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C. Gasquet
P. Witomski
Translated by R. Ryan

Fourier Analysis and Applications

Filtering, Numerical Computation, Wavelets

傅立叶分析和应用

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With 99 Illustrations

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Translator's Preface

This book combines material from two sources: *Analyse de Fourier et applications: Filtrage, Calcul numérique, Ondelettes* by Claude Gasquet and Patrick Witomski (Masson, Paris, second printing, 1995) and *Analyse de Fourier et applications: Exercices corrigés* by Robert Delmasso and Patrick Witomski (Masson, Paris, 1996). The translation of the first book forms the core of this Springer edition; to this have been added all of the exercises from the second book. The exercises appear at the end of the lessons to which they apply. The solutions to the exercises were not included because of space constraints.

When Springer offered me the opportunity to translate the book by Gasquet and Witomski, I readily accepted because I liked both the book's content and its style. I particularly liked the structure in 42 lessons and 12 chapters, and I agree with the authors that each lesson is a "chewable piece," which can be assimilated relatively easily. Believing that the structure is important, I have maintained as much as possible the "look and feel" of the original French book, including the page format and numbering system. I believe that this page structure facilitates study, understanding, and assimilation. With regard to content, again I agree with the authors: Mathematics students who have worked through the material will be well prepared to pursue work in many directions and to explore the proofs of results that have been assumed, such as the development of measure theory and the representation theorems for distributions. Physics and engineering students, who perhaps have a different outlook and motivation, will be well equipped to manipulate Fourier transforms and distributions correctly and to apply correctly results such as the Poisson summation formula.

Translating is perhaps the closest scrutiny a book receives. The process of working through the mathematics and checking in-text references always uncovers typos, and a number of these have been corrected. On the other hand, I have surely introduced a few. I have also added material: I have occasionally added details to a proof where I felt a few more words of explanation were appropriate. In the case of Proposition 31.1.3 (which is a key result), Exercise 31.12 was added to complete the proof. I have also completed the proofs in Lesson 42 and added some comments. Several new references on wavelets have been included in the bibliography, a few of them with annotations. All of these modifications have been made with the knowledge and concurrence of Patrick Witomski.

Although the book was written as a textbook, it is also a useful reference book for theoretical and practical results on Fourier transforms and distributions. There are several places where the Fourier transforms of specific functions and distributions are summarized, and there are also summaries of general results. These summaries have been indexed for easy reference.

The French edition was typeset in Plain TeX and printed by Louis-Jean in Gap, France. Monsieur Albert at Louis-Jean kindly sent me a copy of the TeX source for the French edition, thus allowing many of the equations and arrays to be copied. This simplified the typesetting and helped to avoid introducing errors. My sincere thanks to M. Albert. Similarly, thanks go to Anastis Antoniadis (IMAG, Grenoble) for providing the LaTeX source for the exercises, which was elegantly prepared by his wife. I had the good fortune to have had the work edited by David Kramer, a mathematician and freelance editor. He not only did a masterful job of straightening out the punctuation and other language-based lapses, but he also added many typesetting suggestions, which, I believe, manifestly improved the appearance of the book. I also thank David for catching a few of the typos that I introduced; those that remain are my responsibility and embarrassment.

Robert Ryan
Paris, July 14, 1998

Preface to the French Edition

This is a book of applied mathematics whose main topics are Fourier analysis, filtering, and signal processing.

The development proceeds from the mathematics to its applications, while trying to make a connection between the two perspectives. On one hand, specialists in signal processing constantly use mathematical concepts, often formally and with considerable intuition based on experience. On the other hand, mathematicians place more priority on the rigorous development of the mathematical concepts and tools.

Our objective is to give mathematics students some understanding of the uses of the fundamental notions of analysis they are learning and to provide the physicists and engineers with a theoretical framework in which the "well known" formulas are justified.

With this in mind, the book presents a development of the fundamentals of analysis, numerical computation, and modeling at levels that extend from the junior year through the first year of graduate school. One aim is to stimulate students' interest in the coherence among the following three domains:

- Fourier analysis;
- signal processing;
- numerical computation.

On completion, students will have a general background that allows them to pursue more specialized work in many directions.

The general concept

We have chosen a modular presentation in lessons of an average size that can be easily assimilated ... or passed over. The density and the level of the material vary from lesson to lesson. We have purposefully modulated the pace and the concentration of the book, since as lecturers know, this is necessary to capture and maintain the attention of their audience. Each lesson is devoted to a specific topic, which facilitates reading "à la carte." The lessons are grouped into twelve chapters in a way that allows one to navigate easily within the book.

A progressive approach

The program we have adopted is progressive; it is written on levels that range from the third year of college through the first year of graduate school.

