for elementary and middle grades classroom teachers:** The book describes the types of errors struggling students frequently make with fraction concepts and computation. Teachers, especially those with limited experience teaching this content, should find this knowledge valuable as they teach these concepts.
- **Resource for special education teachers, including those who work with students in coteaching situations:** These teachers—from the elementary level through high school—will find a wealth of material in this book to help them differentiate instruction for individual students or students working in small groups.
- **Resource for teachers and tutors working in developmental labs at the high school and community college levels:** The book provides fresh approaches to help students who have been struggling for years to learn how to work with fractions.
- **Resource for instructional supervisors and curriculum coordinators:** These educators should find this book to be an important tool to use as they work with their teachers.
- **Text that may be used with teachers and preservice teachers in mathematics methods courses, workshops, and professional development programs (including online programs):** Educators at all grade levels should embrace this book as a key source of pedagogical content knowledge that they can directly use in their classrooms. The *Questions for Teacher Reflection* provide opportunities for teacher discussion and/or assignments in such settings.
- **Resource for educators interested in academic research on intervention:** This book provides a handy compilation of important research findings of the past 35-plus years related to intervention for fraction concepts and computation.

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About the Author



David B. Spangler has devoted his entire professional career of about 40 years to mathematics education. After graduating from Northern Illinois University with BS and MS degrees, he began his career as a mathematics teacher in middle school, where he taught in an individualized setting. Later he taught at Triton Community College, where he gained direct experience interacting with struggling students in a developmental math laboratory. He has also worked many years as a mathematics editor and editorial director for major publishers in the el-hi textbook industry.

David is the author of *Strategies for Teaching Whole Number Computation: Using Error Analysis for Intervention and Assessment* (Corwin, 2010), *Mathematics Explorations: Detective-style Activities for the Real World* (Good Year Books, 2008, 2011), and *Math for Real Kids* (Good Year Books, 1997, 2005). He has also written numerous articles for journals, including *Mathematics Teaching in the Middle School*.

Currently David teaches methods courses through National-Louis University and *ActiveMath®* Workshops, a professional development company he cofounded in 1994 (www.activemath.com). The courses, workshops, and in-district training he facilitates address special-needs students, intervention, and hands-on activities. He also provides staff development that specifically addresses the concepts in this book and in *Strategies for Teaching Whole Number Computation*. David has worked with literally thousands of students and teachers during his career.

David is a frequent speaker at mathematics conferences of professional organizations, where he addresses teaching mathematics for meaning, teaching through real-world applications, and teaching through humor. He also delivers after-dinner talks on the subject of mathematical humor. The talk features mathematical blunders from the real world and suggestions for addressing mathematical illiteracy in our society.

As a mathematics educator, David's goal is to teach mathematics for meaning rather than in a way that promotes rote memorization. This book was written to help teachers achieve that goal. Based on his extensive experience in mathematics education, David is uniquely qualified to write this book.

David lives with his wife, Bonnie, in Northbrook, Illinois. They have three grown children, Ben, Jamie, and Joey. He would like to thank all of them for their love, support, and encouragement throughout his entire career.

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