JUNIOR LEVEL

Lessons 1 through 7 are accessible to third-year students. They introduce, at a practical level, Fourier series and the basic ideas of filtering. Here one finds some simple examples that will be re-examined and studied in more depth later in the book. The Lebesgue integral is introduced for convenience, but in superficial way. On the other hand, emphasis is placed on the geometric aspects of mean quadratic approximation, in contrast to the point of view of pointwise representation. The notion of frequency is illustrated in Lesson 7 using musical scales.

SENIOR LEVEL

The reader will find a presentation and overview of the Lebesgue integral in Chapter IV, where the objective is to master the practical use of the integral. The lesson on measure theory has been simplified. This chapter, however, serves as a good guide for a more thorough study of measure and integration. Chapter VI contains concentrated applications of integration techniques that lead to the Fourier transform and convolution of functions. One can also include at this level the algorithmic aspects of the discrete Fourier transform via the fast Fourier transform (Chapter III), the concepts of filtering and linear differential equations (Chapter VII), an easy version of Shannon's theorem, and an introduction to distributions (Chapter VIII).

MASTER LEVEL

According to our experience, the rest of the book, which is a good half of it, demands more maturity. Here one finds precise results about the fundamental relation $\widehat{f * g} = \widehat{f} \cdot \widehat{g}$, the Young inequalities (Chapter VI), and various aspects of Poisson's formula related to sampling (Chapter XI). Finally, time-frequency analysis based on Gabor's transform and wavelet analysis (Chapter XII) call upon all of the tools developed in the first eleven chapters and lead to recent applications in signal processing.

The content of this book is not claimed to be exhaustive. We have, for example, simply treated the z-transform without speaking of the Laplace transform. We chose not to deal with signals of several variables in spite of the fact that they are clearly important for image processing.

Possible uses of time

This book is an extension of a course given for engineering students during their second year at E.N.S.I.M.A.G.[1] and at I.U.P.[2]. We have been confronted, as are all teachers, with class schedules that constrain the time available for instruction. The 40 hours available to us per semester at E.N.S.I.M.A.G. or at I.U.P., which is divided equally between lectures and work in sections, provides enough time to present the essential material.

Nevertheless, the material is very rich and requires a certain level of maturity on the part of the students. We are thus led to assume in our lectures some of the results that are proved in the book. This is facilitated by the partition of the book into lessons, and it is not incompatible with a good mathematics education. The time thus saved is more usefully invested in practicing proofs and the use of the available tools. The material is written at a level that leads to a facility in manipulating distributions, to a rigorous formulation of the fundamental formula $\widehat{f * g} = \widehat{f} * \widehat{g}$ under various assumptions, to an exploration of the formulas of Poisson and Shannon, and finally, to precise ideas about the wavelet decomposition of a signal.

Our presentation contrasts with those that simply introduce certain formulas such as

$$\int_{-\infty}^{+\infty} e^{-2i\pi(\lambda-a)}\, dt = \delta(\lambda - a)$$

out of thin air, where one ignores all of the fundamental background for a very short-term advantage.

Different possible courses

One can work through the book linearly, or it is possible to enter at other places as suggested below:

Juniors
Chapters I, II, and III.

Seniors and Masters in Mathematics
Chapters IV, V, VI, VIII, and IX.

Seniors and Masters in Physics
Chapters VII, X, XI, and XII.

This book comes from many years of teaching students at E.N.S.I.M.A.G. and I.U.P. and pre-doctoral students. In fact, it was for pre-doctoral instruction that a course in applied mathematics oriented toward signal processing

[1]Ecole Nationale Supérieure d'Informatique et de Mathématiques Appliquées de Grenoble (Institut National Polytechnique de Grenoble)

[2]Institut Universitaire Professionnalisé de Mathématiques Appliquées et Industrielles (Université Joseph Fourier Grenoble I)

was established by Raoul Robert. His initiative in this subject, which was not his area of research, has played a decisive role, and the current explosion of numerical work based on wavelets shows that his vision was correct. Our thanks go equally to Pierre Baras for the numerous animated discussions we have had. Their ideas and comments have been a valuable aid and irreplaceable inspiration for us.

The second printing of this book is an opportunity to make several remarks. We have chosen not to include any new developments. We have listed at the end of the book several references on wavelets, which show that this area has exploded during these last years. But for the student or the teacher to whom we address the book, the path to follow remains the same, and the basics must be even more solidly established to understand these new areas of applications. It seems to us that our original objective continues to be appropriate today.

We have made the necessary corrections to the original text, and a book of exercises with solutions will soon be available to complete the project.

Claude Gasquet
Patrick Witomski
Grenoble, June 30, 1994

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